FOREWORD

The idea of publishing this book was born after realising the difficulty involved in establishing a sound library for the school. This is due to the fact that text books are too expensive to afford if every student is to have access to them.

The school administration and the teaching staff have therefore offered to subsidize the cost of text books in terms of service and materials to allow every student to afford these books.

I therefore thank the teachers so much for their unending endeavors in preparing these books with the aim of facilitating our students' learning.

Thank you indeed.

HEADMASTER
FOR ADMINISTRATION

MASTER KEY TO 'O' LEVEL PHYSICS
S.1 & S.2
EIGHTH EDITION
2017

PREFACE

Basic knowledge about our world is rapidly increasing as a result of the advancement in all scientific fields. This expansion results in the application of much of this knowledge in our modern society. Therefore, at no other time in our history has it been as important as it is today for students to understand *physical laws* or *principles* and their applications in daily life. It is the laws and concepts of physics that often help us to describe and give scientific explanation to the both naturally and artificially phenomenon that occur around us and deep in the space beyond our planet.

Secondary School is the starting point for students who select scientific careers in later life. The "*Master Key to Physics*" presents a modern and easy approach to the teaching of physical science at secondary level with specific emphasis on fundamental principles and concepts.

This book helps students to know the *what*, *why* and *how* of physics at secondary level. The contents of the book are based on students' common activities, interests, and experiences. All the concepts and principles dealt with in each unit relate to the central theme of the unit. They offer the student an opportunity to explore the field in depth, and to experience some of the excitement that comes with the discovery of new ideas.

Mathematical problems are one of the more serious difficulties encountered in a course in physics. They always tend to make the subject difficult to students. In this book, as each new kind of problem appears, a detailed example is given. Some of the examples begin with *Steps in solving problems*, which help to the student to develop sense of direction. The steps in these examples are explained into detail. The examples are immediately followed by Self-Check questions to confirm the understanding of the problem. The questions range from simple ones to more difficult ones. Throughout the book there are many simple experiments with plenty of guidance on the results of these experiments.

Every important definition or formula is highlighted or *italicized*.

In calculations, simple numbers have been used to keep the arithmetic as straight-forward as possible. There is orderly flow of information in solving example.

Finally, it is my sincere hope that the users of this will find this it useful and be forced to agree that indeed it is a "*Master Key*" to Physics which is widely believed to be "*unlockable*".

ACKNOWLEDGEMENT

I am grateful to all the people who helped either directly or indirectly in writing of this book. I would like to thank the administration of Light Academy for the encouragement and financial support towards the printing of the book.

I also wish to express my gratitude to my colligues Mr. Hakan Ulus and Mr. Enos for proof reading the book.

Finally, I would like to thank the director Mr. Ilihan Edogan for supporting the idea of writing the book to up lift the standard of the school.

DEDICATION

I dedicate this book to my late parents, Late Dega Nassur and Late Odaya Jenner, may the Almighty Allah rest their souls in eternal peace, my guardian Mr. Yasin Mustafa who has been the driving force behind my education, brothers, sisters and finally to friends who missed my company during the period of compiling the book. May Almighty Allah bless us all!!

TABLE OF CONTENTS

HOW TO USE THIS BOOK

The "**Master Key to Physics**" aims at giving you the opportunity to understand the basic principles of Physics and practice your Physics by giving detailed examples and self-check questions on topics that form the core of all 'O' level syllabuses. Each chapter begins with *chapter outline* and *learning objectives*. The learning objectives tell you about the most important ideas and areas in that chapter. These will help you to have the idea of what you should cover in any topic and prepare for the work on that chapter. And help you to remain focused when revising that chapter. And once you have an idea of what you should cover in any topic, you can start on the self-questions. There are two types of questions:

- Worked examples and Self-Check questions.

The examples and the questions are either from **UNEB** examination papers or are specially devised to fill in gaps and avoid unnecessary repetition. They are all of 'O' level standard, but some are of course easier than others. They are worked a little more detailed than those we would expect from a good candidate. This is to ensure clarity.

With each example you should gain confidence and improve your technique. Once you have studied the examples, try the self-check questions before checking the answers.

Note: The examples can be used in a variety of ways, but a suggested routine is:

- Read the question carefully. So many marks are lost by careless reading which leads to misfiring of the question.
- Without looking at the answer, have a go at answering it in note form with sketch diagrams where necessary. Consult the start of the chapter and your text book at this stage. Try the numerical problems.
- Consult the answer to find out if and where you have gone wrong and go through it very carefully. Follow any algebra through stage by stage. But **do not** just find your mistakes- learn from them.

A common question that an 'O' level student will ask is: Many people say that Mathematics is the mother of Science. What if my mathematics is **not** strong?

If you are having trouble with the mathematics, read through the working very carefully ask someone to explain for you the steps you do not understand. If you consult a mathematics text book remember that it may go into much more detail than you need. But do not worry too much since the examination board is aware of the problem and try to make sure that they are testing mainly your physics and not your mathematics. But of course you do not have to relax, rather work hard to develop your mathematics strongly.

Abbreviations/Symbols

Physics all over the world use a consistent set of symbols and abbreviations. You should be familiar with them. Most of the symbols used in physics are Roman and Greek letters. Some of which are: α *- Alpha,* β *- Beta,* γ *- gamma,* λ *- lambda,* μ *- mu,*
θ *- theta,* η *- eta,* ϕ *-phi,* ρ *- rho, and* π *- pi.*

Study Skills

What kind of learner Are you?

To get the most out of your studies, you should find out how you learn best – that is, what kind of learner you are.

Types of learners

There are three types of learners, namely:

- auditory learner
- visual learner
- kinesthetic learner

- An *auditory learner* learns by listening. If you are an auditory learner, you should find friends to study with, so you can ask questions and discuss the text. If you have to study alone, you can reinforce your learning by reciting the key concepts out loud.
- A *visual learner* learns by reading. If you learn best this way, you should concentrate on finding a quiet place where you can focus on material you are reading.
- A *kinesthetic learner* learns by doing. If you learn best by doing something, you have to be creative in the way in which you study. For example, highlight your text or take notes as you read. Think about practical ways to apply what you are reading about.

LEARNING SKILLS

In this final section, you are going to examine some specific suggestions on how you can make your study time more valuable. Not every method will be appropriate for your study habits. Choose what you think will help you, or adapt some to fit your needs. Ignore those that you feel won't contribute to your understanding of the materials.

SQ3R method

The SQ3R stands for: Survey, Question, Read, Recite, and Review

SQ3R was developed in1941 by Francis P. Robinson of Ohio State University. It's an old system, but it still works, Millions of students have successfully used this system, or a variation of it, to improve their reading and studying. SQ3R stands for Survey, Questions, Read, Recite, and Review. Let's take a look at each one of these elements in SQ3R.

Survey

You first survey the material you are going to study. The purpose of the survey step in SQ3R is to help you become familiar with your text book organization. To survey material you are about to read, look quickly at the following types of features:

- Tiles and other headings.
- Illustration, diagrams, charts, maps and graphs.
- Text printed in highlighted boxes.
- Boldface and italic type.
- Self-check questions.
- Summary, if appropriate.

Scanning these features will give you a good idea of what topics you are about to study.

Questions

The next step in the **SQ3R** method is question. This step requires you to go through the pages you are about to read and turn the headings into questions. Doing this helps direct your reading and your thinking. Then as you read and study the material, you can look for the answers to your questions. For example, look at the first few headings at the beginning of this book. Here is how you can turn them into questions:

- What is physics?
- How important is physics to us or why is physics taught in Secondary Schools?
- What are the meanings of the following words?
    - Hypothesis
    - Scientific law or Scientific Principle
    - Scientific theory

Note: *The better your questions are, the better will be your understanding of the material*.

The 1st R Read

Begin to read the material slowly and carefully, one section at a time. Don't worry about how long it takes. As you read, look for answers to the questions you have just formed under 'Q' above. Make brief notes, and look up any words you don't understand. If you have completed the first two steps (*Survey* and *questions*), the material should seem familiar to you. You are prepared to read the new material more efficiently. You have an idea of the information you are required to learn and you are able to read with clearer intent. You know why you are reading a section and what to focus on. Specifically, mark definitions, formulae, examples, names, principles, rules, and haracteristics.

The 2nd R Recite

Every time you come to a new heading in the text, stop and repeat, either silently or loud, the main point is to help you to remember what you have just read. Recite it from memory or refer to your marginal notes or the information you have highlighted. If you have trouble with this step, reread the section until it becomes clear to you. Reciting the material in your own words is a tremendous aid to learning .It makes it easier to retain the information.

The 3rd R Review

Review any material you read as soon as you can. Review it again before you complete a self-check and gain confidence before you prepare for an examination .This part of **SQ3R** helps to keep information fresh in your mind. I believe by now you should be able to agree that; the SQ3R method is real helping to make your revisions effective.

Note: (i) One way to review is to resurvey the material you have read. Or go over the notes you have made to see if they still make sense. Reread any passages and that you have underlined or highlighted.

(ii) Another method you can use in the review step is to go back over the questions you developed for each heading. See if you can answer them. If not, look back and refresh your memory about that particular topic. Then continue with your review until you are satisfied that you know the material well.

PREPARING FOR EXAMINATION

Revision

Literally, revision means ‘seeing again’ and it is a vital part of learning, though the word often gets used simply to mean preparation for an examination or a test. It is found out that the amount that can be recalled drops dramatically unless the new ideas are revised or reviewed. If one reviews his notes most often, it is possible to get the stage where nearly 100% recall is achievable long after the last review. It is also possible that one can forget every thing after **not** reviewing for a long period of time. The explanations to these facts lie in the changes that take place in the cells of the brain and in the connections between them.

If you follow the suggestions in this study unit, you should have no difficulty when reviewing in preparation for an examination. Use these tools, along with answers to the “self – checks,” to prepare for the examination. Don’t just “read” physics. Make your revision effective by involving the hands in going over the examples, drawing and labeling diagrams, trying to answer the questions on examination past papers. Familiarity with and understanding of the text material, diagrams, examples past paper questions will make *taking the examination* much easier.

TAKING EXAMINATIONS

Begin by surveying the examination questions. Read the directions/instructions carefully and be sure to follow them exactly. Do the easy questions first. Skip any you are unsure of. While taking the examination, maintain a positive attitude. If you feel negative thoughts creeping in, say to yourself, “I’ve studied hard.”

Types of questions

There are two types of questions:

- Multiple Choice
- Structured and semi-structure

Multiple Choice questions

Before you do anything on the question paper,

- First write your name and then read the instructions carefully.
- Read the questions very carefully. Careless reading of the instructions and the questions often leads to loss of marks. Carefully reading of the instructions and the questions help you to know exactly what you are to do and how to do it.

For multiple type questions, try to answer from your mind before you look at the answers.

- Read each answer thoroughly even if you think you have already spotted the correct one. Note that some answers may be correct. But the question always requires you to choose the most correct answer. Be aware the wrong answers are always plausible.
- Be active: sketch a diagram; write down a relationship, do anything that helps you think clearly.
- Do not be worried by too many of one letter since they are random.
- Be absolutely sure to get the easier questions right, by not rushing them. The questions carry the same marks.
- Finally if in doubt guess, illuminate the patently wrong answers first. If you can whittle the choice down to two you have a 50% chance of being right.

For Structured and semi-structured questions:

- Use simple straightforward English.
- Use clear-labeled diagrams.
- Where necessary leave space so you can add answers that occur to you later.
- Explain all your working and put in correct ***units***. Remember wrong units led to loss of marks.
- Round off your final answers to a sensible number of significant figures.
- Check your work as you continue.
- Look at the marks given besides the question to gauge how much detail is likely to be expected and, in the case of papers where you answer on the question paper, look at the space available. For example, six marks probably mean six important ideas are needed in the answer.

Terms used in setting examination questions

For getting high marks in structured and semi-structured questions, you have to understand the meanings of the following common terms used in setting the questions.

Define State precisely the meaning of terms or words. E.g. define the term density.

State: Name, mention or give the condition of a thing. E.g. state the S.I unit of pressure.

Describe: Give a detailed account or representation of something in words, e.g. describe an experiment to verify Hooke's law.

Identify: Recognize or prove something as being a certain. E.g. the factors that affect density of a substance.

Name: Write down, mention or state. E.g. name the parts labeled in a diagram.

Determine: Ascertain something after some observation, solve a problem of find out. E.g. determine the refractive index of a glass block.

Explain: Give a detailed account of something. E.g. explain what happens to a ball bearing released in a viscous fluid in a tall cylinder.

Discuss: Give points and general description in writing either for or against a statement. E.g. discuss the effect of temperature on density of water.

Suggest: Put forward an idea. E.g. suggest what would happen to pressure of a gas if the volume of the gas is reduced.

Calculate: Work out a problem mathematically. E.g. calculate the focal length of a converging lens.

Outline: Give the main points. E.g. outline the steps followed in determining the relative density of a substance.

Distinguish: Make, show or recognize differences. E.g. distinguish between density and relative density of a substance.

CHAPTER ONE

INTRODUCTION TO PHYSICS

LEARNING OBJECTIVES

By the end of this chapter, you should be able to:

1. Define: - *Physics,*
- *Matter and*
- *Energy.*

2. State: - *The aims and objectives of teaching physics.*

3. Define: - *Hypothesis,*
- *Scientific Theory and*
- *Scientific law or principle.*

4. State: - *the states of matter and give examples in each case.*

1.1 INTRODUCTION TO PHYSICS

At secondary level, science is divided into:
- Biological Science and
- Physical Science

Biological Science **-** *is the branch of Science which deals with the study of living things.*

Physical Science - *is a branch which deals with the study of non-living things.*

Physical Science is further divided into *Physics* and *Chemistry*.

Physics

Physics is a branch of physical science, which deals with the study of matter in relation to energy.

Matter - *is anything that occupies space and has mass.*

Energy - *is the ability to do work.*

In physics, the main topics studied include:
- Mechanics, Optics (Light), Waves, Electricity & Magnetism, and Modern Physics.

1.2 Aims and Objectives of teaching Physics

(a) Aims

(i) To make a society that knows about physics and enjoys the fruits of physics. E.g. Physics teachers, Engineers, Electricians, Electronic (T.V, Computer, Radio, Watch etc) technicians, medicine doctors.

(ii) To make a society that understands everyday phenomena (happenings) both naturally and artificially and their scientific explanations.

(iii) To produce a large number of people capable of harnessing natural resources e.g. minerals, scientifically and technically in an innovative way for the service of the society.

(iv) To produce an effective team workers for the advancement of knowledge.

(b) Objectives

(i) To make the student aware of the effects of scientific discoveries and knowledge on everyday life through the application of physics.

(ii) To help the student understand the world around us.

(iii) To enable the student to develop an experimental attitude of the mind by performing experiments and to equip him/her with the techniques of performing the experiments, making correct observations, recording the values and drawing appropriate conclusion from the experimental data.

(iv) To prepare the student to pass examinations in physics for further studies.

(v) To familiarise the student with the scientific Hypothesis, Theories, Principles and laws.

1.3 Hypothesis, Theory and Law (Principle)

(a) Hypothesis - *is a scientific idea put forward and is still in the stage of experimental investigation.*

However, if proved to be correct and therefore generally accepted, it then becomes a scientific law or principle.

(b) Scientific Law or Principle

A Scientific Law or Principle is a generalised statement of observed facts.

(c) Scientific Theory:

A Scientific Theory is an idea put forward to explain the existence of a scientific laws or principle.

1.4 Characteristics of Materials

All the materials/substances that exist in the universe either naturally or artificially are scientifically called *matter*. Every matter has some characteristic properties which make it different from others.

Examples of such properties are: State, Colour, Density, Smell, etc.

These properties are called the *physical properties* of matter. Using a simple classification, based on the physical properties of matter, matter can be classified into several classes. However, it is convenient to first classify matter according to the *physical states* and then further classification to be considered after words.

States (Phases) of Matter

Matter exists in three states namely:

(i) Solid, e.g. stone, wood etc.

(ii) Liquid e.g. water, paraffin etc.

(iii) Gas e.g. oxygen, nitrogen etc.

Self-Check 1.0

1. (a) Define the following terms.

    (i) Physics (ii) Matter (iii) Energy

    (b) State any **three** aims and **three** objectives of teaching physics in secondary schools.

2. Define the following terms

    (a) Hypothesis (b) Scientific law or scientific principle

    (c) Scientific theory

3. State the states of matter. In each case, give ***two*** examples.

CHAPTER TWO
MEASUREMENT

LEARNING OBJECTIVES

By the end of this chapter, you should be able to:

1. *Explain how to use:*
- *the meter rule, the Vernier callipers and the micrometer screw gauge for measuring length,*
- *beam balance, spring balance and electric balance for measuring mass and*
- *Stop watches, watches for measuring time.*

2. *Recognise and state:*
- *the S.I units/other units for the three fundamental quantities.*

3. *Convert other metric units to S.I unit and vice versa.*

4. *Express numbers in scientific notation or standard order.*

5*.* *Use the formula for calculating: Area and Volume.*

2.1 Measurement: **-**is essential in physics. Before any measurement is taken, the *quantity* to be measured and its *unit* must be specified.

(a) Physical Quantities of Matter

These are the measurable properties of matter E.g. length, mass, speed etc. They are expressed in terms of a numerical *value* and a *unit*. There are so many quantities that can be measured in physics. However, some of the quantities are defined by using basic quantities.

(b) Basic Quantities

These are quantities of matter that are used to define other quantities of matter. The basic quantities of matter are:
- Length,
- Mass and
- Time.

Since these quantities are used to define other units, they are referred to as *fundamental quantities*.

(c) S.I system of Units

The S.I system of units is an International System of units based on the MKS (metre-kilogram-second) system. It is abbreviated as **SI** in all languages and is derived from French Le Système International d'Unités.

The basic quantities and their S.I units are shown in the table 2.1 below.

Basic quantity	Name of Unit	Symbol of SI Unit
Length	Metre	m
Mass	Kilogram	kg
Time	Second	s

Table 2.1

NB: When writing the names or symbols of the units, note that:
- Symbols of units have **no** plural forms.

e.g. we write: 2 kg **not** 2 kgs
5 m **not** 5 ms

(d) Prefixes

A prefix is a word or letter placed before another. Examples of prefixes are: micro (μ), milli (m), centi (c), deci (d), kilo (k), mega (M), giga (G) and tera (T).

Prefixes can be used with the units for measuring quantities.

E.g.	*kilo* with *gram*	forming	*kilogram* (kg),
	kilo with *meter*	forming	*kilometer* (km),
	milli with *meter*	forming	*millimeter* (mm),
	kilo with byte	forming	*kilobyte (kB),*
	mega with *byte*	forming	*megabyte* (MB),
	giga with byte	forming	*gigabyte* (GB) etc.

(e) Multiples and Sub-multiples

Multiples and sub-multiples are shown in tables 2.2 and table 2.3 below.

Multiples	Prefix	Symbol
10^3	Kilo	k
10^6	Mega	M
10^9	Giga	G
10^{12}	Tera	T

Table 2.2 Multiples

Sub-multiples	Prefix	Symbol
10^{-3}	milli	*m*
10^{-6}	micro	μ
10^{-9}	nano	*n*
10^{-12}	pico	*p*

Table 2. 3 Sub-multiples

2.11 Instruments used for measurement

The devices used for measuring quantities are called *instruments*. Those used to measure the basic quantities are given in the table 2.4 below.

Basic quantity	Instruments
Length	- Metre rule, Tape measure, Vernier calipers and Micrometer screw gauge
Mass	- Electronic balance, Beam balance, Spring balance and triple balance.
Time	- Stop watch and Watch

Table 2.4 Instruments for measuring the basic quantities of matter

2.12 Measurement of Length

Length is measured by using the *Tape measure*, *Metre rule*, *Vernier calipers* and the *Micrometer screw gauge*.

The choice of the instrument to be used depends on:

- The size of the distance to be measured (how long and short the distance is) and
- The accuracy to which the measurement is needed.

(i) Measurement of Long distance

Lang distances are measured using, the *metre rule* and the *tape measure*. Both instruments are calibrated (graduated) in metres (m), centimeters (cm) and millimeters (mm).

1 m = 100 cm and 1 cm = 10 mm

∴ 1 m = 100 x 10 = 1 000 mm

The larger unit is kilometer (km). **1 km = 1 000 m**

Note: *Due to wear at the edges, when using the metre rule, it is advisable to start measuring from the 10 cm mark and deducting 10 cm from the total measurement.*

(ii) Measurement of Small and very small distances

For *small distances*, the Vernier calipers or the engineers calipers is used. While for *very small distances* such as the diameter of copper wire, thickness of paper, the micrometer screw gauge is used. Both the Vernier calipers and the micrometer screw gauge give readings with reasonable accuracy.

(a) The Vernier Caliper

A Caliper is a mechanical device used to determine small lengths with reasonable accuracy.

Types of Vernier Callipers

There are two types of Vernier Callipers

(i) Simple caliper (commonly called Engineer's caliper) and

(ii) Complex vernier

(i) Simple caliper

Simple calipers have two movable parts/legs of some desired shape and length to meet the surfaces whose separation is to be measured. The adjusted width between the leg tips is then placed against some length scale. E.g metre rule and the reading taken.

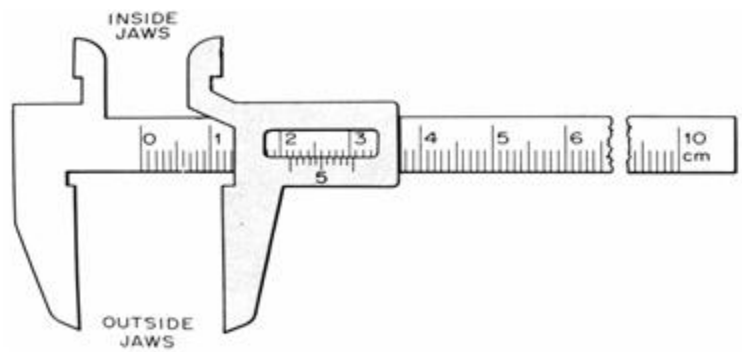
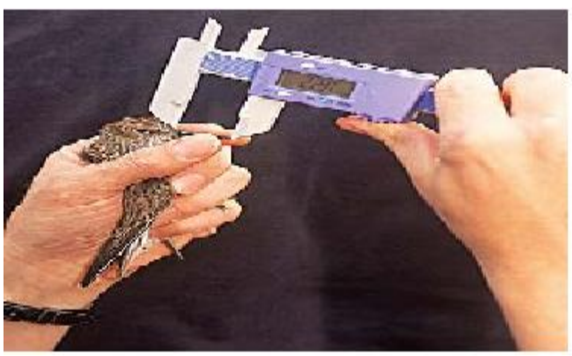


Figure 2.1 Engineer's caliper *Figure 2.2 The mechanical Vernier calipers*

For measuring the internal diameter of a pipe, the caliper is turned the other way round. This way, the tips of the jaws point outward.

(ii) Complex vernier



The more complex vernier caliper has two types of scale that allow direct reading of the adjusted width between the jaws. The types of the complex vernier are mechanical and digital. Figure 2.2 below shows the mechanical vernier.

Figure 2.3 Showing a digital vernier being used to measure length of a bird

How to use the Vernier calipers
The movable jaw is adjusted until it grips the object to be measured and then the reading is taken as described below.

How to read the Vernier

(i) ***The Digital Vernier calipers***

Once the vernier is adjusted to the width of the object to be measured, a reading is taken directly from a small screen engraved on it.
E.g. in figure 2.3, the length of the bird's bick is 2.20 cm.

(ii) ***The Mechanical Vernie***

For the mechanical vernier, the reading is taken in three steps:

Step I Read and record the main scale reading at the zero mark of the vernier scale to an accuracy of one millimeter.
E.g. 2.1 cm.

Step II Read and record the vernier scale reading at the position on the vernier where a mark on it is coincident (i.e. coincides) with a mark (division) on the main scale in ***<u>tenths</u>*** of millimeters.
E.g. let the 6^{th} vernier division coincide with a mark on the main scale. In tenths, 6 becomes $\frac{6}{10}$ = 0.6 mm = 0.06 cm

Step III Get the sum of the two readings (i.e add the main scale reading and the vernier scale reading to get the total reading).

Main scale reading	=	2.10 cm
Vernier scale reading	=	+ 0.06 cm
Total reading	=	**2.16 cm**

Example 1
The S.1 students of **TLA** measured the thickness of a desk top during physics lesson and found that the main scale reading before the zero mark of the vernier scale to be 4.4 cm. Find the thickness of the desk top if the 4^{th} vernier mark coincides with one of the marks on the main scale.

Solution

Main scale reading	=	4.40 cm
Vernier scale reading $4^{th} = \frac{4}{100}$	=	0.04 cm
Total reading	=	**4.44 cm**

Note: *Instead of dividing the vernier reading by 10 to get the answer in mm and then by 10 to change to cm, we can divide the value directly by 100 to get the answer once in cm.*

Example 2
Find the readings of the verniers shown figures 2.4 (i) and (ii) below.

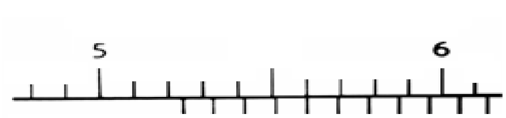


Fig. 2.4 (i)

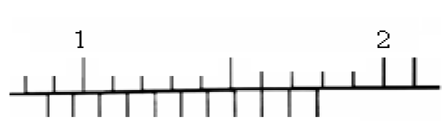


Figure 2.4 (ii)

Solution: (i) Main scale reading $= 5.20$ cm

Vernier scale reading $4^{th} = \frac{4}{100}$ $= + 0.04$ cm

Total reading **= 5.24 cm**

(ii) Main scale reading $= 0.80$ cm

Vernier scale reading $8^{th} = \frac{8}{100}$ $= + 0.08$ cm

Total reading **= 0.88 cm**

(b) The Micrometer screw gauge

The Micrometer screw gauge is an instrument used to measure very small distances.
The Micrometer screw gauge is calibrated in mm on the sleeve and some small divisions on the thimble scale.
There are two types of thimble readings:

(i) One with 50 divisions on the thimble scale and

(ii) The other with 100 divisions on the thimble scale.

However, the two types give the same reading when used to measure the same distance.
The diagram of the micrometer screw gauge is shown in the figure 2.5 below.

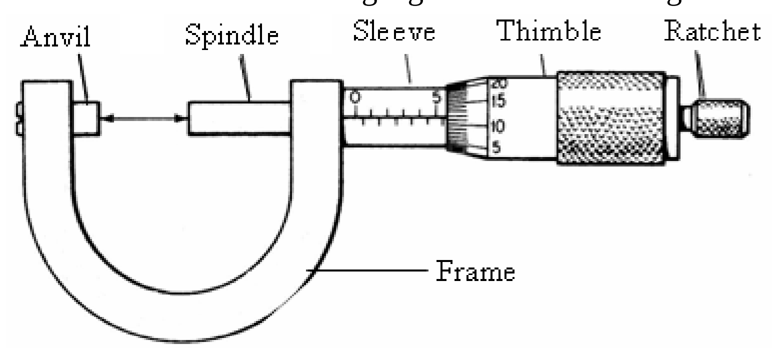


Figure 2.5 Micrometer screw gauges

How to use the Micrometer screw gauge

- Place the object whose thickness is to be measured in between the jaws (the anvil and the spindle) of the micrometer.
- Rotate the ratchet clockwise until the jaws touch the object. As soon as the object is gripped tight enough, it starts to slip, making a characteristic sound.
- Take the reading in three steps as shown below.

Step I: Read and record the reading on the sleeve scale at the edge of the thimble in millimeters and half millimeters. E.g. 4.0 mm.

Step II: Read and record the reading on the thimble scale opposite to the centerline on the sleeve scale (i.e. where a division on the thimble scale coincides with the centerline on the sleeve scale) in ***hundredths*** of millimeters.

E.g. Let the 33^{rd} division coincide with the centerline.

In hundredths, 33 becomes $\frac{33}{100} = 0.33$ mm

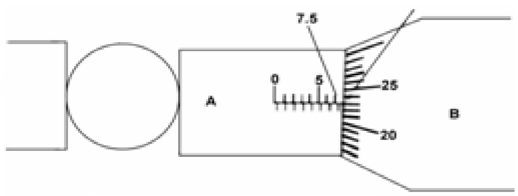
Step III Get the sum of the two readings i.e. sleeve scale reading and the thimble scale reading.

Sleeve scale reading	=	4.00 mm
Thimble scale reading	=	+ 0.33 mm
Total reading	=	**4.33 mm**

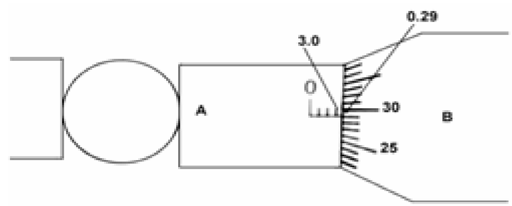
Note: *If the answer is required in cm or m, you convert it as required.*

Example 1

Find the reading on the micrometer screw gauge shown in the diagrams below.



(a) Figure 2.6



(b) Figure 2.7

Solution

(a)

Sleeve scale reading	=	3.00 mm
Thimble scale reading, $\frac{29}{100}$	=	+ 0.29 mm
Total reading	=	**3.29 mm**

(b)

Sleeve scale reading	=	7.50 mm
Thimble scale reading, $\frac{23}{100}$	=	+ 0.23 mm
Total reading	=	**7.73 mm**

SELF-CHECK 2.0

1. (a) Draw and label the diagram of the Vernier calipers.

   (b) Find the thickness of a text book measured using a vernier caliper if the main scale reading is 24 mm and the 8th vernier mark coincides with one of the marks on the main scale.

   (c) Find the reading on the verniers shown in the diagrams below.

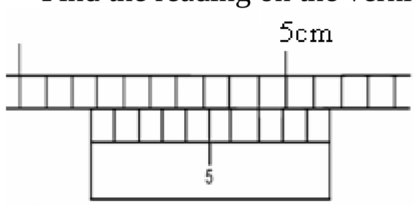


Figure 2.9 (i)

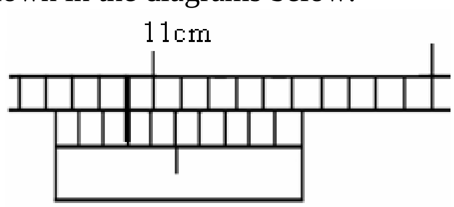


Figure 2.9 (ii)

2. (a) Draw and label the diagram of the micrometer screw gauge.

(b) Find the reading on the micrometer screw gauge shown in figures 2.10 below.

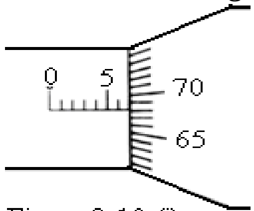


Figure 2.10 (i)

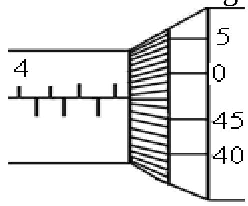


Figure 2.7 (ii)

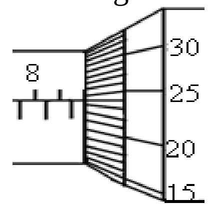


Figure 2.8 (iii)

2.13 Measurement of Area (Two dimensions)

Regular Surfaces

The area of regular surfaces is found by measuring any two of the following dimensions and then applying the appropriate formula.

- Length (*l*), width (*w*), height (*h*), side (*s*), radius (*r*) and diameter

Units of area

The SI unit for area is *square metre* (m^2).

Other units are: - mm^2, cm^2, km^2 and hector.

$1\ m^2 = 100 \times 100 = 10{,}000\ cm^2$

$1\ cm^2 = 10 \times 10 = 100\ m\ m^2$

$1\ m^2 = 1000 \times 1000 = 1{,}000{,}000\ mm^2$

Table 2.5 below shows the common regular surfaces and their respective formulae.

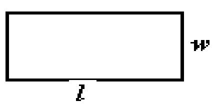
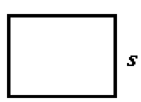
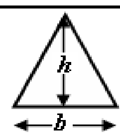
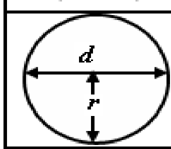
Figure	Name	Dimensions	Formula
	Rectangle	*l and w*	$A = lw$
	Square	*s*	$A = s^2$
	Triangle	*b and h*	$A = 1/2bh$
	Circle	*r and d*	$A = \pi r^2$

Fig. 2.5

(c) Measurement of Volume (Three dimensions)

(i) Regular Solids

The volume of regular solids is determined by measuring the dimensions and then applying an appropriate formula.

Units of Volume

The SI unit of volume is metre cubed, (m^3).

Other units are: mm^3, cm^3.

$1\ m^3 = 1\ 000\ 000\ cm^3$

$1\ cm^3 = 0.000\ 001\ m^3$

The table below shows the common regular solids and their respective formulae.

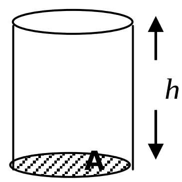
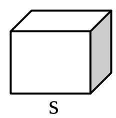
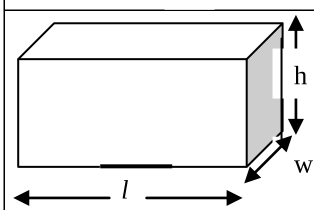
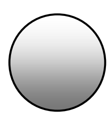
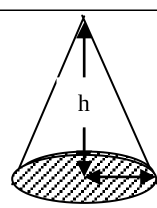
Figure	**Name**	**Dimensions**	**Formula**
	Cylinder	*r, or d & h*	$V = Ah$ or $V = \pi r^2 h$ or $V = \frac{1}{4}\pi d^2 h$
	Cube	*s*	$V = s3$
	Cuboid	*l,w,h*	$V = lwh$
	Sphere	*r or d*	$V = \frac{4}{3}\pi r^3$ or $V = \frac{1}{6}\pi d^3$
	Cone	*r & h*	$V = \frac{1}{3}\pi r^2 h$

Table 2.6

(ii) Irregular Solids

The volume of irregular solids is determined by using displacement method. In this method, the solid is fully or wholly immersed in a liquid and the volume of the liquid displaced is measured. This method operates on the principle that "*A body fully or wholly immersed in a fluid (liquid) displaces its own volume*".

Apparatus/requirements used to measure the volume of irregular solid are:

(i) Measuring cylinder, water and a piece of thin silk thread.

(ii) Measuring cylinder, overflow (displacement) can, water and a piece of silk thread.

Measuring the volume of irregular solid

1. Using a measuring cylinder

- Fill a measuring cylinder with water.
- Read and record the initial reading.
- Tie the irregular object with a piece of thin silk thread and lower it carefully into the water in the cylinder until it is fully immersed.
- Shake it gently to remove any air bubbles.
- Read and record the final reading.

Diagram showing measurement of volume of irregular object

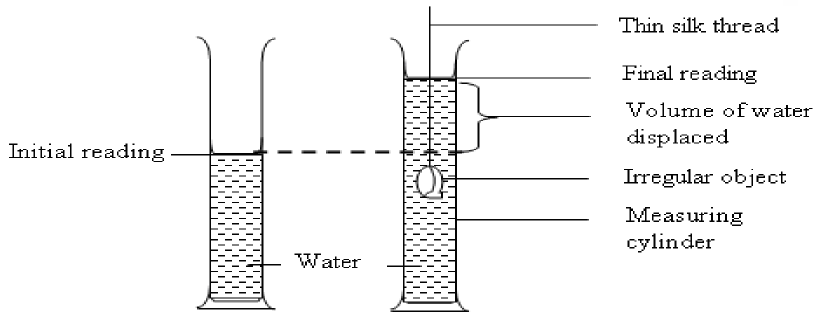


Figure 2.11 (ii)

Before immersing After immersing

The volume is calculated from the formula:

Volume of irregular object = Final reading - Initial reading

2. Using an Overflow can and measuring cylinder

- Fill an overflow can with water until water flows out through the spout and wait until the water ceases dripping and then place a dry measuring cylinder below the spout.
- Tie the irregular object with a piece of thin silk thread and lower it carefully into the water in the overflow can until it is fully immersed.
- Shake it gently to remove any air bubbles.
- Wait until the water ceases dripping into the measuring cylinder.
- Read and record the volume of the displaced water in the cylinder.

Diagram showing measurement of volume of irregular object

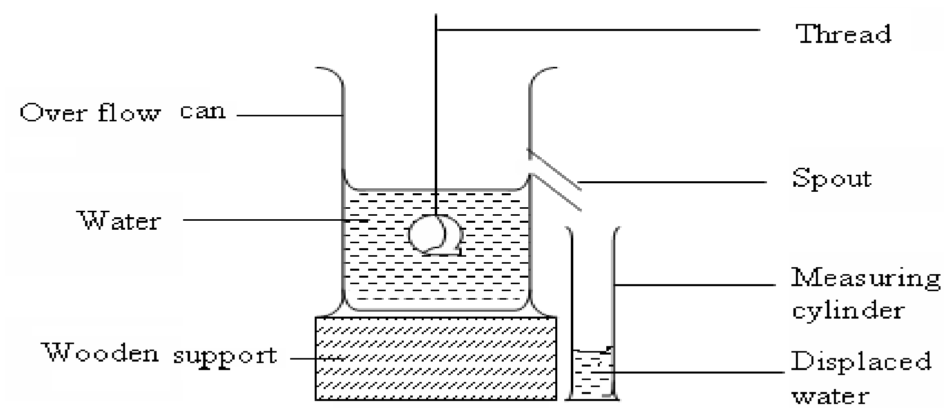


Figure 2.10

Volume of object = Volume of displaced water

(iii) Measurement of volume of Liquids

The volume of a given liquid is determined by using a measuring cylinder.

The liquid is carefully poured into the measuring cylinder and the volume is read off by placing the eye level in line with the bottom of the meniscus of the liquid surface in order to avoid error due to parallax.

Diagram showing correct reading of measuring cylinder

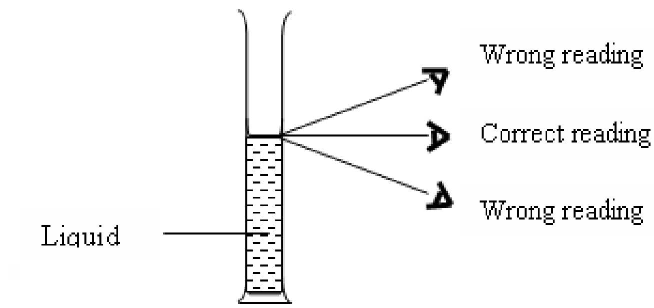


Figure 2.13

Specified volumes of liquids are accurately measured by using specific instruments such as: - Burette, Pipette and Syringe.

Note: *The first two are commonly used in Volumetric analysis in Chemistry practical*.

(iv) Gases

The volume of gases is measured by using a syringe. The gas syringe is connected to the gas supply with a help of rubber tubing. Due to the gas pressure the piston moves backwards and the required volume is noted from the scale.

Unit for measuring Volume

The SI unit for volume is metre cubed (m^3)

$$1\ m^3 = 1\ 000\ 000\ cm^3$$

$$1\ dm^3 = 1\ 000\ cm^3$$

$$1\ \text{litre} = 1\ dm^3$$

$$\therefore \quad 1\ \text{litre} = 1\ 000\ cm^3$$

2.2 Measurement of Mass

Mass is a measure of the amount of matter in an object. Or it is the quantity of matter in a body. It is constant everywhere. That is, it does not vary from place.

The mass of an object is commonly measured by comparing it with a standard (known) mass.

The instruments used to measure mass are:

- Beam balance,
- Triple balance,
- Lever arm,
- Electric balance,
- Chemical balance and
- Spring balance.

The unit for measuring mass

The SI unit for mass is the kilogram (kg).

Other units include: milligrams (mg), grams (g) and tonne (ton).

1 kg = 1 000 g

1 g = 1 000 mg

1 ton = 1 000 kg

2.3 Measurement of Time

Time is measured using: watches and stopwatches (both mechanical and digital).

Due to accuracy and easy in reading, the digital stopwatches are preferred. They measure to 0.01 second.

Units for measuring time

The SI unit for time is *second* (*s*).

Other metric units include:

- Minute (*min*), hour (*hr*), day, week, month, year, decade, century, millennium.

1 hr = 60 min

1 min = 60 s

1 hr = 60 x 60 = 3 600 s

$\therefore$ **1 hr = 3 600 s**

Worked Examples

Conversion from one unit to another

1. Convert the following as required.

(a)	(i)	5 000 m to km.	(ii)	60 m to cm
(b)	(i)	2 kg to gram.	(ii)	500 g to kg.
(c)	(i)	24 hr to seconds.	(ii)	30minutes to seconds.
	(iii)	1800 s to hours.	(iv)	900 s to minutes.

Solution

(a) (i)

1 km = 1000 m

y = 5000 m

1000 m x y = 1 km x 5000 m

$$y = \frac{1km \ x \ 5000 \ m}{1000 \ m}$$

$\therefore$ **y = 5 km**

(ii)

1 m = 100 cm

60 m = r cm

1 m x r = 60 m x 100 cm

$$r = \frac{60 \ m \ x \ 100 \ cm}{1m}$$

$\therefore$ **r = 6000 cm**

(b) (i)

1 kg = 1000 g

2 kg = t

1 kg x t = 2 kg x 1000 g

$$t = \frac{2 \ kg \ x \ 1000 \ g}{1 \ kg}$$

$\therefore$ **t = 2000 g**

(ii)

1 kg = 1000 g

z = 500 g

1000 g x z = 1 kg x 500 g

$$z = \frac{1kg \ x \ 500 \ g}{1000 \ g}$$

$\therefore$ **z = 0.5 Kg**

(c) **(i)**

$$1 \text{ hr} = (60 \text{ x } 60) \text{ s}$$
$$24 \text{ hr} = t$$
$$1 \text{ hr x } t = 24 \text{ hr x } (60 \text{ x } 60) \text{ s}$$
$$t = \frac{24\,hr \text{ x } (60 \text{ x } 60\,s)}{1\,hr}$$
$$\therefore \mathbf{t = 86\ 400\ s}$$

(ii)

$$1 \text{ min} = 60 \text{ s}$$
$$30 \text{ min} = v$$
$$1 \text{ min x } v = 30 \text{ min x } 60 \text{ s}$$
$$v = \frac{30 \text{ min} \text{ x } 60\,s}{1 \text{ min}}$$
$$\therefore \mathbf{v = 1\ 800\ s}$$

(iii)

$$1 \text{ hr} = 3600 \text{ s}$$
$$h = 1800 \text{ s}$$
$$3600 \text{ s x } h = 1 \text{ hr x } 1800 \text{ s}$$
$$h = \frac{1\,hr \text{ x } 1800\,s}{3600\,s}$$
$$\therefore \mathbf{h = \tfrac{1}{2}\ hr}$$

(iv)

$$1 \text{ min} = 60 \text{ s}$$
$$q = 900 \text{ s}$$
$$60 \text{ s x } q = 1 \text{ min x } 900 \text{ s}$$
$$q = \frac{1 \text{ min} \text{ x } 900\,s}{60\,s}$$
$$\therefore \mathbf{q = 15\ min}$$

Self-Check 2.1

1. Convert the following as required.

(a)	(i)	10, 000 m to km.	(ii)	20 m to cm
(b)	(i)	25 kg to gram.	(ii)	2000 g to kg.
(c)	(i)	12 hr to seconds.	(ii)	¼ hr to seconds.
(d)	(i)	20 000 cm^3 to m^3	(ii)	50 m^3 to cm^3

2.4 Significant Figures

Measurements are always given correct to a certain number of significant figures.
To determine the number of significant figures, in a measurement the following rules may be useful.

(i) All non-zero digits (1, 2, 3, 4, 5, 6, 7, 8, 9) are significant.
E.g. 5.23cm has 3 significant figures (3 sgf)

(ii) All zeros between non-zero digits are significant.
E.g. 4.002 has 4s.g.f.

(iii) All zeros to the right of a decimal point and following a non-zero digit are significant.
E.g. 62.00 has 4 s.g.f.

(iv) A zero before a decimal point and zero(s) after a decimal point but before a non-zero digit are not significant.
E.g. 0.2, 0.005, 0.000004 all have 1 s.g.f.

NB: *When handling calculations, the final answer should always be given with only as many significant figures as that number involved in the calculation which has the least number of significant figures.*

Example:

A rectangular block of wood measures 5.24 cm by 3.64 cm by 0.63 cm respectively. Calculate the volume of the block of wood.

Solution: l = 5.24 cm, w = 3.64 cm, h = 0.63 cm.

Applying Volume $= lwh$

$= 5.24 \times 3.64 \times 0.63$

$= 12.016368$

$\mathbf{= 12\ cm^3}$

***Explanation**:* *Since the measurent with the least number of sgf is 0.63, which is 2 sgf, the final answer must have 2 sgf.*

Self-Check 2.2

1. How many significant figures are there in each of the following numbers:

   (a) 4.02 **(b)** 0.008 **(c)** 8 600 **(d)** 1 049 **(e)** 0.0002

2. The following values were taken as part of a set of experimental data: 25.57 cm and 8.48 mm. Find the sum of the two figures.

3. A water tank measures by 4.5 m by 3.25 m by 5.5 m. Calculate giving the answers with the correct number of sgf.

   (a) Base area **(b)** Volume of the tank.

2.5 SCIENTIFIC NOTATION (Exponential or Standard Notation)

Scientific Notation **is** *a short way of writing or expressing very large or very small numbers using powers of 10.*

The number is expressed in the form: $M \times 10^n$

Where: M - is a positive number ($1 \leq M < 10$.

n - is $\pm$ integer (number)

(a) Determination of the arithmetic sign of *n*

The arithmetic sign of n is determined by using any one of the following simple rule.

(i) The direction of the movement of the decimal point.

(ii) The value of the number given.

Rule:

(i) When the decimal point is moved from right to left, the power of 10 is positive and when moved from left to right, the power of 10 is negative.

(ii) When the value of the number given is greater than 10, the power of 10 is positive and negative when the number is less than 1 (i.e. when the number begins with 0.…).

(iii) When the value of the number given is 1 or greater than 1 but less than 10, the power of 10 is zero.

Examples

Express the following numbers in exponential notation.

(i) 25000 **(ii)** 0.000024 **(iii)** 250 x 4 **(iv)** ¼ x 1200

Solution

(i) 25 000

In (i), the decimal point is not shown but its position is understood. For such numbers, the position of the decimal number is always after the last digit. For this case the decimal point is after the last zero. We move the decimal point from that position and put it in between the first two digits in order to make it less than 10 but greater than 1.

To get the power of 10, we count the number of digits between the original position of the decimal point and the new position. Then the number becomes **= 2.5 x 10^4**

(ii) 0.000024

In (ii), the position of the decimal point is clearly seen. We move it from left to right and put it between the first two non-zero digits in order to make it less than 10 but greater than 1.

To get the power of 10, we again count the number of digits between the original position of the decimal point and the new position. Then the number becomes **= 2.4 x 10^{-5}**

For the case of (iii) and (iv), we first work out the answers and then use the same procedure to express the answers.

(iii) 250 x 4 = 1 000
= 1 x 10^3

(iv) ¼ x 1200 = 3 000
= 3 x 10^3

Try (iii) and (iv)? ***Answers*** (i) 4200 (iii) 0.085

(b) Working out numbers with powers of 10

In working out numbers expressed using powers of ten, we use the following rules.
(i) *When multiplying powers of ten, add the exponents together.*
(ii) *When dividing powers of ten subtract the exponents.*

Examples

Work out the following numbers

(a) $10^4 \times 10^8$ **(b)** $10^6 \times 10^{-3}$ **(c)** $\frac{6\ x10^6}{2\ x10^3}$ **(d)** $\frac{1.5\ x10^5}{2\ x10^{-2}}$

Solution:

(a) $10^4 \times 10^8 = 10^{(4+8)}$
$= \mathbf{10^{12}}$

(b) $10^6 \times 10^{-3} = 10^{(6+-3)}$
$= \mathbf{10^{3}}$

(c) $\frac{6\ x10^6}{2\ x10^3} = \frac{6\ x10^{(6-3)}}{2}$
$= \mathbf{3\ x10^3}$

(d) $\frac{1.5\ x10^5}{2\ x10^{-2}} = \frac{1.5\ x10^{(5-{}^-2)}}{2}$
$= 0.75 \times 10^{(5+2)}$
$= 7.5 \times 10^{-1} \times 10^7$
$= 7.5 \times 10^{(-1+7)}$
$= \mathbf{7.5 \times 10^6}$

Self-Check 2.3

Write the following numbers in scientific notation.

(a) (i) 0.000222 (ii) 0.0025 (b) (i) 5620 (ii) 75000

(c) (i) 120 million (ii) 20 (d) (i) 5 (ii) 55 x 20

(e) (i) 25 km in m (ii) 10 g in kg.

Self-Check 2.4

1. Which of the following is a fundamental quantity?
   A. Time B. Density C. Volume D. Area

2. What does a beam balance measure?
   A. Area B. Mass C. Length D. Density

3. Which one of the following is not a method of science?
   A. Measurement B. Observation
   C. Experimentation D. Presentation

4. Which one of the following are not matter?
   I. Steam II. Pencil III. Light IV. Space
   A. I and II B. II and III C. III and IV D. I and IV.

5. How many cubic centimeters are there in a litre?
   A. 500 B. 100 C. 2000 D. 1000

6. The sides of a black board are 2m and 5m, What is the surface area in m^2 ?
   A. 0.1 B. 10 C. 1 D. 7

7. How many mm^2 are in 0.032 dm^2.
   A. 3.2 B. 320 C. 32 D. 0.0032

8. Find the correct expression.
   A. Litre is a unit of length.
   B. 1 A is equal to 1 ten-thousandth of a micron.
   C. A day is equal to one complete rotation of the earth.
   D. A graduated cylinder is used to measure volume.

9. What is equivalent to 5 minutes?
   A. 30s B. 60s C. 120s D. 300s

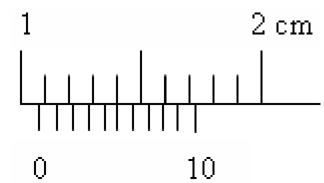
10. How many minutes are there between 05: 30 and 21:15?
    A. 715 B. 945 C. 1595 D. 900

11. The width of a meter rule is accurately measured by a
    A. micrometer screw gauge B. vernier caliper
    C. tape measure D. meter rule

12. A set of apparatus that is suitable for measurement of the volume of an irregular object includes;
A. Over flow can, measuring cylinder, irregular object and a string.
B. Measuring cylinder, irregular object, over flow cans, flask
C. Overflow can,. Irregular objects, string, retort sand and burette
D. Burette, overflows can, irregular object, a string, measuring cylinder, and retort stand.

13. Convert 25cm^3 into m^3
A. 02.5×10^5 B. 2.5×10^2 C. 2.5×10^{-1} D. 2.5×10^{-5}

14. The figure shows vernier calipers. The diameter of the object is



A. 1.05 cm B. 1.06 cm C. 1.56 cm D. 1.60 cm

15. Three of the fundamental physical quantities are:
A. Density, mass and time
B. Length, time and mass
C. Length, time and weight
D. Volume, density and mass

That was a good revision test. Do not be unduly worried if you made a slip or two in your working. Try to avoid doing so, of course, but you are doing fine.

Now on to the next Chapter.

CHAPTER THREE

DENSITY AND RELATIVE DENSITY

LEARNING OBJECTIVES

By the end of this chapter, you should be able to:

1. *Define: - Density and state its S.I unit and the other unit.*
2. *Determine experimentally the densities of:*
   - *Regular and Irregular Solids,*
   - *Liquids and*
   - *Gases/Air.*
3. *Solve numerical problems on density and density of mixtures.*
4. *Define: - Relative Density (R.D)*
   - *Determine experimentally the R.D of;*
     - *Solids and*
     - *Liquids.*
5. *Solve numerical problems on relative density of solids and liquids.*

3.0 Density

Definition**:** *Density is defined as mass per unit volume of a substance.*

Mathematically, it is expressed as: $$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\boldsymbol{\rho} = \frac{\text{m}}{\text{v}}$$

(a) S.I Unit

The SI unit of density is kg/m^3 (kgm^{-3}). It is a derived unit. I.e. a unit derived from the units of the quantities in the formula of density.

(b) Derivation of the unit

From Density $= \dfrac{\text{m}\,(\text{kg})}{\text{v}\,(\text{m}^3)}$ we have;

$$= \frac{kg}{m^3}$$

Conveniently expressed as $= kg/m^3$ **or** kgm^{-3}

The smaller unit of density is g/cm^3 (g cm^{-3}).

Note*: The density of a substance is subject to variation depending on the prevailing physical factors.*

3.1 (a) Factors that affect density

There are two physical factors that affect the density of a substance. These are:

- *Temperature* and *Pressure*.

(i) Effect of Temperature on density of substances

Substances expand and contract when their temperatures changes. The expansion and the contraction cause increase and decrease in the volume. Since, density is the ratio of mass to volume of a substance and then there will be change in the value of the density.

Effect of High temperature

At high temperature, a substance absorbs heat energy and expands and the volume is increased. Since the mass remains constant, the value of the density (i.e. the ratio of mass to volume) becomes low i.e. the density becomes low.

Effect of low temperature

At low temperature, a substance loses heat energy and contracts. The volume decreases. Dividing the constant mass by the reduced volume gives a higher density than the density under normal conditions.

(ii) Effect of Pressure on Density of substances

Pressure as a determining factor of density mostly affects the density of gases.

High pressure

High pressure squeezes the gas molecules into a smaller volume. If a given mass of a gas is contained within a smaller volume, the ratio of mass to volume results to a higher density.

Low pressure

At low pressure, the gas molecules occupy a larger volume and therefore, the density becomes low.

Note: *As a result of the above factors, when stating densities of gases, the temperature and pressure values **must** be stated as well.*

The table 3.1 below shows the densities of some common substances/ materials.

Substance	Density in kg m^{-3}
(a) Solids: - Ice	920 (9×10^2)
- Aluminium	2,700 (2.7×10^3)
- Copper	8,900 (8.9×10^3)
- Lead	11,300 (1.13×10^4)
(b) Liquids: - Water	1,000 (1×10^3)
- Paraffin, gasoline	800 (8×10^2)
- Mercury	13,600 (1.36×10^4)
Gases (at *stp) - Air	1.30
- Hydrogen	0.09
- Oxygen	1.43
- Carbon dioxide	1.98

Table 3.1

Note: *(i) The above density values are **not** be memorised except for water.*

(ii) ***stp** *- stands for Standard Temperature and Pressure.*

(b) Uses of Density

Density is used to:

(i) Identify materials.
(ii) Determine the purity of a material.
(iii) Choose light gases for filling meteorological balloons.

(c) Importance of density

The densities of materials are important to architectures and engineers in the design of structures. For example, aircrafts and overhead cables for the transmission of electricity are made of aluminium alloy. This is because aluminium has low density (i.e. is light) and is quite strong.

Worked Examples

Calculating density

Steps in problem solving

Before solving any problem, ask your self the following questions.

- *What is asked in the question?*
- *What information is given to help solve the problem?*
- *What are the equation(s) to solve the problem?*
- *Are units of the quantities given matching?*

These questions can only be answered when you collect the data.

1. A glass stopper has a volume of 16 cm^3 and a mass of 40 g. Calculate the density of the glass stopper in: (i) g/cm^3 (ii) kg/m^3

Solution

Data: $m = 40$ g, $v = 16\ cm^3$, $\rho = ?$

(i) Density $= \frac{m}{v} = \frac{40}{16}$ **= 2.5 g/cm³**

Note: For (ii), the units of mass and volume in the data are small units. But you are required to get the answer in kg/m^3. This means that the mass must be in kg and the volume in m^3. So first convert the mass from gram to kg and volume from cm^{-3} to m^3.

Converting the units: (ii) Mass: $40g = \frac{40}{1000} = 0.04\ kg = 4.0 \times 10^{-2}\ kg$

Vol; $16\ cm^3 = \frac{16}{1000000} = 0.000016 = 1.6 \times 10^{-6}\ m^3$

Now calculate the density: $\boldsymbol{\rho} = \frac{m}{v} = \frac{4.0\ x\ 10^{-2}}{1.6\ x\ 10^{-6}}$

$\mathbf{= 2.5 \times 10^3\ kg/m^3}$

2. The mass of 24.4 cm^3 of mercury is 332 g. Find the density of mercury.

Solution: **Data**: $m = 166$ g, $v = 12.2\ cm^3$, $\rho = ?$

$$\rho = \frac{m}{v} = \frac{332}{24.4} = \underline{\underline{\mathbf{13.6\ g/cm^3}}}$$

3. 25 cm^3 of aluminium has a mass of 67.5 g. Calculate its density in
(a) g/cm^3 (b) kg/m^3

Solution: **Data**: (a) $v = 25\ cm^3$, $m = 67.5\ g$, $\rho = ?$

$$\rho = \frac{m}{v} = \frac{67.5}{25} = \mathbf{2.7\ g/cm^3}$$

(b) First convert the mass and the volume into their respective SI units.

⇒ *Converting the **mass***:

$$1\ kg = 1\ 000\ g$$
$$m = 67.5 g$$
$$= \frac{1 \times 67.5}{1000}$$
$$\therefore \mathbf{m = 0.0675\ kg}$$

⇒ *Converting the **volume***:

$$1\ m^3 = 1\ 000\ 000\ cm^3$$
$$v = 25\ cm^3$$
$$= \frac{1 \times 25}{1000\ 000}$$
$$\therefore \mathbf{v = 0.000\ 025\ m^3}$$

Now applying the formula $\rho = \frac{m}{v}$ we have;

$$= \frac{0.0675}{0.000025}$$
$$\therefore \rho = \mathbf{2700\ kg/m^3} \text{ or } \mathbf{2.7 \times 10^3\ kgm^{-3}}$$

Calculating mass

4. The density of copper is 8.9 g/cm^3. What is the mass of 100 cm^3 of copper?

Solution: Data: $\rho = 8.9\ g/cm^3$, $v = 100\ cm^3$, $m = ?$

$$\rho = \frac{m}{v}$$
$$8.9 = \frac{m}{100}$$
$$m = 8.9 \times 100$$
$$\therefore \mathbf{m = 890\ g}$$

Calculating volume

5. Calculate the volume of a block of expanded polystyrene of mass 400 g if its density is 16 kg/m^3.

Solution ***Data***: m = 400 g, ρ = 16 kg/m^3, v = ?

Note: *The mass must be converted to kilograms to match the density unit***.**

$$\mathbf{m} \quad \mathbf{=} \frac{400}{1000} = 0.4 \text{ kg}$$

Rearranging the formula for calculating density we have:

$$\text{v} \quad = \frac{m}{\rho} \quad = \frac{0.4}{16} \quad \mathbf{= 0.025\ m^3}$$

3.2 Measurement of Density

(a) To find the density of a Regular Solid

- Measure the dimensions of the solid object i.e. length, width, height or diameter, using an appropriate instrument.
- Calculate the volume of the object from the appropriate formula. Say volume of object = *y* m^3
- Find the mass of the object using a triple balance. Say mass = *x* kg.
- Calculate the density from the formula

$$\textit{Density} \quad = \frac{\text{m}}{\text{v}}$$

$$\therefore \textbf{ Density} \quad \mathbf{=} \frac{x}{y}\, kg\, m^{-3}$$

Examples

1. A cuboid of wood of mass 20 g measures 5 cm by 4 cm by 2 cm. Find its density.

Solution: **Data**: m = 20 g, l = 5 cm, w = 4 cm, h = 2 cm, ρ = ?

Notice from the data that mass is given and volume is not given. Therefore, we first get the volume by substituting the dimensions in the formula of volume of a cuboid.

$$\text{v} \quad = lwh = 5 \times 4 \times 2 = 40 \text{ cm}^3$$

Now calculate the density, $$\rho \quad = \frac{\text{m}}{\text{v}} \quad = \frac{20}{40} \quad \mathbf{= 0.5\ g/cm^3}$$

2. A spherical metal made of aluminium weighs 90.477 g in air. If the diameter of the sphere is 4.0 cm^3, find the density of the sphere. (Take π = 3.14)

Solution: **Data**: m = 90.477 g, d = 4.0 cm (r = 2 cm), v = ?, ρ = ?

$$v \quad = \frac{4}{3}\pi r^3 = \frac{4 \times 3.14 \times 2^3}{3} = \frac{4 \times 3.14 \times 2 \times 2 \times 2}{3} = \underline{\underline{33.49 \text{ cm}^3}}$$

Applying the formula $$\rho \quad = \frac{\text{m}}{\text{v}} \quad = \frac{90.477}{33.49} \boldsymbol{= 2.7\ g/cm^3}$$

(b) To find the density of an Irregular Solid

- Pour water in a measuring cylinder and record the first reading of the water level, say x cm^3
- Tie the irregular solid with a piece of thin silk thread and carefully immerse it into the water in the measuring cylinder.
- Read and record the second reading of the water level, say y cm^3.
- Find the volume of the irregular solid from the formula:

Volume of object = (Second reading – First reading)

= (y – x) cm^3

- Determine the mass of the solid on a triple balance, say mass = z g.

Calculation;

Results: Mass of object in air = z g

Volume of object = (y – x) cm^3

Using the formula

$$\text{Density} = \frac{\text{m}}{\text{v}} = \frac{z\,g}{(y-x)\,cm^3} = \left(\frac{z}{y-x}\right) g/cm^3$$

Example

When a piece of irregular stone of mass 164.5 g was immersed in 300 cm^3 of water in a measuring cylinder, the level of water rose to 370 cm^3. Calculate the density of the stone.

Solution: Data: Density of stone, ρ = ?

Mass of stone in air = 164.5 g

Initial reading of water = 300 cm^3

Final reading = 370 cm^3

Volume of stone = Final reading – Initial reading

= 370 – 300

= 70 cm^3

$$\text{Density} = \frac{\text{m}}{\text{v}}$$

$$= \frac{164.5}{70}$$

$$\therefore \rho = \mathbf{2.35\ g/cm^3}$$

(b) To find the density of liquid e.g. Paraffin

Procedure

- Weigh an empty beaker on a triple balance, say x grams.
- Pour a known volume, v, of the liquid in the beaker.
- Weigh the beaker and the liquid (paraffin). Let the total mass be y grams.
- Calculate the density as below:

Results:

Mass of empty beaker	= x g
Mass of beaker + Paraffin	= y g
Mass of paraffin only	= $(y - x)$ g
Volume of paraffin	= v cm^3

$$\text{Density} = \frac{m}{v}$$

$$= \frac{(y - x)}{v}\ gcm^{-3}$$

Example

An empty beaker weighs 120 g in air. And weighs 180 g when filled with 75 cm^3 of methylated spirit. Find the density of the methylated spirit.

Solution:

Mass of empty beaker	= 120 g
Mass of beaker + Paraffin	= 180 g
Mass of paraffin only	= (180 – 120) g
Volume of paraffin	= 75 cm^3

$$\text{Density} = \frac{m}{v}$$

$$= \frac{60}{75}$$

$\therefore$ **Density of methylated spirit = 0.8 gcm^{-3}**

3.2 Density of Mixtures

A mixture is a substance that consists of two or more substances physical combined together.

Mixtures are obtained by mixing two or more substances physically. In dealing with the calculations of density of mixtures, the following assumptions are made.

- The constituents of the mixture do not react with one another.
- The total mass of the mixture is the sum of the masses of the constituents.
- The total volume of the mixture is the sum of the volumes of the constituents.

The density of mixtures is calculated from the formula.

$$\text{Density of mixture} = \frac{Mass\ of\ mixture}{Volume\ of\ mixture}$$

N.B (a) The density of the mixture lies between the densities of its constituents.

(b) Calculations on density of mixtures are of two types.

(i) Where the masses and volumes of the constituents are given directly.

(ii) Where either the masses and densities are given but volumes not given.

Or volumes and densities are given but masses not given.

In the case of the (b) (i), the formula for calculating density of mixture is applied directly after getting the total mass and total volume of the mixture.

While for the case of (b) (ii), we first use the formula for calculating density to get the quantities which are not given. There after we apply the formula of density of mixtures.

(a) Calculating density of a mixture when the masses and the volumes of the constituents are given

Example 1

100 cm^3 of fresh water which weighs 100g is mixed with 100cm^3 of sea water which weighs 103g .Calculate the density of the mixture?

Hint: ***For this type of question, we get the total mass and total volume of the mixture and then substitute the values in the formula of density of mixtures. See examples below.***

Solution:

Mass of fresh water = 100 g
Mass of sea water = 103 g
Mass of the mixture = 100 g + 103g
= 203 g
Volume of fresh water = 100 cm^3
Volume of sea water = 100 cm^3
Volume of the mixture = 100 cm^3 + 100 cm^3
= 200 cm^3

Density of mixture $= \frac{Mass\ of\ mixture}{Volume\ of\ mixture}$

$= \frac{203}{200}$

$\therefore$ **Density of mixture = 1.015 g/cm^3**

(b) Calculating density of a mixture when either masses or volumes of the constituents are not given

Example 2

0.0018 m^3 of fresh water of density 1000 kg/m^3 is mixed with 0.0022 m^3 of sea water of density 1,025 kg/m^3 . Calculate the density of the mixture.

Solution:

Mass of fresh water = ?
Volume of fresh water = 0.0018 m^3
Density of fresh water = 1000 kg/m^3

Mass of sea water = ?
Volume of sea water = 0.0022 m^3
Density of fresh water = 1,025 kg/m^3

Hint: *Note that the masses are not given*. So u*se the formula of density of a substance to get the masses of the constituents first and then follow the steps in example 1 above.*

$\Rightarrow$ *Calculating mass of fresh water:* $\rho = \frac{m}{v}$

$m = \rho v$
$= 1000 \times 0.0018$

$\therefore$ Mass of fresh water = 1.8 kg

$\Rightarrow$ *Calculating mass of sea water:* $\rho = \frac{m}{v}$

$m = \rho v$

$= 1000 \times 0.0018$

$= 1.025 \times 0.0022$

$\therefore$ **Mass of fresh water** **= 2.255 kg**

Since the masses and the volumes of the constituents are now known, the density of the mixture can calculated from the data below.

Density of the mixture

Mass of fresh water = 1.8 kg
Mass of sea water = 2.255 kg
Mass of the mixture = 1.8 + 2.255
= 4.055 kg
Volume of fresh water = 0.0018 m^3
Volume of sea water = 0.0022 m^3
Volume of the mixture = 0.0018 m^3 + 0.0022 m^3
= 0.004 cm^3

$$\text{Density of mixture} = \frac{Mass\ of\ mixture}{Volume\ of\ mixture}$$

$$= \frac{4.055}{0.004}$$

$\therefore$ **Density of mixture = 1.013 kg/m^3**

Note: ***The detailed steps are to make you to understand how to solve problems in this topic. But in examination you are required to present your work brief and clear.***

3.3 RELATIVE DENSITY (R.D)

Definition

Relative density of a substance is defined as *the ratio of the density of a substance to the density of water.*

Mathematically, R.D is expressed as:

$$\text{Relative density (R.D)} = \frac{Density\ of\ substance}{Density\ of\ water}$$

If the masses of equal volume of a substance and water are found, then this relation takes the form

$$\text{Relative density (R.D)} = \frac{Mass\ of\ any\ volume\ of\ a\ substance}{Mass\ of\ an\ equal\ volume\ of\ water}$$

In a normal weighing, the mass of a substance is proportional to the weight, so it is also true to say;

$$\text{Relative density (R.D)} = \frac{Weight\ of\ any\ volume\ of\ a\ substance}{Weight\ of\ an\ equal\ volume\ of\ water}$$

NB: *R.D has no unit since it is a ratio of the same quantity i.e. densities, masses or force (weigh)t as a result the units cancel.*

3.31 Measurement of Relative Density (R.D)

(a) To measure the density of Liquid

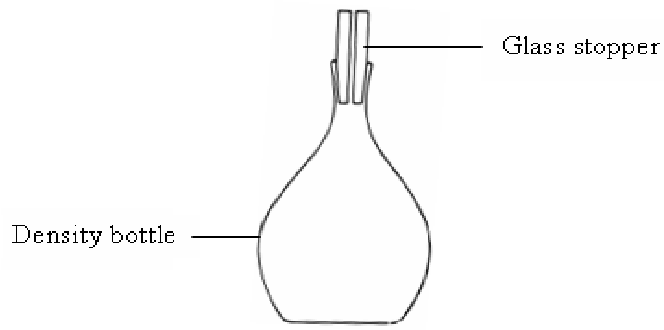


Figure 3.2

The R.D of a liquid is measured by using a *density bottle*. The density bottle has glass stopper with a fine hole through it, so that, when it is filled fully with the liquid and the stopper inserted, the excess liquid rises through the fine hole and runs down the outside.
So long as the bottle is used to the same liquid level at the top of the hole, it will always contain the same volume of whatever liquid is filled in it provided the temperature remains the same.

Experiment to determine the R.D of liquid e.g. Paraffin

Procedure

- Weigh the density bottle when empty.
- Fill the bottle full with the paraffin.
- Wipe the paraffin that runs out through the hole and weigh the bottle and the paraffin.
- Empty the bottle and clean it thoroughly.
- Refill the bottle with water to the same level and weigh it after wiping the water that flows out.

Calculate the R.D as shown below.

Mass of empty bottle = x g
Mass of bottle full of liquid = y g
Mass of bottle full of water = z g
Mass of liquid = $(y - x)$ g
Mass of water = $(z - x)$ g

Applying the formula

$$\text{Relative density of liquid} = \frac{Mass\ of\ a\ substance}{Mass\ of\ equal\ volume\ of\ water}$$

$$= \frac{(y-x)\,g}{(z-x)\,g}$$

$$\therefore \text{R.D} = \frac{(y-x)}{(z-x)}$$

(b) Precautions

To obtain accurate result, the following precautions should be taken.

(i) The outside of the bottle must be wiped dry before weighing.

(ii) The bottle must **not** be held by the neck with warm hand otherwise some liquid may be lost due to expansion.

Worked Examples

Steps in problem solving

Before solving any problem, ask your self the following questions.

- What is asked in the question?
- What information is given to help solve the problem?
- What are the equations to solve the problem?

These questions are answered when you collect the data.

Example 1

A density bottle was used to measure the relative density of a liquid and the following results were obtained.

Solution: Mass of empty density bottle = 30 g
Mass of bottle full of liquid = 110 g
Mass of bottle full of water = 130 g

Mass of liquid = 110 – 30
= 80 g
Mass of water = 130 – 30
= 100 g

$$\text{Relative density of liquid} = \frac{\textit{Mass of a substance}}{\textit{Mass of equal volume of water}}$$

$$= \frac{80\ g}{100\ g}$$

$\therefore$ **R.D of liquid = 0.8**

Example 2

The mass of an empty density bottle is 46.00 g. When fully filled with water it weighs 96 g. And when full of a liquid of unknown R.D. it weighs 86 g.

Calculate: (i) the R.D of the liquid.
(ii) the density of the liquid.

Solution: **(i)** Mass of empty bottle = 46 g
Mass of bottle full of liquid = 96 g
Mass of bottle full of water = 86 g

Mass of liquid = 86 – 46
= 40 g
Mass of water = 96 – 46
= 50 g

$$\text{Relative Density of liquid} = \frac{\textit{Mass of liquid}}{\textit{Mass of water}}$$

$$= \frac{40\ g}{50\ g}$$

$\therefore$ **R.D of liquid = 0.8**

(ii) R.D of liquid = 0.8, density of water = 1000 kgm^{-3}

$$\text{Relative density (R.D)} = \frac{Density\ of\ a\ subs\tan ce}{Density\ of\ water}$$

$$0.8 = \frac{Density\ of\ a\ subs\tan ce}{1000}$$

Density of liquid = 0.8 x 1000

$\therefore$ **Density of liquid = 800 kgm^{-3}**

(c) To measure the Relative Density of Solid

The R.D of solid/liquid substances can best be measured by applying *Archimedes's Principle* to be discussed in detail later.

Archimedes's Principle stats that:

When a body is wholly or partially immersed in a fluid, it experiences an upthrust equal to the weight of fluid displaced.

Procedure

- Suspend the solid whose relative is to be determined from a spring balance by means of light string in air and record its weight.
- Immerse the solid wholly in water and record its apparent weight.

Results:

Let: Weight of solid object in air = W_a N

Weight of solid object in water (Apparent weight) = W_w N

Calculation:

Upthrust (Loss in weight of object) = Weight of water displaced

= Weight in air – Apparent weight

$\therefore$ Upthrust = $(W_a - W_w)$ N

But Volume of water displaced = Volume of the solid immersed

From Relative Density $= \dfrac{Weight\ of\ subs\tan ce}{Weight\ of\ an\ equal\ volume\ of\ water}$

$$= \frac{W_a}{(W_{a1} - W_w)}$$

I.e. Relative Density (R.D) $= \dfrac{Weight\ of\ subs\tan ce\ in\ air}{Upthrust\ in\ water}$

Note: ***For details on R.D of solids, see chapter 10.***

Examples

1. A piece of aluminium weighs 80 N in air and 50.37 N when completely immersed in water. Calculate the relative density of glass.

Solution: (Remember the steps in problem solving!!)

Weight of solid object in air = 80 N

Weight of solid object in water (Apparent weight) = 50.37 N

Weight of water displaced = Upthrust in water = 80 – 50.37 N

= 29.63 N

$$\text{Relative Density} = \frac{Weight\ of\ substance\ in\ air}{Upthrust\ in\ water}$$

$$= \frac{80}{29.63}$$

$$= 2.699966$$

$\therefore$ **Relative Density = 2.7**

SELF-CHECK 3.0

1. To calculate the density of an object, which one of the following must be known?
   I. Height II. Volume III Area IV Mass V Weight
   A. I and II B. II and V. C. III and IV D. II and IV A

2. A block of wood 10m x 5m x 4m has a mass of 80 000 kg. What is the density of this wood?
   A. 2000kg/m^2 B. 4000kg/m^2 C. 200 kg/m^2 D. 400 kg/m^2

3. The density of gold is 19.3 g/cm^3. What is the mass of 10cm^3 gold?
   A. 19.3 g B. 0.193g C. 1.93g D. 193g

4.. What is the mass of the copper cube having each side 2cm? (take c_{opper} = 9g/cm^3)
   A. 0.18g B. 72g C. 180g D. 36g

5. What is the volume of 60g wood ? (d_{wood} = 0. 6 g / cm^3)
   A. 10cm^3 B. 36cm^3 C. 100cm^3 D. 360cm^3

6. Study the table below and use it to spot the correct answer.

Material	Density (g/cm^3)	Mass (g_)
K	3	60
L	9	180
M	6	360
N	5	200

From the values shown in the table which material has the biggest volume?

A. K B. L C. M D. N

7. What is the volume and mass of the block which measures by 2m, by 3m by 5m if its density is $1500kg/m^3$?

A. $50m^3$; 75 000 kg B. 100 m^3; 75 000 kg
C. $30m^3$; 75 000 kg D. 30 m^3; 75 000 kg

8. Two litres of corn oil has a mass of 1. 85kg. What is the density of the oil?

A. $1850kg/m^2$ B. $925kg/m^2$ C. $185kg/m^2$ D. $92.5kg/m^2$

9. If an object of volume $0.02m^3$ weighs 500 N in a liquid of density $2000kg/m^3$, what is the weight in air?

A. 900 N B. 1000 N C. 400 N D. 600 N

10. Which one of the following is the SI unit of density?

A. kgm^3 B. kg/m^{-3} C. g/cm^3 D. kg/m^3)

11. If 10g water and $10cm^3$ alcohol are mixed what will be the mass of the mixture?
($d_{alcohol} = 0.80 g/cm^3$)

A. 18g B. 20g C. 16g D. 19g

12. A tin containing 5 litres of paint has a mass of 8.5kg. The mass of the empty tin is 2.0kg, the density of the paint is

A. $1.3kg\ m^{-3}$ B. $1.3x10^3kg\ m^{-3}$ C. $1.7x10^3kg\ m^{-3}$ D. $2.1x10^3kg\ m^{-3}$

13. A rectangular block of tin is 0.5m long and 0.01m thick. Find the width of the block if its mass and density are 0.45kg and 9000 kg m^{-3} respectively.

A. $0.005x0.45x9000m$ B. $\frac{0.45}{9000x0.005}$

C. $\frac{0.005}{0.45x9000}m$ D. $\frac{0.45x0.005}{9000}m$

14. A box of dimensions 0.2m by 0.3m by 0.5m is full of a gas of density $200kg/m^3$. The mass of the gas is

A. $3x10^{-2}kg$ B. $6.0x10^0kg$ C. $2x10^2kg$ D. $6.7x10^3kg$

15. A piece of material of mass 200g has a density of $25kgm^{-3}$. Calculate its volume in m^3.

A. $\frac{200}{25}$ B. $\frac{200}{1000x25}$ C. $\frac{1000x25}{200}$ D. $\frac{1000x200}{25}$

16. Two solid cubes have the same mass but their edges are in the ratio 4:1. What is the ratio of their densities?

A. 1:4 B. 1:8 C. 1:16 D. 1: 64

17. A tin containing $6x10^{-3}$ m^3 of paint has a mass of 8kg. If the mass of the empty tin with the lid is 0.5kg, calculate the density of the paint in kgm^{-3}

A. $\frac{8x0.5}{6x10^{=3}}$ B. $\frac{7.5}{6x10^{-3}}$ C. $\frac{8x10^{6}}{6x10^{-3}}$ D. $\frac{8.5x10^{6}}{6x10^{-3}}$

18 A tank 2 m tall base area of 2.5 m^2 is filled to the brim with a liquid which exerts a force of 40000N at the bottom. Calculate the density of the liquid.

A. $\frac{4000}{25x2x10}kgm^{-3}$ B. $\frac{40000}{2.5x2x10}kgm^{-3}$ C. $\frac{40000}{25x2x10}kgm^{-3}$ D. $\frac{40000}{2.5x2}kgm^{-3}$

19. The following reading were recorded when measuring the density of a stone; Mass of the stone = 25g, volume of water = 25 cm^3, volume of water and stone = $35cm^3$ .What is the density of the stone?

A. $\frac{25}{10}g/cm^3$ B. $\frac{35}{30}g/cm^3$ C. $10g/cm^3$ D. $\frac{25}{35}g/cm^3$

20. Liquid Y of a volume $0.40m^3$ and density 900 kg/m^3 is mixed with liquid Z of volume 0.35 m^3 and density 800 kg/m^3. Calculate the density of the mixture.

A. $800kg/m^3$ B. 840 kg/m^3 C. 850 kg/m^3 D. 900 kg/m^3

CHAPTER FOUR
PARTICULATE NATURE OF MATTER

LEARNING OBJECTIVES

By the end of this topic you should be able to:

1. *Define: - Matter.*
2. *State: (i) - The states of matter.*
   (ii) - The differences between the states of matter.
3. *(a) State: - Kinetic Theory of Matter.*
   (b) Describe an experiment (Brownian Motion) to proof the kinetic theory
   (c) The effect of heat on Brownian Motion.
   (d) Use the Kinetic Theory to explain change of state.
4. *List and define the Properties of Matter.*
   - *Molecular forces (Cohesion and Adhesion),*
   - *Diffusion, Capillarity,*
   - *Surface tension, and*
   - *Elasticity.*
5. *Describe experiments to show:*
   - *Diffusion in liquids and gases.*
   - *Surface tension.*
   - *The effect of detergents on surface tension.*

4.1 Matter

***Definition*:** *Matter is any thing that occupies space and has mass.*

States (Phases) of Matter

There are three states of matter namely:

(iv)	Solid,	e.g.	stone, wood etc.
(v)	Liquid	e.g.	water, paraffin etc.
(vi)	Gas	e.g.	oxygen, nitrogen etc.

Each of these states is made up of so many tiny particles called atoms and molecules. The arrangement of these particles and the magnitude of the forces holding them makes one state different from other states.

Note: ***Examples of things which are not matter are:- Light, Sound and Heat.***

4.11 Differences in the three states of matter

(a) Solids

(i) The molecules are closely packed and are arranged in a regular pattern called *lattice*.
(ii) The forces holding the molecules are strong as such
(iii) The molecules are not free to move.
(iv) The molecules vibrate about a fixed position.
(v) Solids have a definite shape and volume.
(vi) There is very little diffusion in solids. E.g. Naphthalene.
(vii) Due to the close packing of the particles, solids cannot be compressed.

(b) Liquids

(i) The particles in liquid are fairly close to each other and are in irregular pattern.
(ii) The forces holding the molecules are weak.
(iii) The molecules move randomly throughout the liquid.
(iv) Liquids have definite volume but no definite shape. They take the shape of the container in which they are placed and then acquire a definite volume.
(v) Liquids have slow diffusion.
(vi) They cannot be compressed.

(c) Gases

(i) The particles in a gases are far apart from each other.
(ii) The forces holding the molecules are very weak.
(iii) The particles move randomly and at comparatively high velocities.
(iv) Gases have no definite shape and no definite volume. They fill the whole container they are placed in.
(v) There is high rate of diffusion in gases.
(vi) Gases are quite easy to compress. This is because there are large spaces between the particles.

The diagrams showing a model representation of the three states of matter

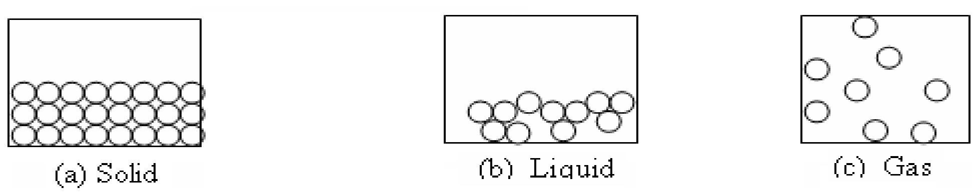


Figure 4.0

Note: *The diagrams in figure 4.0 show that the molecules are of the same size for all the three states. However, is not the case. Even for the same state, particle size is different for different substances.*

4.2 Kinetic Theory of Matter

The kinetic theory of matter states that:

1. Matter is made up small particles called *atoms or molecules*.
2. The molecules are in a constant random motion.
3. The speed of movement of the particles increases when temperature increases.
4. The particles (molecules) in gases move faster than those of liquids and those of liquids move faster than those of solids.

4.21 Assumptions of Kinetic Theory of Gases

1. Molecules of gases move in straight line with a very great speed or velocity until they collide with each other or with the walls of the container in they are placed. Pressure of a gas is due to collisions of the gas molecules with the walls of the container. This pressure increases as the temperature increases.
2. The volume of the gas molecules is negligible compared to the total volume of the container; so the molecules can be taken to be points of negligible volume.
3. The forces of attraction between the gas molecules are negligible.
4. The average kinetic energy of the molecules is a measure of the temperature of the gas.
5. The collisions of the molecules are perfectly elastic. (i.e. When molecules collide, there is no loss in kinetic energy).

4.22 The proof of Kinetic Theory of Matter

The kinetic theory of matter can be proved by:

(i) Brownian Motion and
(ii) Diffusion.

Experiment 4.0 To demonstrate kinetic theory of matter using *Brownian motion.*

Apparatus/Requirements
A smoke cell, microscope, source of light, glass rod or convex lens, smouldering cigarette or paper.

Procedure
- Place the source of light at a small distance such that the rays are incident on to the window of the glass cell.
- Place the glass rod or converging lens between the source of light and the smoke cell.
- Adjust the object distance such that the light rays are in focus in the smoke cell thus strongly illuminating it.
- Fill the smoke cell with some smoke from a smouldering paper.
- View the smoke particles from above using a microscope as show in the figure below.

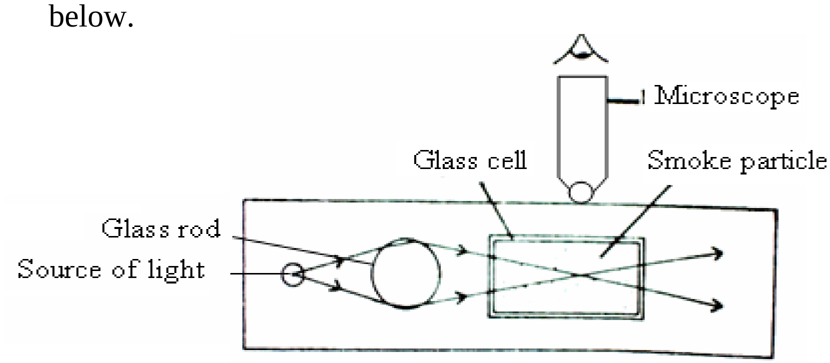


Figure 4.1

Observation
The white specks of the smoke particles are seen moving with constant random motion.

Explanation
The random motion of the smoke particles is due to the bombardment with the air molecules which are in a constant random motion. The air molecules can not be seen, only their effects can be seen.

NB: *If the smoke cell is replaced by a glass container containing water with some pollen grains suspended in it, the pollen grains will also be seen moving with a constant random motion.*

Effect of heat on Brownian motion
When the temperature of the smoke cell is increased, the smoke particles are seen moving faster.

Explanation

Due to the increase in temperature, the molecules acquire kinetic energy. The kinetic energy is directly proportional to the velocity of the particles (i.e. from K.E = $\frac{1}{2}mv^2$) hence increased velocity.

4.23 Effect of Heat on Matter

(a) Heating Solid

When a solid is heated, its particles acquire kinetic energy and vibrate more violently. The particles continue to vibrate until a point is reached when the vibration overcome the *binding force* (forces of attraction between the particles). The crystalline structure corrupts and the particles become mobile. At this point the solid *melts* or *fuses* (i.e. changes to liquid at a constant temperature called *melting point*. The process is called *melting* or *fusion.*

On further heating of the liquid, the molecules of the liquid acquire increased kinetic energy. The kinetic energy continues to increase until they overcome the forces of attraction between the molecules. At this point the liquid *boils* (i.e. changes to vapour at a constant temperature called the *boiling point*. And the process is called *boiling* or *evaporation*.

(b) Effect of Cooling

When a gas or vapour at a higher temperature is cooled, the kinetic energy of the particles also reduces. Since the kinetic energy of the particles is directly proportional to their velocity, the average velocity (speed) of the particles gradually decreases until the forces of attraction between the particles build up and the particles come closer i.e. they condense. At this point, the gas changes to liquid at a constant temperature. The process is called condensation.

Cooling the liquid further causes more loss in kinetic energy until eventually the particles settle to form a solid at a constant temperature called *freezing point*. The process involved is called *freezing* or *solidifying*.

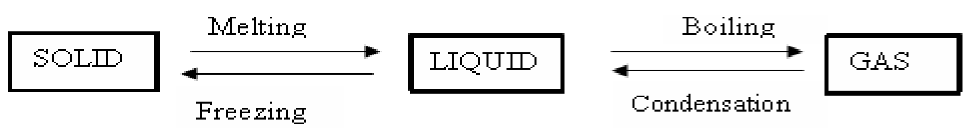


Figure 4.2

For example when ice is heated, it changes to liquid water. On heating the water, it changes to vapour. If the vapour is cooled, it condenses to liquid. On further cooling of the liquid it changes to ice.

4.3 Properties of Matter

The properties of matter are as a result of the behaviour of its molecules. The properties resulting from the behaviour of molecules include:

(i) Molecular forces,
(ii) Diffusion,
(iii) Capillarity,
(iv) Surface tension and
(v) Elasticity.

4.31 Molecular forces

A molecular force refers to the attractive force that exists between molecules. The molecules may be of the same substance or of different substances.

Types of molecular forces

There are two types of molecular forces namely:

- Cohesion and
- Adhesion.

(i) Cohesion

Cohesion is the force of attraction between the molecules of the same kind.
E.g. The force of attraction between water molecules.

(ii) Adhesion

Adhesion is the force of attraction between molecules of different substances.
E.g. The force of attraction between water molecules and glass molecules.

The magnitude of cohesion and adhesion determine:

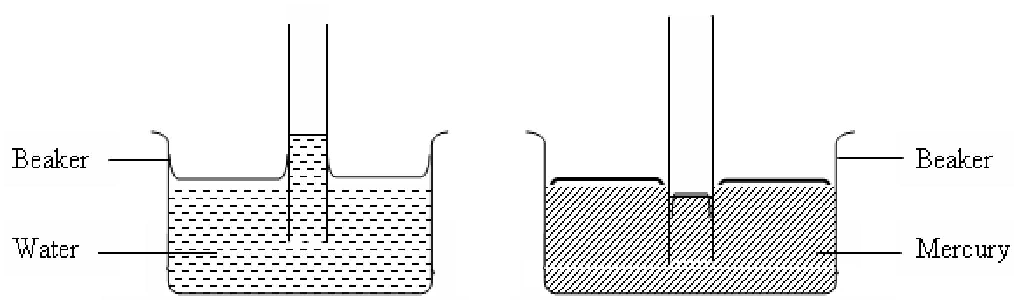
- The shape of liquid meniscus when in contact with other substances.
- Ability to wet substances and
- The rise or fall in a capillary tube.

For example:

(i) Water in a glass tube wets glass. This is because the adhesion is greater than cohesion. The result is that the meniscus of water curves upwards as shown in figure 4.3

(ii) Mercury in a glass tube does not wet glass because cohesion is greater than adhesion. The result is that the meniscus of mercury curves downward. See figure 4.3 (ii)

Diagrams showing the meniscus of water and mercury



(i)
Water rises in capillary tube
(Adhesion > Cohesion)

(ii)
Mercury falls in capillary tube
(Cohesion > adhesion)

Figure 4.3

4.32 Diffusion

Diffusion is the spreading of molecules of one substance in to the molecules of another substance. Or

Diffusion is the movement of molecules from a region of high concentration to a region of low concentration.

Factors which determine the rate of diffusion depends

The rate of diffusion depends on the following factors:

(i) Size of the diffusing molecules,

(ii) Molecular weight

(iii) Temperature

(iv) Pressure

(v) Size of the pore (in the porous material) across which the molecules diffuse

The table below shows the factors which determine the rate of diffusion and there explanations.

Factor of rate of diffusion	Explanation
i) Size of the molecules	Bigger molecules occupy larger space than smaller molecules. As a result it is difficult for them to pass through porous material with small pores. Therefore, low rate of diffusion.
ii) Molecular weight	Lighter molecules diffuse faster than the "massive" molecules.
iii) Temperature	Temperature determines the amount of kinetic energy in a body. Since kinetic energy is directly proportional to velocity of a body, therefore, at high temperature gas molecules gain kinetic energy and diffuse faster than when at low temperature. Therefore the rate of diffusion is higher at high temperature than at low temperature.
iv) Pressure	Pressure mainly affects the rate of diffusion of gases. At higher pressure, the gas molecules are squeezed in a small space. This makes the molecules to collide frequently and move faster than at low pressure. Therefore, rate of diffusion is high at high pressure than at low pressure.
v) Size of the pore in the porous material	A large pore allows many molecules to pass through in a unit time. While a small pore allows very few molecules to pass through. As a result the rate of diffusion is high when the size of the pore is large and low when the size of the pore is small.

Figure 4.1

Diffusion is extremely slow in solids, average in liquids and very fast in gases. For example a small amount of perfume or ammonium gas placed at one corner of a room spreads quickly and fills the whole room.

Experiment 4.2 To show diffusion in liquid e.g. Water

Apparatus/Requirements
A beaker, Crystal of potassium permanganate, water and a glass tube.

Procedure:
- Fill a beaker with clean water.
- Drop a fairy large crystal of potassium permanganate in the middle of the bottom of beaker with a help of a glass tube.
- Leave the set up to stand for some time.

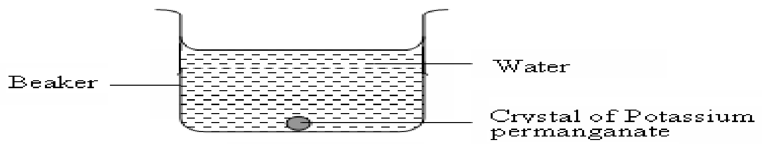


Figure 4.4

Observation
The crystal dissolved and spread through out the water forming a purple solution.

Explanation
The molecules of the potassium permanganate moved slowly moved in to the molecules of water to give the purple colour. We say that the molecules of potassium permanganate diffused into the molecules of water.

Experiment 4.3 To show diffusion in gases

Apparatus/Requirements
Porous pot, manometer, water, a delivery tube, hydrogen gas and air.

Procedure
- Arrange the apparatus as shown in the diagram below.

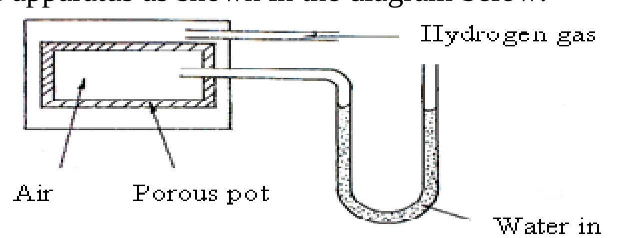


Figure 4.5

- Supply hydrogen gas into the space surrounding the porous pot and observe the water levels in the arms of the manometer.

Observation: The level of the liquid in the left arm of the manometer falls, while that in the right arm rises.

Explanation:
The hydrogen molecules increase the pressure inside the porous pot. And the pressure acts on the water surface in the limb of the manometer thus pushing the water level downward.

When the hydrogen supply is stopped, the situation is reversed. This is because the hydrogen that diffused into the porous pot diffuses out faster than the air outside porous pot diffuses into the pot.

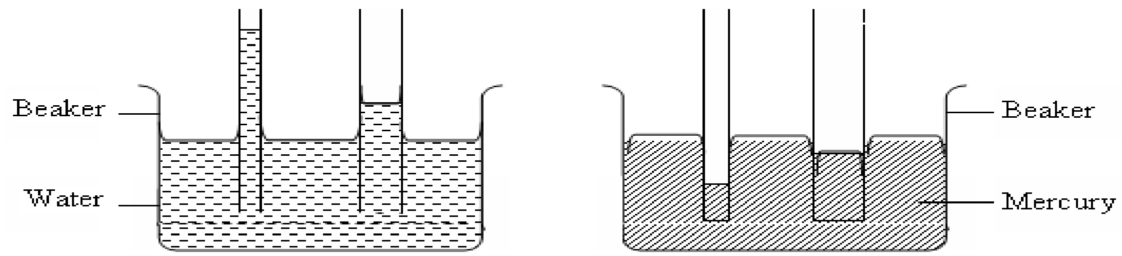
NB: *Brownian motion and diffusion, in liquid or in gases is a strong evidence of kinetic theory of matter. That is, it shows that matter is made up of small particles and that the particles are in a constant random motion.*

4.33 Capillarity

Capillarity is the rise or fall of liquid in a capillary tube.

Capillary rise is due to adhesion being greater than cohesion. Examples of liquids that have capillary rise when a capillary tube is dipped into them are: *Water*, *paraffin* and *diesel*. While *capillary fall* is due to cohesion being greater than adhesion. Example of liquid that has a capillary fall is mercury.

The diagrams in figure 4.6 show capillary rise and fall in glass tubes of different diameters



(i)
Water rises higher in a tube of smaller diameter than in the tube of larger diameter

(ii)
Mercury falls deeper in tube of smaller diameter than in the tube of larger

Figure 4.6

Uses of Capillarity

It helps:
- i) Fuel e.g paraffin to rise up in wicks of stoves and lamps.
- ii) Water to move up tree trunks to the leaves.
- iii) Blotting paper to absorb liquids.

4.34 Surface tension

Surface tension is the force on a liquid surface that makes the liquid surface to behave as if it is covered with thin elastic membrane.

Experiment 4.4 To show surface tension in a liquid e.g. water

Apparatus/Requirements: A glass trough, bloating paper, a needle or pin and water.

Procedure
- Fill a beaker with clean water.
- Place a filter paper or bloating paper on the surface in the trough.
- Drop a needle on the bloating paper.
- Leave the apparatus to stand for some time as shown in figure 4.7 below.

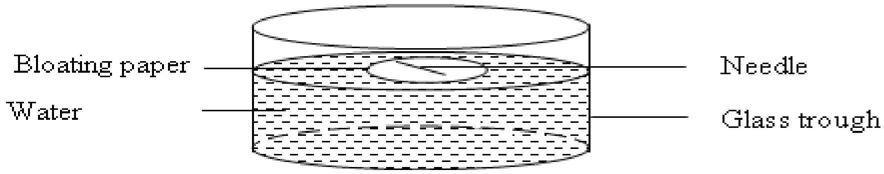


Figure 4.7

Observation
The filter paper absorbs water and sinks to the bottom of the trough.
The needle is seen floating on the water surface.

Conclusion
The surface of liquid water behaves as if it is covered with a thin elastic skin or membrane.

How to reduce or get rid of surface tension
Surface tension in liquids is reduced or got rid of by adding the following.

(i) Soap solution,
(ii) Detergent solution,
(iii) Oil.

These solutions get rid of surface tension by dissolving process.

Experiment 4.5 To show the effect of detergent solution on surface tension

Repeat the experiment 4.4 above.

When the needle is floating, carefully add a detergent solution with a help of syringe to the water and observe.

Observation: The needle quickly sinks to the bottom.

Conclusion: Surface tension can be reduced by detergent solution.

4.35 Elasticity

Elasticity is a property of a material that enables the material to return to its original size and shape after deformation.

Materials that have elasticity (*e.g. rubber, foam, metal wires*) are said to be elastic. While those which do not have this property (*e.g. wood, glass, paper*) are said to be inelastic.

Self-Check 4.0
SECTION A

1. When a crystal of potassium permanganate is carefully placed at the bottom of a beaker containing water, it spreads uniformly in the water after some days. This is due to:
A. diffusion B. capillarity
C. surface tension D. Brownian motion

2. Soap is used to wash clothes because it
A. increases capillarity in the clothes
B. reduces capillarity in the clothes
C. increases surface tension allowing water to penetrate the dirt easily.
D. reduces surface tension allowing water to penetrate the dirt easily.

3. Brownian motion experiment shows that molecules of gasses are
A. Stationary B. in motion in one direction only
C. In constant random motion D. More closely packed than molecules in liquid.

4. When mercury is spilt on glass it forms small spherical droplets because its
A. Density is high. B. Surface tension makes its surface elastic.
C. Molecules are small D. Cohesive force is greater than adhesive force with the glass.

5. A needle may float on a clean water but sinks when some detergent is added to water because the detergent
A. reduces the density of water.
B. increases adhesive force between the needle and water molecules.
C. lowers the surface tension of water.
D. makes water surface slippery.

6. The particles in a solid at room temperature are
A. Close together and vibrating. B. Close together and stationary.
C. Far apart and moving at random. D. Close together and moving at random.

7. Mercury forms spherical drops when spilt on a wooden bench because it
A. is very viscous B. has a high density
C. has a high cohesive force D. has a low surface tension

8. Water wets glass because
A. adhesive forces between water and glass molecules are greater than cohesive forces
B. adhesive forces between water and glass molecules are more than cohesive forces
C. surface tension forces between water and glass molecules are more than adhesive forces.
D. Surface tension forces are less than cohesive forces.

9. The forces which hold the molecules in water drop together are called…
A. Surface tension B. Adhesive
C. Cohesive D. Electrostatic forces

10. When water spreads on a glass plate, the forces between it's molecules and glass molecules are due to
A. Surface tension B. Adhesion C. Cohesion D. Viscosity.

11. In a Brownian motion experiment, the
A. smoke particles are seen moving about with uniform velocity.
B. motion observed is caused by the air molecules colliding with the smoke particles.
C. Size of particles are found to increase the motion.
D. smoke cell has a vacuum within it.

12. When smoke is introduced in a smoke cell and observed under a microscope, it is observed as particles moving at random. This is mainly because the particles
A. are hot B. collide with one another
C. collide with air molecules D. collide with the walls of the smoke cell

13. Which of the following statements is incorrect when a tin containing air tightly sealed is heated?
A. the average speed of molecules increases
B. the molecules of air hit the walls of the tin harder
C. The molecules of the air strike the walls of the tin less often.
D. The pressure inside the tin increases.

14. When a room is filled with smoke, the smoke tends to concentrate
A. around the walls B. close to the walls
C. close to the roof D. midway between the roof and the floor

15. Capillary rise in a tube dipped in water is due to
A. Surface tension B. adhesive force being greater than cohesive force
C. high vapour pressure D. Atmospheric pressure acting on the surface of the water

16. Which of the following statements about states of matter is/are true?
(i) a liquid has a definite volume but no a definite shape.
(ii) a vapour has no definite volume and no definite shape.
(iii) a solid has a definite volume and shape.
(iv) a gas has a definite volume and shape.
A. (i) and (iii) only B. (i) only C. (i), (ii) and (iii) only D. (iii) only

17. When viewing Brownian motion in a smoke cell the observer sees:
A. Air molecules moving in a random motion
B. air molecules vibrating regularly
C. air molecules colliding with each other
D. Smoke particles in a random motion.

18. Which of the below is a matter?
A. Heat B. Sound C. Air D. Light

19. Which of the following statements below is *wrong*?
A. Matter is made up of atoms. B. Solid, liquid and gas are states of matter.
C. Solids and liquids have definite shape and volume. D. Gas molecules move freely.

20. The diagrams in figure show two capillary tubes standing in a trough of mercury and two capillary tubes standing in a trough of water. The correct order of arrangement of the tubes in order of increasing height of the liquid column is;

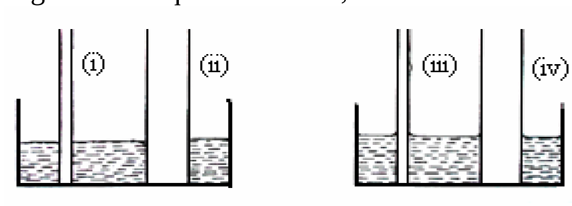


Fig. 4.8

A. (i), (ii), (iv),(iii) B. (ii), (i), (iv), (iii) C. (ii), (iv), (i), (iii) D. (iii), (iv), (i), (ii)

SECTION B

21. Describe the relationship between molecules of liquids, gases and solids in terms of:
 (a) the arrangement of the molecules through out the bulk of the material,
 (b) the separation of the molecules,
 (c) the motion of the molecules and
 (d) The forces of attraction between the molecules.

22. (a) (i) What is meant by the term *diffusion*?
 (ii) State factors on which diffusion depends.
 (b) Describe an experiment to show diffusion in liquids.
 (c) A porous pot containing air is connected to a water manometer. Explain what happens if hydrogen is let in the space surrounding the as shown in the figure.
 (d) (i) Describe a simple experiment to show surface tension in water.
 (ii) State two factors, which affect surface tension.

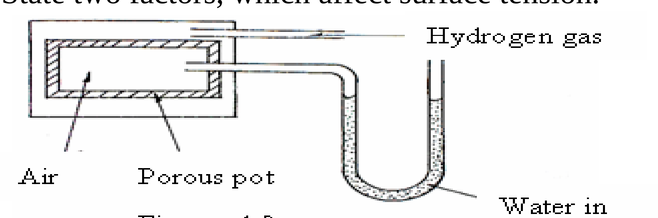
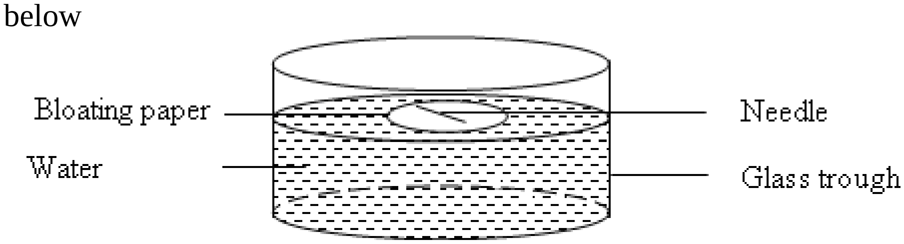


Figure. 4.9

23. A pin is placed on a bloating paper, which is on the surface of water as shown in figure 4.10 below



 (a) Explain what happens after some time.
 (b) Explain what happens when some soap solution is carefully added to the water.

24. Draw a well labelled diagram you would use to describe Brownian motion.
(a) How is the motion of the smoke particles best described?
(b) What accounts for the motion of the smoke particles?
(c) The motion is viewed using bigger smoke particles.
What difference in the motion would this lead to.
Give reason for the difference.

25. The diagram in figure 4.11 shows an arrangement for observing Brownian motion.

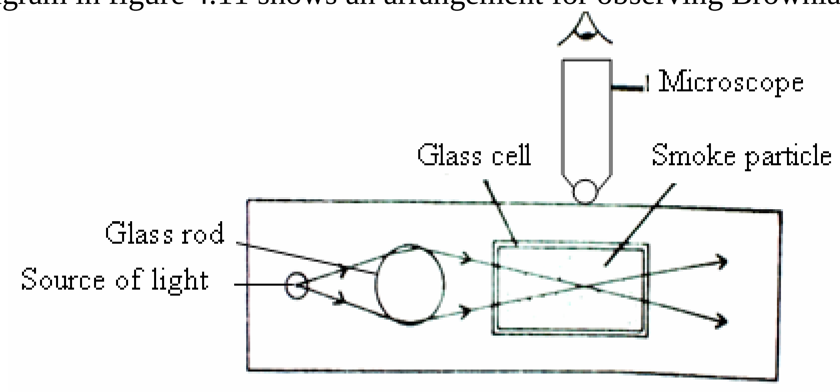


Figure 4.1

(a) Explain:
(i) The observation made.
(ii) What will be observed when the glass cell temperature is raised.
(b) State one factor which determines the rate of diffusion of a gas.

26. (a) Distinguish between cohesion and adhesion.
(b) Sketch diagrams to show the level of liquid in a capillary tube that is immersed in a liquid which has greater;
i) Cohesion than adhesion
ii) Adhesion than cohesion

27. (a) Define surface tension.
(b) Describe a simple experiment to show the existence of surface tension in water.
(c) Explain the following observations as fully as you can.
(i) A small needle can be floated on the surface of water, but if a drop of detergent is added to the water the needle sinks.
(ii) Damp courses are used in modern houses.
(iii) Gases can easily be compressed but liquids cannot.
(iv) Diffusion occurs more easily in a gas than in a liquid.

28. (a) State the kinetic theory of matter.
(b) Describe experiments, in each case, to show:
(i) Diffusion in liquids
(ii) Diffusion in gases.
(c) Use kinetic theory to explain the result of your experiment demonstrating the diffusion in gases.

4.4 SIZE OF A MOLECULE

Molecules are so small that we cannot measure their sizes accurately. The approximate size of a molecule can only be estimated through an experiment.

Experiment 4.6 To estimate the size of a molecule

Apparatus/Required
Cooking oil (solute), solvent, clean glass trough, water, burette and lycopodium powder.

Procedure

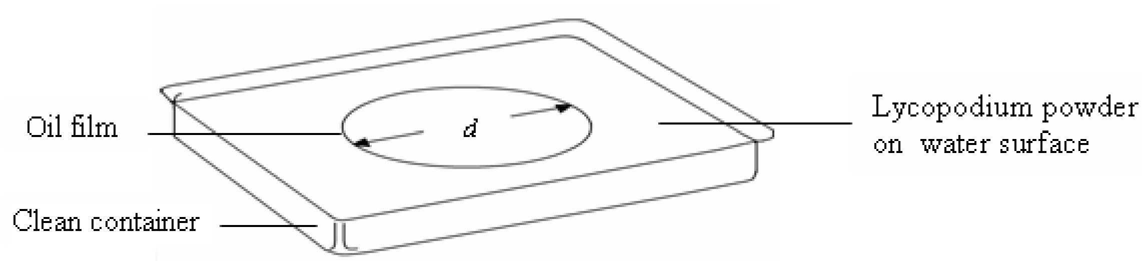
(i) Dissolve a small volume, x cm^3, of vegetable oil (*solute*) in a larger volume, v cm^3, of petroleum ether (*solvent*) to form a total volume, V cm^3, of oil-petroleum ether solution.

(ii) Fill a clean (grease free) trough with water.

(iii) Sprinkle lycopodium powder uniformly to cover the surface of the water in the trough.

(iv) Drop a small volume, y cm^3, of the oil-petroleum ether solution from a burette on to the water surface covered with the lycopodium powder. The oil repels the powder leaving a clear oil film.

(v) Measure and record the diameter, d, of the circular oil film.



Calculations

(a) To find the volume of the oil film

Volume of oil (solute) dissolved = x cm^3

Volume of petroleum ether used = v cm^3

Vol. of solution dropped on water = y cm^3

Total volume of solution, V, = $(x + y)$ cm^3

If the total volume, $(x + y)$, cm^3 of oil-petroleum ether solution contains x cm^3 of oil, then the volume, V, of oil contained in y cm^3 $= \left(\frac{x}{(x+y)}\right) \times y$ cm^3

Note *that from the above expression, we can write the form for calculating the volume of oil film as:*

$$\textbf{Volume of the oil film, V} = \left(\frac{\textit{Vol. of oil dissolved}}{\textit{Total vol. of solution}}\right) \times \textit{Vol. of solution dropped}$$

(b) To find the thickness, *t*, of the oil film (i.e. size of a molecule)

The thickness, *t*, is found by using any of the following expressions for finding volume of a cylinder.

(i) $V = At$ from Volume = Base area (A) x thickness (t)

(ii) $V = \pi r^2 t$ from $A = \pi r^2$

(iii) $V = \frac{1}{4}\pi d^2 t$ from $2r = d,\ r = \frac{d}{2},\ \therefore\ r^2 = \left(\frac{d}{2}\right)^2 = \frac{d^2}{4}$,

$\therefore V = \frac{1}{4}\pi d^2 \boldsymbol{t}$

Substituting for V in the above formulae, we have the following expressions.

V in (i) $At = \left(\frac{x}{(x+y)}\right) \text{ x } y \qquad \therefore\ t = \frac{1}{A}\left(\frac{x}{(x+y)}\right) \text{ x } y\ cm$

V in (ii) $\pi r^2 t = \left(\frac{x}{(x+y)}\right) \text{ x } y \qquad \therefore\ t = \frac{1}{\pi r^2}\left(\frac{x}{(x+y)}\right) \text{ x } y\ cm$

V in (iii) $\frac{1}{4}\pi d^2 t = \left(\frac{x}{(x+y)}\right) \text{ x } y \qquad \therefore\ t = \frac{4}{\pi d^2}\left(\frac{x}{(x+y)}\right) \text{ x } y\ cm$

Assumptions made in the estimation of the size of a molecule

(i) The oil film is assumed to be one molecule thick.

(ii) Each molecule is assumed to be a perfect sphere.

(iii) The spaces between the molecules in the oil film are assumed to be negligible.

(iv) The oil film is assumed to form a complete circle.

Example 1

1 cm^3 of oleic acid was dissolved in 999 cm^3 of alcohol to form 1000 cm^3 of solution. A 1 cm^3 drop of the solution was put on a water surface sprinkled with lycopodium powder. The alcohol dissolved in the water leaving the acid to spread forming a patch of diameter of 28 cm^3.

(a) Calculate the volume of oleic acid in the 1 cm^3 drop of the solution.

(b) Estimate the size of oleic acid molecule.

(c) Why was lycopodium powder used?

Solution

Volume of oleic acid (solute) dissolved = 1 cm^3

Volume of alcohol (solvent) = 999 cm^3

Volume of solution dropped on water = 1 cm^3

Total volume of solution, of solution = (1 + 999) cm^3

= 1000 cm^3

If the total volume, 1000 cm^3 of oleic-alcohol solution contains 1 cm^3 of oleic acid, then the volume, 1 cm^3 of oleic acid , of oil contained in 1 cm^3

$$= \left(\frac{1}{1000}\right) \text{ x } 1\ cm^3 = \frac{1}{1000} = 0.001\ cm^3$$

The volume of the oleic acid in 1 cm^3 of the solution = 0.001 cm^3.

(b) **Data**: d = 28 cm, volume, V, of oleic acid = 0.001 cm^3, $\pi = \frac{22}{7}$, t = ?

Using the formula $V = \frac{1}{4}\pi d^2 t$

$$0.001 = \frac{1}{4} \times \frac{22}{7} \times 28^2 \times t$$

$$t = \frac{0.001 \times 4 \times 7}{22 \times 28 \times 28}$$

$$= \frac{0.028}{17{,}248}$$

$$\therefore \boldsymbol{t} \mathbf{= 1.62 \times 10^{-6}\ cm\ or\ 1.62 \times 10^{-8}\ m}$$

Alternatively you can first find the radius and then substitute the value in the formula

$V = \pi r^2 t$ From d = 28 cm, r = ½d $= \frac{1}{2} \times 28 = 14$ cm

$$V = \pi r^2 t$$

$$0.001 = \frac{22}{7} \times 14 \times 14 \times t = \frac{0.001 \times 7}{22 \times 14 \times 14} = \mathbf{1.62 \times 10^{-6}\ cm\ or\ 1.62 \times 10^{-8}\ m}$$

(c) *To give clear circular patch of the oleic film.*

Self-Check Question 4.1

1. An oil drop of volume 10^{-3} cm^3 forms a patch of an area of 0.785 cm^2 on water surface during an experiment to estimate size of a molecule. If the film is 1 molecule thick, what is the size of the molecule?
A. 4.06×10^{-4} cm B. $7.85\text{X}10^{-4}$cm C. $9.53\text{x}10^{-4}$cm D. $1.27\text{X}10^{-3}$cm

2. The diagram in figure 4 shows an arrangement for determining the size of an oil molecule.

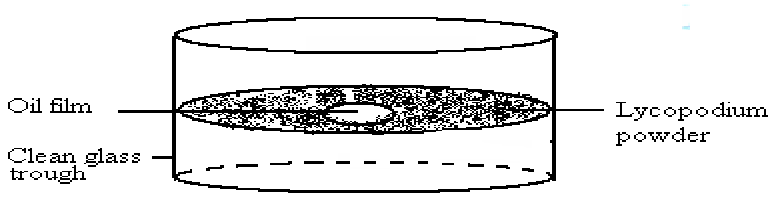


Fig. 4

(a) State two assumptions made in the experiment
(b) If 1.8 x 10^{-4} cm^3 of oil spreads to form a patch of area 150 cm^2.
Calculate the thickness

3. A solution is made by dissolving 1 cm^3 of cooking oil in 199 cm^3 of the methanol. When 0.004 cm^3 of the solution is dropped on the surface of water, an oil film of diameter 12 cm is obtained.

(i) Calculate the volume of the cooking oil in the film.

(ii) Estimate the thickness of a molecule of the cooking oil

(iii) State any assumption made in (b) (i).

4. In an experiment to estimate the size of a molecule of olive oil, 0.12 mm^3 of the solution was dropped on a clean water surface in a trough. The oil spreads to form a circular patch of an area of 1.0×10^4 mm^2. Estimate the size of a molecule of olive oil.

5. Suppose an oil drop on water surface sprinkled with lycopodium powder has a volume of 0.10 mm^3 and forms a film of an approximate radius 10 cm, calculate the thickness of the oil film.

CHAPTER FIVE
FORCE

LEARNING OBJECTIVES

By the end of this chapter, you should be able to:

1. (a) Define: - Force.
(b) State and define: the SI unit of force.
2. List the types of forces.
3. State the effects of force.
4. (a) Define acceleration due to gravity
(b) Explain why acceleration due to gravity varies from place to place
(c) Describe an experiment to determine acceleration due to gravity, g.
5. (a) Define: - Mass and Weight.
(b) State: - The difference between mass and weight.
(c) Use: - W = mg, to solve numerical problems.
6. Define: - Scalar and vector quantities and give examples for each.
7. Solve numerical problems involving combination vectors (forces).

5.1 Force

Force is either a push or pulls that acts on an object. Or

Force *is that which changes a body's state of rest or uniform motion in a straight line*.

A force cannot be seen and described like an object, but the effects of force on an object can be seen. Just like we can not see wind but we can feel and see the effects in a flying flag or kite or in a tree bending.

(a) SI Unit of force

The SI unit of force is called *Newton* (N) and is defined as follows:

A Newton is the force required to give a unit mass (mass of one kilogram) an acceleration of one metre per Second Squared.

(b) Representation of force

Force is a vector quantity i.e. it has both magnitude and direction.
It is represented by a straight line with an arrow at the tip showing the direction in which it acts.

$\longrightarrow$ ***F*** N

(c) Effects produced by a Force

A force acts in a particular direction and may have any of the following effects on an object.

It can:
(i) Make a stationary object to move.
(ii) Increase the speed of a moving object.
(iii) Decrease or slow down the speed of a moving object or bring a moving object to a rest.
(iv) Change the direction of a moving object.
(v) Deform (change the shape of) an object.

(d) Types of Forces

There are various types of forces; some of the common ones include the following:

(i)	Gravitational force	- *is the force which pulls a body towards the centre of the earth.*
(ii)	Frictional force	- *is the force which opposes relative motion between two surfaces in contact.*
(iii)	Centripetal force	- *is a force which constrains a body to move in a circular path or orbit. Its direction is towards the centre of the circle.*
(iv)	Magnetic force	- *is a force in magnets that causes motion as a result of attraction or repulsion.*
(v)	Electrostatic force	- *is a force that causes attraction or repulsion in an electric field due to static charges.*
(vi)	Elastic force	- *is a force in a stretched spring or rubber cord.*
(vii)	Upthrust	- *is an upward force that acts on a body immersed in a fluid (a liquid or a gas).*
(viii)	Cohesive force	- *is a force of attraction between molecules of the same kind. E.g force of attraction between water molecules.*
(ix)	Adhesive force	- *is a force of attraction between molecules of different kinds. E.g force of attraction between water molecules and glass molecules.*
(x)	Surface tension	- *is the force on a liquid surface which makes the liquid surface to behave like a stretched elastic membrane or skin.*

5.2 (a) Gravitational force

An object released from a height falls down to the ground. This indicates that there is a force acting on the object directed to the centre of the earth. This downward pull is called *gravitational force*.

Gravitational force is the force which pulls a body towards the centre of the earth.

It makes unsupported objects to fall until they reach the ground. The force of gravity pulls all objects at a particular place on the surface of the earth with the same acceleration of free fall called *acceleration due to gravity*.

(b) Acceleration due to gravity, *g*

Acceleration due to gravity, *g*, *is the acceleration with which the force of gravity pulls all objects on or near the surface of the earth towards its centre.*

The average value of ***g*** on the surface of the earth is 9.8 ms^{-2}, but it varies slightly from place to place.

(c) The variation of *g*

The acceleration due to gravity is not constant everywhere. It varies from place to place. For example g is 9.78 ms^{-2} at the equator, whereas at the poles it is 9.83 ms^{-2}. There are two main causes of this variation.

(i) Variation of g with altitude

Acceleration due to gravity depends on the distance from the centre of the earth. The earth is not a perfect sphere. It is oval in shape; as such places are far away from the centre of the earth while others are near to the centre of the earth. For example the *equatorial radius* of the earth is greater than the *polar radius*. That is the distance from the centre of the earth to the equator is greater than the distance from the centre of the earth to the pole. Therefore a body at the equator is slightly further away from the centre of the earth and consequently feels a smaller gravitational attraction. While a body at the poles is nearer to the centre of the earth and therefore experiences a greater gravitational force.

(ii) Rotation of the earth

Because the Earth rotates, its gravitational pull on the body at the equator provides the body with a centripetal acceleration. This effect does not apply at the pole.

In other words, we say that: The value of the Acceleration due to gravity, *g*, depends on the distance from the centre of the earth to that place. The smaller the distance, the higher the value of *g,* and vice versa. Since the earth is not a perfect sphere, the distances from the centre of the earth to different points on the surface of the earth is different. Therefore the value of, g, varies from place to place. Thus, in places near to the centre of the earth like the north and south poles, the value of, g, is greater than at equator. It is also greater at sea level than at higher altitudes such as on top of mountains.
The acceleration due to gravity on the moon is about one-sixth of that of the earth.

(i.e. $\frac{1}{6}$ x 9.8 = 1.6 ms^{-2}).

5.3 Mass and Weight of a Body

(a) Mass

Mass is the quantity of matter in a body.
It is constant everywhere. I.e. does not vary from place to place.

SI unit of mass

The SI unit of mass is the kilogram (kg).

(b) Weight (W)

Weight is the gravitational pull on a body by the earth. Or *Weight is a measure of the pull of gravity on a body.*
The direction of the weight of a body is always towards the centre of the earth.

SI unit of weight

Weight is a kind of force; therefore its SI unit is *Newton*.

Weight is calculated from the formula:

Weight = Mass x Acceleration due to gravity

In symbols, $\boldsymbol{W = mg}$

The weight of a body varies from place to place.

Explanation

From the formula W = mg, the weight of a body is directly proportional to the acceleration due to gravity. And since the *g* varies from place to place due to the variation in the distance from the centre of the earth, so does the weight of a body.

Note: Basing on the above explanation, it means that in places near to the centre of the earth like the *North Pole* and *South Pole*, the weight of a body is greater than when at the equator. It is also greater at sea level than at places higher than the sea level.

(c) The Difference between Weight and Mass

Mass	Weight
(i) Quantity of matter in a body	- The pull of earth on a body
(ii) Constant everywhere	- Varies from place to place
(iii) SI unit is kilogram (kg)	- SI unit Newton (N)
(iv) A scalar quantity	- A vector quantity
(v) Measured using beam balance or spring balance or electric balance calibrated in grams or kilograms	- Measured using spring balance or electric balance calibrated in Newtons.

Worked Examples

1. Calculate the weight of the following:
   (a) A box of mass 50 kg (b) A boy of mass 2.5 kg
   (c) A bull of mass 200 kg (d) A stone of mass 12 g

2. Find the weight of an astronaut whose mass is 75 kg on:
   (a) The earth
   (b) The moon (Take $g = 10\ ms^{-2}$ and g on the moon is $\frac{1}{6}^{th}$ of g the earth)

3. Find the masses of the weights: (a) 400 N (b) 10 N

Solution

1. (a) m = 50 kg, g = $10\ ms^{-2}$, W = ?

$$W = mg = 50 \times 10$$
$$\therefore \mathbf{W = 500\ N}$$

(b) m = 2.5 kg, g = $10\ ms^{-2}$, W = ?

$$W = mg = 2.5 \times 10$$
$$\therefore \mathbf{W = 25\ N}$$

(c) m = 200 kg, g = $10\ ms^{-2}$, W = ?

$$W = mg = 200 \times 10$$
$$\therefore \mathbf{W = 2000\ N}$$

(d) m = 12 g, = $\frac{12}{1000}$ kg, g = $10\ ms^{-2}$, W = ?

$$W = mg = 0.012 \times 10$$
$$\therefore \mathbf{W = 0.12\ N}$$

2. (a) m = 75 kg, g = $10\ ms^{-2}$, W = ?

$$W = mg = 75 \times 10$$
$$\therefore \mathbf{W = 750\ N}$$

(b) m = 200 kg, g = $10\ ms^{-2}$, W = ?

$$W = mg = 75 \times \frac{1}{6} \times 10 = 120\ N$$
$$\therefore \mathbf{W = 125\ N}$$

3. (a) W = 400 N, g = $10\ ms^{-2}$, m = ?

$$W = mg$$
$$400 = m \times 10$$
$$m = \frac{400}{10}$$
$$\therefore \mathbf{m = 40\ kg}$$

(b) W = 10 N, g = $10\ ms^{-2}$, m = ?

$$W = mg$$
$$10 = m \times 10$$
$$m = \frac{10}{10}$$
$$\therefore \mathbf{m = 1\ kg}$$

Self-Check 5.0

1. What is the mass of a man on the earth if his mass on the moon 60kg.

A. 6kg B. 10kg C. 60kg D. 360kg

2. Assume that you are taking measurements with a spring balance (dynamometer), where can you get the greatest reading for the same object?

A. At the centre of the earth B. On the moon

C. At the equator D. At the poles.

3. What is the name of any push or pull exerted?

A. Mass B. Force C. Friction D. Tension

4. What do we call the pull of gravity on an object?

A. Mass B. Weight C. Moment D. Tension

5. Which one of the following is **not** a measuring tool?

A. Equal-arm balance B. Dynamometer C. Lever D. Ruler

6. Which one of the following is the unit of weight?

A. Newton B. Kilogram C. Meter D. Ton

7. A mass of 60kg weighs 600N on the earth and 100N on the moon. What is the mass and weight of an object on the earth if it weighs 50N on the moon?

A. 60kg mass, 600N weight B. 10kg mass, 60N weight

C. 30kg mass, 300N weight D. 5 kg, mass, 100N weight

8. Which one of the following is a force?

A. Energy B. Mass C. Weight D. Speed

9. Which one of the following statements is **not** correct?

A. Force can change the speed of an object.

B. Force can change the shape of an object.

C. Force can change the direction of motion.

D. Force can change the mass of an object.

10. Which one of the following are SI units of mass and weight?

A. g and n respectively B. N and kg respectively

C. kg and g respectively D. kg and N respectively

5.4 FRICTION

Friction is a force that opposes relative motion between two surfaces in contact with one another.
It results from two surfaces rubbing against each other or moving relative to one another.
It can hinder the motion of an object or prevent an object from moving at all.

The strength of frictional force depends on:

- The nature of the surfaces that are in contact,
- The magnitude of the force pushing them together and
- The weight of the object or objects.

In cases involving fluid friction, the force depends upon

- The shape of the object moving through the fluid,
- Viscosity and
- Speed of an object as it moves through the fluid.

(a) The Causes of Friction

Friction is thought to be caused by:

(i) Small irregularities on the rough surfaces becoming interlocked. E.g. rubbing two files together.

(ii) The attractive force between the surface molecules. This factor seems to play major role than interlocking of the small irregularities.

(b) Types of friction

There are two types of friction, namely;

(i) Static friction and

(ii) Dynamic (Sliding or kinetic) friction.

1. Static friction

Static friction is the frictional force between two surfaces that are not sliding over each other.

(a) Facts about Static Friction

- Static friction occurs between stationary objects.
- It prevents an object from moving against a surface.
- It is the force that keeps an object e.g. a book from sliding off a desk, even when the desk is slightly tilted, and that allows you to pick up an object without the object slipping through your fingers.
- Static friction depends on:
  - The coefficient of static friction (μ_s) between the surfaces of the objects in contact,
  - The nature of the surface and
  - The normal reaction (R) of the object.

NB: *In order to move something, you must first overcome the force of static friction between the object and the surface on which it is resting.*

Experiment 5.1 To Measure Static friction

Static friction can be measured using the apparatus shown in figure 5.1 below

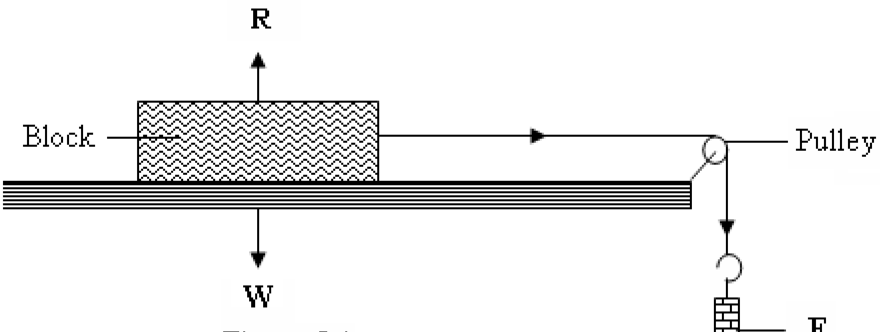


Figure 5.1

- Place a wooden block on a table and connected it by a light string passing over a smooth pulley on to a mass hanger.
- Gradually increase force, **F**, by first adding 50 g and when the block is about to slide, continue adding small standard (known) 20 g or 10 g on to the mass hanger until the block just begins to slide or move.
- Read and record force, **F,** (the total mass on the mass hanger) at the point when the block just begins to slide.

Observations

At first the block remained at rest as the force is increased.

After some time, i.e. at certain value of the force on the pan, the block just begun to slip or slide in the direction shown on the diagram.

Explanation

When force, **F,** on the mass hanger was increased the frictional force that opposes the motion of the block also increases.

As more and more weights were added to the mass hanger, the frictional force reached its maximum value for the two surfaces in contact and begun to slide.

Result

The maximum value of the frictional force is equal to the total weight, **F**, on the mass hanger. This maximum frictional force is called *limiting friction.*

(b) Limiting friction

Limiting friction *is the force of friction between two surfaces when they are at the verge of sliding over each other.*

The force of friction between an object and a surface can be calculated from the formula:

$$F = \mu R$$

Where: F = the force of friction,

μ = the coefficient of friction between the object and the surface,

and R = the normal reaction.

(c) Factors which determine the limiting friction

Limiting friction depends on:

(i) The coefficient of static friction (μ_s) between the object,

(ii) The nature of the surface and

(iii) The normal reaction (R) of the object.

These factors are demonstrated by attaching a spring balance calibrated in newtons to two blocks of wood of different weights as shown in the examples A and to the same block B below.

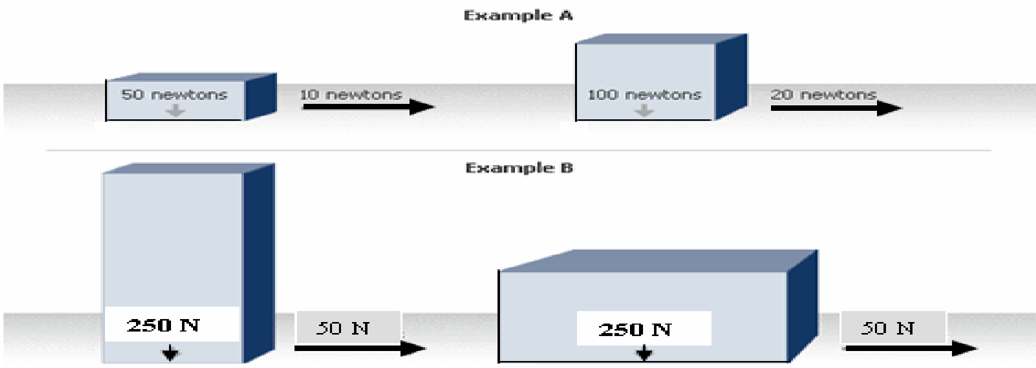


Figure 5.2

In example A, illustrates that limiting friction depends on weight. It shows that if it takes 10 N to slide a block of weight 50N on the floor, it will take 20 N of force to slide a block of weight twice that of 50 N (i.e.100 N).

While example B illustrates that friction is independent of the area in contact. That is it does not depend on the size of surface area in contact between the objects. It shows that the limiting friction required to move 250 N for the different surface areas.

(d) Coefficient of static friction

Coefficient of static friction is the ratio of limiting friction to the normal reaction between surfaces. It is denoted by a symbol, μ, pronounced mew.

Experiment 5.2 To determine the value of coefficient of friction, μ

Apparatus: A wooden block, a string, mass hanger, heavy standard weight (masses), slotted masses of 50 g and 10 g, a smooth pulley and a weighing balance.

Procedure

Arrange the apparatus as shown in figure 5.3 below.

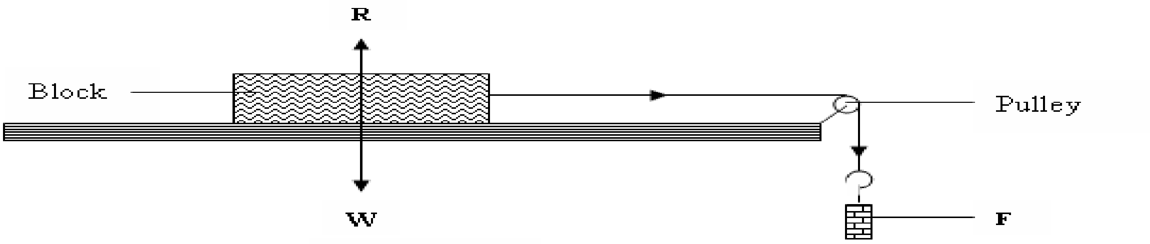


Figure 5.3

a) Weigh the block of wood and record its mass, M_b, in kg.
b) Gradually increase force, **F**, by first adding 50 g and when the block is about to slide, continue adding small standard (known) 20 g or 10 g on to the mass hanger until the block just begins to slide or move.
c) Record the total mass, **m**, hence the force **F**, on the mass hanger.

d) Load the block of wood by adding a heavy standard mass M = 1 kg.
e) Repeat the procedure (b) to (c).
f) Now repeat the procedures (b) to (d) for values of M = 2, 3, 4, 5 and 6 kg.
g) Enter your results in the table 5.1 below.

Table of results

(M_b + M)/kg	**W =(M_b + M)g = R /N**	**m/kg**	**F = mg/N**
M_b + 0			
M_b + 1			
M_b + 2			
M_b + 3			
M_b + 4			
M_b + 5			
M_b + 6			

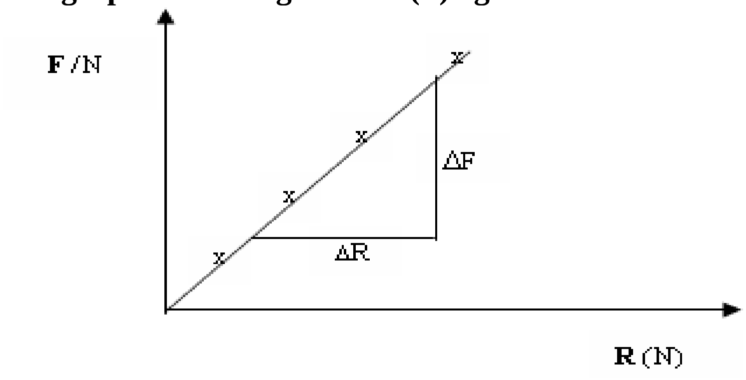
Table 5.1

h) Plot a graph of F against R (normal reaction = W).
i) Find the slope of the graph.

Note that the value of **F** is equal to the total weight on the mass hanger which is equal to the *limiting friction* for each total weight.

If the experiment is accurately performed, the graph in the figure below is obtained.

The graph of limiting friction (F) against normal reaction



The slope of the graph is called the *coefficient of friction, μ, and is given by the formula.*

$$\text{Coefficient of static friction,} \quad = \frac{Change\ in\ Limiting\ \ friction\ (F)}{Change\ in\ Normal\ \mathrm{Re}\,action(R)}$$

$$\mu_s \quad = \frac{\Delta F}{\Delta R}$$

$\therefore$ *Limiting friction,* $\quad \boldsymbol{F} \quad \boldsymbol{=} \quad \boldsymbol{\mu R}$

Note that:

- *Since normal reaction R, is always greater than the limiting friction F, i.e. $R > F$, the value of μ is always less than 1.*

Worked Examples

Example 1

A block of wood of mass 5 kg is placed on a table top. Find the limiting friction if the coefficient of friction is 0.5. (Take g = 10 m/s^2).

Solution

Data: $\mu = 0.5$, $m = 5$ kg, $g = 10$ m/s^2, Limiting friction, F = ?, R = ?

Hint: *Limiting friction is calculated from the formula F* **= *μR.*** *But R is not given in the data. This should not make the whole thing difficult to you. Always remember that R = W. So using the formula W = mg, find the value of W and then finally use the formula* ***F* = *μR*** *to get F as shown below.*

First calculate W: W = mg

= 5 x 10

= 50 N

From Weight = Reaction $\therefore$ R = 50 N

Now that you have known R, you can now use the formula;

Limiting friction, $F = \mu R$

= 0.5 x 50

$\therefore$ **F = 25 N**

Self-check

Try to check your understanding by answering the following questions.

1. A chalk box of mass 400 g is placed on a table. Find the limiting friction if *$\mu = 0.2$, ($g = 10\ ms^{-2}$).* ***Answer: 0.8 N***

2. A block of wood of mass 2 kg is placed on a table. Find the limiting friction if *$\mu = 0.4$, ($g = 10\ ms^{-2}$).* ***Answer: 8 N***

2. Kinetic or Sliding friction

The kinetic or sliding friction is the frictional force between any two surfaces that are moving relative to each other.

(a) Facts about kinetic friction

- ❖ It resists the motion of an object as it moves along a surface.
- ❖ Sliding friction occurs between objects as they slide against each other.

(b) Factors on which Sliding friction depends

Like the limiting friction, sliding friction depends on:

- The weight of the sliding object,
- The nature of the surfaces in contact and
- The coefficient of kinetic friction.

Experiment 5.3 To Measure sliding friction

Sliding or kinetic friction can be measured using the apparatus shown in figure 5.4 below

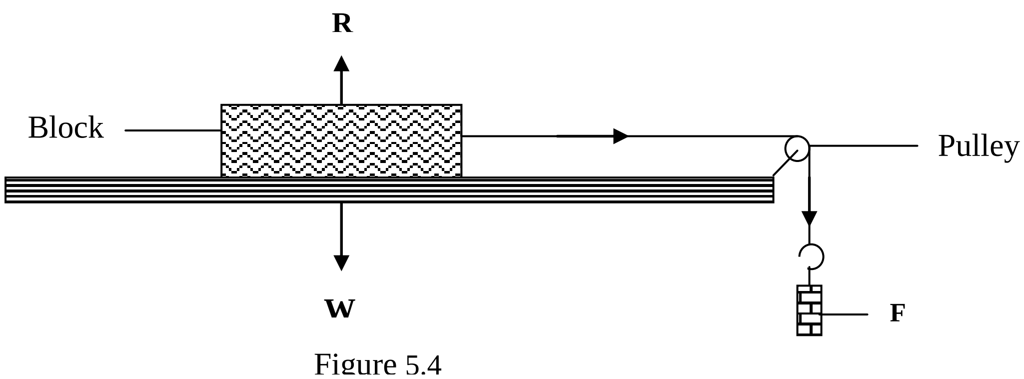


Figure 5.4

- Place a wooden block on a table and connected it by a light string passing over a smooth pulley on to a mass hanger.
- Gradually increase force, **F**, by first adding 50 g and 20 g or 10 g on to the mass hanger until the block is about to slide.
- Give the block a push.
- Read and record force, **F,** (the total mass on the mass hanger).

Result

The maximum value of the frictional force is equal to the total weight, **F**, on the mass hanger. This maximum frictional force is called *sliding or kinetic friction,* $\mathbf{F}_k$.

Note that:

1. *The limiting friction is always greater than the sliding friction.*
2. *The procedure for the measurement of kinetic friction and the coefficient of kinetic friction, μ_k, are the same as for static friction. The only difference is that a push is required for the determination of the kinetic friction at the point when the body is about to slide.*

5.6 The angle of friction

When a block of wood is placed on an inclined plane at an angle of friction α, such that the block is just on the point of slipping as shown in the diagram below.

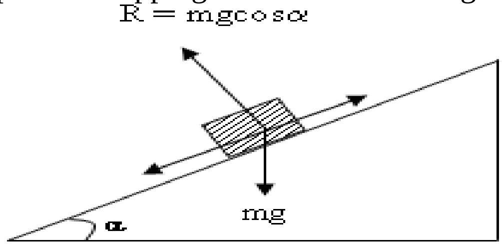


Figure 5.5

From Coefficient of friction, $\mu = \dfrac{Limiting\ friction}{Normal\ Reaction\ between\ surfaces}$

$$\mu = \frac{F}{R}$$

$$F = \mu R \qquad \text{.................................} \quad (1)$$

$$\mu = \frac{mg\sin\alpha}{mg\cos\alpha} \qquad \text{But } \frac{mg\sin\alpha}{mg\cos\alpha} = \text{Tan}\ \alpha$$

$$\therefore \mu = \text{Tan}\ \alpha \qquad \text{.................................} \quad (2)$$

Note: That the tangent of the angle of friction is equal to the coefficient of friction.

Substituting equation (1) in equation (2) we have,

$$\boldsymbol{F = Tan\alpha R}$$

Note: *Do not worry if you do not understand the steps in the derivation of the above formula. You will understand it when you cover trigonometry in mathematics in S.2.*

5.7 (a) Laws of friction

(i) Friction always opposes motion

(ii) Friction depends on the nature of the surface in contact.

(iii) Friction - is much less on smooth surface and is much greater on rough surface.

(iv) The limiting friction and sliding friction are both proportional to the normal reaction i.e. $F \propto R$

(b) Advantages of Friction

Friction is useful in our daily life. This is seen in the following:

1. **Motion**

Friction allows motion of objects, as seen in the following:

(i) *Walking* Friction between the foot and the ground allow a person to walk.

In a slippery place, there is no friction. The result is that movement becomes difficult.

(ii) *Clutch* Friction between clutch plates allows the engine to drive the wheels.

(iii) *Tyres* Friction between the tyres and the road surface allows the torque of the wheels to push cars or bicycles along the road.

(iv) *Brakes* Friction between the brake lining and the brake drum halts the vehicle.

2. **Ladder**

Friction between the foot of the ladder and the ground prevents it from slipping off.

3. **Lubrication**

Liquid friction helps to spread oil round an axle.

However, despite these advantages, friction has disadvantages as listed below.

(c) Disadvantages of Friction

(i) Friction between the moving parts of machines reduces the efficiency of machines and often causes much of the work done by the effort to be wasted.

(ii) It causes surfaces to wear off. E.g. tyres and soles of shoes.

(iii) It also causes unnecessary noise and heat.

Where friction becomes a disaster, we can reduce it by applying some methods.

5.8 Ways of reducing friction

1. Lubrication

In lubrication, oil or grease is introduced between surfaces that sliding over one another. The oil film keeps rough surfaces apart, thus preventing them from coming into direct contact.

2. Ball bearings

Bearing is a mechanical device for decreasing friction in a machine in which a moving part bears—that is, slides or rolls while exerting force—on another part.

Steel ball bearings are used for example in an axle and shaft of the bicycle then the axle and shaft. When the ball bearings are used, the axle and shaft do not slide but roll over one another. *Rolling friction* is less than the sliding friction; therefore friction is lessened by means of ball bearing.

3. Rollers Friction can be reduced by placing rollers between the two surfaces in contact.

4. **Smoothening of surfaces in contact**
   Friction can also be reduced by making rough surfaces smooth where necessary.

5. **Streamlining** Streamlining refers to shaping of an object such that the layer of air easily slides off. Notable examples are seen in:
   - aeroplane, submarines, boats, fish, birds etc.

5.9 Fluid Friction

Fluid Friction is a retarding force which acts on a body moving through a fluid.

It is due to the internal friction existing between layers of fluids of a liquid in motion. It is caused by the attraction between the molecules of one layer and the molecules of another layer. The frictional resistance of a fluid (liquid or gas) opposing motion of a body moving through it is called viscosity.

Factors which determine the magnitude of fluid friction

1. Unlike friction between solids' viscosity is proportional to the surface area and the velocity of the object moving through the fluid.
2. *Nature of fluid*. Viscosity depends on the type of fluid for example glycerin, engine oil, and natural honey flow much less easily from a vessel than liquids such as petrol, paraffin and water.
3. *Temperature*: Viscosity of liquids decreases as a temperature of liquid increases while that of gases increases with temperature.

NB: *Liquids which have high viscosity e.g. glycerin, engine oil are called vicious liquids and are used to make lubricants.*

5.10 Scalar and Vector Quantities

(a) Scalar Quantity

A scalar quantity is a quantity which has only magnitude. They only give the amount of things. Examples scalar quantities are:

- Mass	e.g.	5 kg of sugar
- Time		2 hours
- Volume		4 litres
- Density		$0.8 g/cm^3$
- Temperature		50 ºC

(b) Vector Quantity

A vector quantity is a quantity which has both magnitude and direction. Examples of vector quantities are:
- Force or weight,
- Velocity,
- Acceleration,
- Momentum.

5.11 Representation of vectors

Vectors are represented by an arrow headed line drawn to scale is called a vector. A vector has the following properties:

- An application point (where it is applied).
- A magnitude (size i.e. how big or small it is).
- A direction.

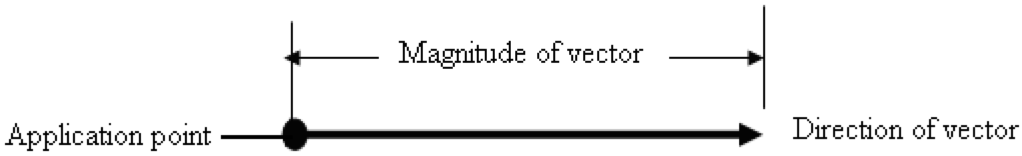


Figure 5.6

5.12 Addition of Forces

When two or more forces act on a body their combined effect called, *resultant force.* The resultant force on the body depends on their relative directions, (i.e. the angle, θ, between the forces).

Resultant force

Resultant force is a single force which has an effect of two or more forces acting at the same point.

The forces which form a resultant force are called *component forces*.
Resultant force is found by using the following equation:

Resultant force = Sum of forces

$$\text{I.e} \quad \overrightarrow{F_{r\,r}} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} + \ldots$$

Points to note: Since force is a vector, the direction is important. Therefore,

- ❖ If force(s) to the right are taken to be positive, then the one(s) to the left is/are negative. Like wise,
- ❖ If force(s) acting to north or east is/are taken to be positive, then the one(s) acting to south and west respectively is/are taken to be negative.
- ❖ The number of **F**s to the right of the above formula = to the number of forces acting on a body.

(a) Forces acting in a straight line

The resultant force due to forces acting in a straight line is got by addition or subtraction of the forces depending on their directions.

(i) Two forces acting on a straight in the same direction (θ = 0°)

Consider two forces F_1 and F_2 acting on body P as shown in figure below.

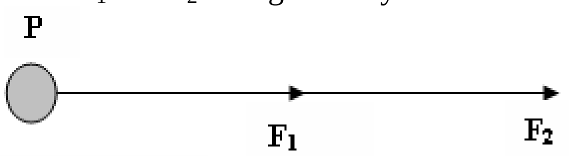
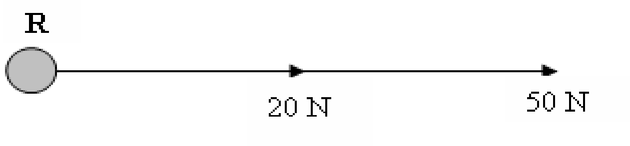


Figure 5.7

Since the forces are acting in the same direction, they are taken to be positive.
The resultant force, R, is got by addition of the forces.

$$\text{i.e} \quad \overrightarrow{F_r} = \overrightarrow{F_1} + \overrightarrow{F_2}$$

Example 1
Two forces 20 N and 50 N act on body R in the same direction as shown in the diagram below. Find the resultant force.



Data: $F_1 = 20$ N, $F_2 = 50$ N, $F_r = ?$

$$F_r = \vec{F_1} + \vec{F_2}$$
$$= 20\text{ N} + 50\text{ N}$$
$$\therefore \mathbf{F_r} = \mathbf{70\ N}$$

(ii) Two forces acting on a straight in opposite directions (θ = 180°)

Consider two forces F_1 and F_2 acting on body M in opposite directions as shown in figure 4.8 below.

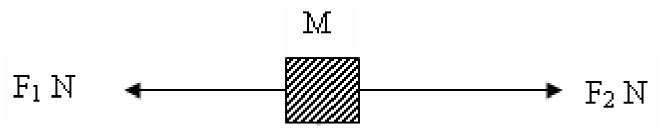


Fig. 5.8

Since the forces are acting in opposite directions, one is taken to be positive and the other negative.

Let F_1 be positive and F_2 be negative.

The resultant force, given by using the equation:

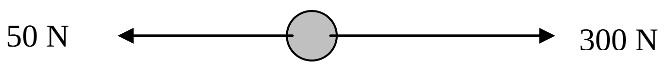
$$F_r = F_1 + F_2$$
$$= F_1 + {}^{-}F_2$$

$$\therefore F_r = F_1 - F_2$$

Therefore, the resultant force, F_r, $= (F_1 - F_2)$ in the direction of F_1.

Example 2
Two forces 300 N and 50 N are acting on a body Q as shown in figure 4.9 below. Find the resultant force.

Solution



$F_1 = 300$ N, $F_2 = 50$ N

Since the forces are acting in opposite directions, one is taken to be positive and the other negative.

Let 300 N be positive and 50 N be negative.

The resultant force, given by using the equation:

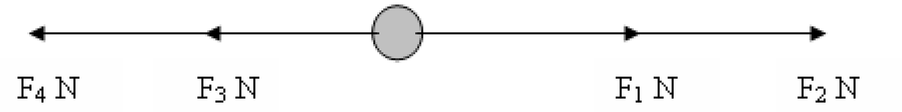
$$\begin{aligned} F_r &= F_1 + {}^-F_2 \\ &= 300 + -50 \\ &= 300 - 50 \\ \therefore \mathbf{F_r} &= \mathbf{250\ N} \end{aligned}$$

Therefore the resultant force is 250 N in the direction of 300 N (to the right).

NB: *Instead of taking one force to be positive and the other negative, you may simply subtract the smaller force from the larger one.*

(iv) More than two forces acting on a straight in opposite directions (θ = 180°)

Consider four forces F_1, F_2, F_3 and F_4 acting on body S as shown in the diagram below. Find the resultant force.



Solution

For the case such as above where there are more than two forces acting on a body in opposite direction, follow the following steps.

Step I: Get the resultant forces in the directions shown in the diagram above.

Resultant force *towards right*, $R_r = (F_1 + F_2)$ N

Resultant force *towards left*, $R_l = (F_3 + F_4)$ N

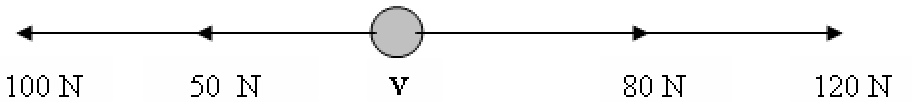
Step II: Get the final resultant force, R, of the two resultant forces above.

Resultant force, $R = R_r - R_l$

$= [(F_1 + F_2) - (F_3 + F_4)]$ N

Example 3

Four forces are acting on body V as shown in the diagram below. Find the resultant force.



Solution

Resultant force acting towards right R_r $= (F_1 + F_2)$ N
$= 80\text{ N} + 120\text{ N}$
$= 200\text{ N}$

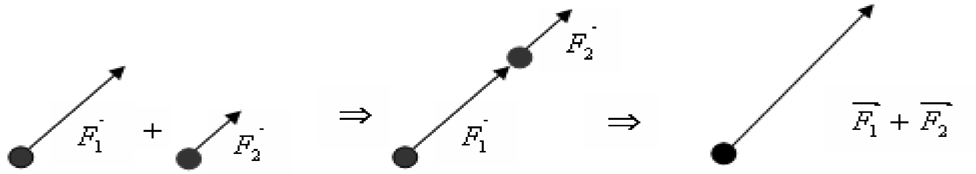
Resultant force acting towards left R_l $= (F_3 + F_4)$ N
$= 100\text{ N} + 50\text{ N}$
$= 150\text{ N}$

Resultant force, R, acting on V $= R_r - R_l$
$= 200\text{ N} - 150\text{ N}$

= 50 N acting towards right.

(b) Parallel forces

The resultant vector for two or more vectors can be represented by a *head-to-tail* addition of arrows. For example if two or more forces act in the same direction, the resultant force is obtained as shown below.



***NB**: The little arrow over F indicates that we are dealing with vector (a quantity for which direction is important).*

c) Forces acting at right angles to each other (θ = 90°)

When two forces act on a body at an angle to each other, the resultant force can be obtained by applying any of the following laws.

- The triangle law of forces and
- The parallelogram law of forces.

(i) Triangle law of forces

The law states that "*if a body is acted upon by two forces that are at an angle to each other, the resultant of the forces is the third line drawn to complete the triangle".*

For example, if two forces of equal or unequal magnitude, one directed due east and the other directed due north act on an object at rest but free to move in the direction of the resultant force, the magnitude of the resultant force is shown by the length of the arrow representing the resultant force.

Consider the diagram in figure 5.9 below in which two forces F_1 and F_2 act on a body. F_1 acts due north and F_2 act due east as shown below.

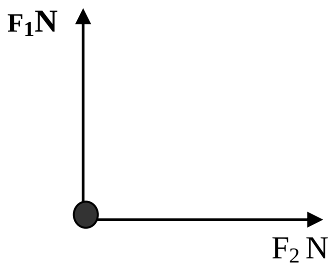


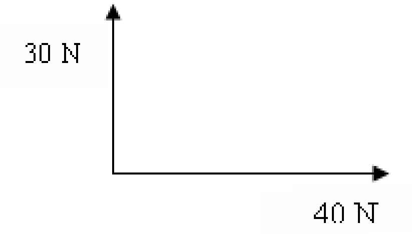
Figure 5.9

Using the method of *head-to-tail* addition of vectors at right angle, the right angle triangle is completed by using the tail of F_1 on the head of F_2 as shown in the diagrams below and the resultant force, $F_{r,}$ is got by applying Pythagoras theorem as shown in the diagrams below.

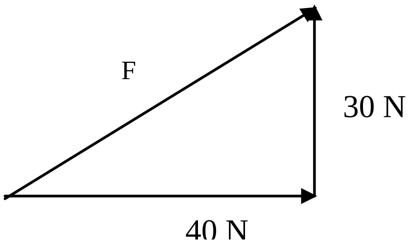
Worked Examples

1. Two forces 30 N and 40 N act on a body of mass 20 kg at right angle to each other. Calculate;
(a) The resultant force
(b) The acceleration of the body.

Solution: (a) A sketch diagram showing the forces



Applying the Triangle law of forces and head-to-tail addition of vectors, we complete the diagram as below.



Using Pythagoras theorem

$$F^2 = 40^2 + 30^2$$

$$\sqrt{F^2} = \sqrt{40^2 + 30^2}$$

$$= \sqrt{1600 + 900}$$

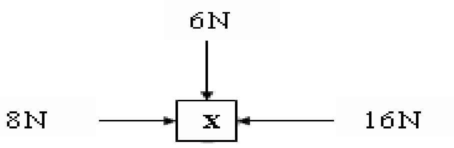
$$= \sqrt{2500}$$

$$\therefore \mathbf{F} = \mathbf{50\ N}$$

(b) F = 50 N, m = 20 kg, a = ?

$$F = ma \quad \Rightarrow \quad a = \frac{F}{m} = \frac{50}{20} = \mathbf{2.5\ ms^2}$$

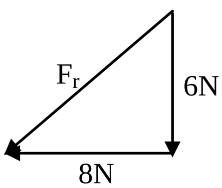
2. The diagram in the figure shows 3 forces of 8N, 6N and 16N acting on a particle X. Find the resultant force magnitude of the resultant force is



Solution

Step I First get the resultant force between 8N and 16N, since they are on a straight line.
Resultant force = 16 N – 8 N
= 8N acting to left (the direction of the larger force).

Step II Join the two forces using a head to tail and complete it to form a right angle triangle. Then find the hypogenous by using Pythagoras Theorem as in example 1 above.



Using Pythagoras theorem

$$F_r^2 = 6^2 + 8^2$$

$$F_r = \sqrt{6^2 + 8^2}$$

$$= \sqrt{36 + 64}$$

$$= \sqrt{100}$$

$$\therefore \mathbf{F_r} = \mathbf{10\ N}$$

(ii) The parallelogram law of forces.

If two forces, acting at one point on the same object, are arranged in magnitude and direction by the sides of a parallelogram drawn from the point, their resultant is represented in both magnitude and direction by the diagonal of the parallelogram drawn from the point.

In this method, the vectors to be added are represented by arrows joined *tail-to-tail* instead of *head-to tail* as before. The resultant is obtained by completing the diagonal of the parallelogram as shown in figure 5.10 below.

Composition and Resolution of Vectors

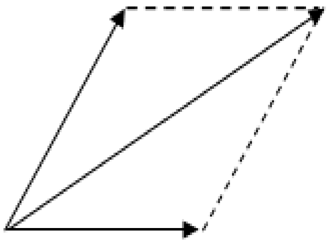


Figure 5.10

Composition of vectors is the substitution of two or more vectors with a single vector which has the same effect as all the separate vectors combined.

Resolution of vectors is the separation of a single vector into two vectors acting in definite directions upon the same point. Each of the two vectors is called a *component* of the single vector. One is the horizontal component and the other is the vertical component.

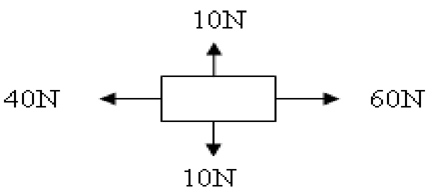
Self-Check Questions 5.1

1. Force is given by the product of

A. displacement and velocity B. displacement and mass

C. acceleration and mass D. velocity and mass

2. Forces of 60N, 10N, 40N and 10N act on a body as shown in figure 5.11. In which direction does the body move?



A. upwards B. downwards

C. to the left D. to the right

3. Two forces of 5N and 12N act on a body at right angles. Find their resultant

A. 7 N B. 13 N C. 17 N D. 169 N

4. A Newton is defined as the

A. unit of force.

B. force which produces an acceleration of $1ms^{-2}$

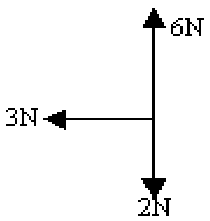
C. force which gives a mass of 1 kg an acceleration of $1ms^{-2}$

D. force which gives any mass an acceleration of $1\ ms^{-2}$

5. Two forces of 3N and 4N act at point at right angles to each other. The magnitude of their resultant force is

A. 25N B. 7N C. 5N D. 1N

6. In the diagram in the figure, the magnitude of the resultant force acting on a body P is



A. 1N

B. 5N

C. 7N

D.11N

7. A force of 1N acts on a mass of 0.05kg initially at rest. Its acceleration is
A. 0.05 ms^{-2}. B. 2 ms^{-2}. C. 0.5 ms^{-2}. D. 20 ms^{-2}.

8. Find the force required to give a mass of 500g an acceleration of 2×10^{-2} ms^{-2}.
A. 1×10^{-2} N B. 1×10^{-1} N C. 1×10^{2} N D. 1×10^{4} N.

9. In which of the following situations is minimum friction force required.
A. Sliding down a slope B. Walking along a road.
C. Leaning a ladder against a wall D. Designing brake blocks for bicycle.

10. Which one of the following are true statements about friction?
(i) It does not oppose friction.
(ii) It causes wearing of surface.
(iii) It decreases as weight of a body decreases.
(iv) It can be reduced by applying oil between surfaces.
A. (i) only. B. (i) and (iii) only. C. (ii), (iii) and (iv). D. All

11. Which of the following physical properties changes when a body is moved from the earth to Pluto (outermost planet of the solar system)?
A. mass B. volume C. weight D, density

12. Which of the following group consists of vectors only
A. momentum, acceleration, time, energy
B. speed, velocity, displacement, energy
C. displacement, velocity, acceleration, force
D. velocity, work, power, momentum

13. A force of 10N acts on a body and produces an acceleration of 2ms^{-2}. The mass of the body is
A. 0.2 kg B. 5.0 kg C. 20.0kg D. 50.0kg

14. A stone of mass 2.5g is thrown with an average force of 5.0N. Find the average acceleration in m/s^2.
A. 5.0×10^{-4} B. 2.0×10^{-3} C.25×10^{2} D. 2.0×10^{3}

15. Two forces act on chest, one of them is 50N and due northeast, and the other one is 75N and due south west. What is the resultant force?
A. 125 N due Northwest B. 125N due southwest
C. 25N due North West D. 25 N due south west

16. Which of the following sets contains only vector quantities?
A. Weight, displacement, acceleration, magnetic field.
B. Energy, electric field, momentum, distance.
C. Mass, velocity, force, speed.
D. Specific heat capacity, power, time, volume.

17. A block of mass 10 kg accelerates uniformly at a rate of 3 ms^{-2} along a horizontal table when a force of 40N acts on it. Find the frictional force between the block and the table
A. 10N B. 13.3N C. 30N D. 70N

18. Which one of the following quantities is the odd one out?
A. force. B. velocity. C. momentum. D. kinetic energy.

19. Find the force that acts on a body of mass 0.05kg accelerating at 20 m/s^2
A.1.0N B.10N C.100N D.400N

20. A box is pulled horizontally with a force of 100N inclined at an angle 50° to the horizontal as shown in the figure 5.12. The effective force, in Newton, is

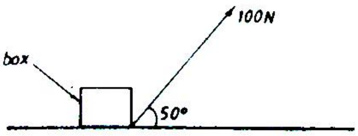


Figure 5.12

A. 100/sin
B. 100sin 50^0
C. 100/cos50^0
D. 100cos50^0

21. A block of mass 8kg slides on a rough horizontal surface under the action of a force of 48N as shown in the figure below. If the block moves with an acceleration of 5 ms^{-2}, calculate the frictional force on the block

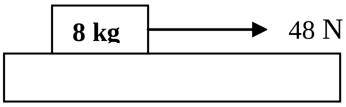


Figure 5.13

A. 6N
B.8N
C. 40N
D. 48N

22. What is the force that slows down or stops sliding?
A. Weight B. Gravity C. Friction D. Tension

23. A force of 20N acts on an object and there is 5N frictional force between the object and the surface. What is the net force on the object and its direction?
A. 15 N in the direction of 20 N
B. 15 N in the direction of the frictional force.
C. 25 N in the direction of 20 N
D 25 N in the direction of frictional force.

24. Which one of the following is a vector quantity?
A. Force B. Time C. Temperature. D. Mass

25. Three forces 25N, 30N, 40N act on object on the same line either in the same direction or in opposite directions. Which one of the following can not be the resultant of these forces?
A. 5N B. 15N C. 35N D. 95

SECTION B

(*Where necessary take g = 10 ms^{-2}***)**

26. **(a**) Distinguish between weight and mass of a body.

(b) The force of gravity on the moon is one-sixth of that on the earth. Determine the weight of a 12 kg mass on:

(i) The earth, (ii) The moon?

(c) Explain why the weight of a body on earth may vary from place to place.

27. (a) (i) What is meant by dynamic friction?

(ii) Describe, with the aid of a diagram, an experiment to determine the limiting friction between two surfaces in contact.

(iii) Sate the any **two** factors, which affect friction.

(b) Give **two** applications of friction.

28. **(**a) What is a vector quantity?

(b) Two forces of 30N and 40N act perpendicularly on an object of mass 10 kg as shown in the figure. Calculate:

(i) The magnitude of the resultant force on the object.

(ii) The acceleration of the object.

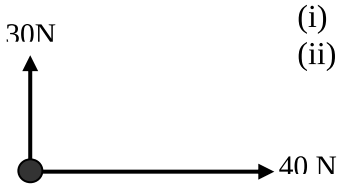
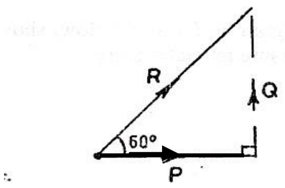


Figure 5.14

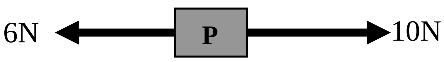
(c) The figure shows the resultant R of two forces P and Q. R makes an angle of 60^0 with a horizontal and P is 50N. Find the magnitude of:



(i) Q

(ii) R

(d) Two forces of 6N and 10N act at the same time on a body P of mass 500g as shown in the figure 5.15. Find the:



Figure

(i) Resultant force on P.

(ii) Acceleration of P.

29. A body of mass 5 kg is acted upon by two forces of 3 N and 4 N at right angles to one another. Find:

(a) By calculation the resultant of the two forces.

(b) The acceleration of the body.

CHAPTER SIX

TURNING EFFECTS OF FORCES

LEARNING OBJECTIVES

By the end of this chapter, you should be able to:

1. (a) Define: - Moment.
(b) State: - The SI unit of moment.
2. (a) State : - The Principle of moments.
- The conditions for a body to be in equilibrium.
(b) Define: - Centre of gravity.
(c) Describe: - Experiments to determine the centre of gravity of:
- Regular body e.g lamina,
- Irregular lamina.
3. (a) State: - Applications of Principle of moments
(b) Describe: - Experiments:
- To determine the mass of a uniform body.
- To determine the mass/weight of an object.
4. (a) Define: - The terms Stability and equilibrium.
(b) State: - The types of equilibrium.
- Factors which determine the type of equilibrium.
(c) Describe: - How to make a body stability

6.1 TURNING EFFECTS OF FORCES

The turning effects of forces are seen in the following actions:

- Opening or closing - a door, window, lid of a container e.g. a Jeri can, turning on a tap etc.
- Doing or undoing a nut using a spanner.
- Two kids playing on a see-saw,
- etc.

(a) Factors determining the effect of a turning force

There are two factors which determine the effect of a turning force. They include:

(i) The magnitude of the force.
(ii) The perpendicular distance from the line of action of the force to the pivot.

The combined effect of the above factors is called *moment*.

(b) Moment

Moment of a force about a point is the product of the force and the perpendicular distance from the line of action of the force to the point.

(c) The S.I unit of moment

S.I unit of moment = Nm (Newton metre)

Consider the diagram in figure 6.1 below in which a uniform metre rule is balanced on a knife edge at its centre.

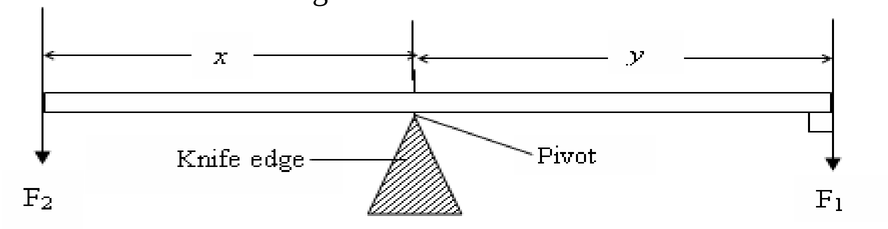


Figure 6.1

The moment about the pivot due to forces, F_1 and F_2, is got by using the formula:

Moment = Force x Perpendicular distance from the pivot to the line of action of the force

Moment due to F_1 = F_1 x y
= F_1y Nm (Clockwise moment)

Moment due to F_2 = F_2 x x
= F_1x Nm (Anti-Clockwise moment)

If the two moments are equal, i.e = $F_1y = F_1x$, the body is said to be in equilibrium according to the *principle of moments*.

6.11 The Principle of Moment:

The Principle of Moment states that:

When a body is in equilibrium, the sum of clockwise moments about any point is equal to the sum of anti-clockwise moments about the same point.

(a) A Couple or Torque (Parallel forces)

A couple refers to two equal and opposite parallel forces whose lines of action do not meet.

A couple produces rotation and can only be stopped or balanced by an equal and opposite couple.

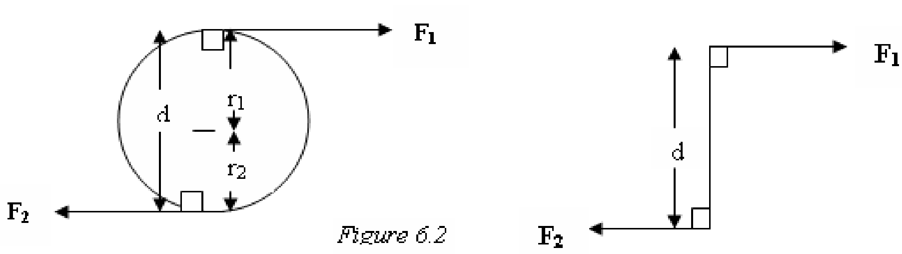


Figure 6.2

Where: $r_1 = r_2$ = radius of the circle $r_1 + r_2 = d$ = diameter of the circle

$F_1 = F_2$

Characteristics of a body under the action of a couple

(i) The resultant force on the body is zero.
(ii) The turning effects of the forces cause a rotational effect.
(iii) The forces act in opposite directions.

(b) Moment of a couple (Torque)

The moment of a couple is equal to the product of one force and the perpendicular distance between the two forces.

The moment due to the couple in figure 6.1 above is given by the formula:

Moment of a couple = *One force* x *the perpendicular distance between the forces*
= F_1 x d *Nm*

Or *Moment of a couple* = *One force* x *the perpendicular distance between the forces*
= F_2 x d *Nm*

Or *Moment of a couple* = $(F_1r_1 + F_2r_2)$ *Nm*

6.12 Parallel Forces in Equilibrium

(a) Conditions for a body in equilibrium

The following are the conditions for a body to be in equilibrium when under the action of a number of parallel forces.

(i) *The sum of forces acting in one direction is equal to the sum of forces acting in the opposite direction. I.e. the net resultant force on the body is zero.*

(ii) *When a body is in equilibrium, the sum of clockwise moments about any point is equal to the sum of anti-clockwise moments about the same point.*

Consider the diagrams in figures 6.3 and 6.4 showing a uniform body in equilibrium under the action of parallel forces below for conditions *(i)* and *(ii)* above.

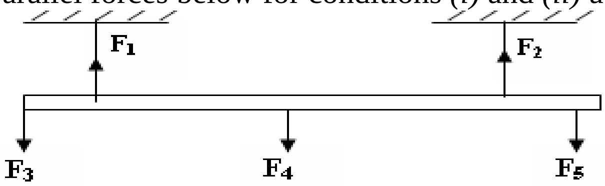


Fig. 6.3

In the diagram, three forces (F_3, F_4 and F_5) act downward and two forces (F_1 and F_2) act upward and since the body is in equilibrium, then

⇒ Using condition (i) above we have:

$$\left(\text{Sum of Upward forces}\right) = \left(\text{Sum of Downward forces}\right)$$

$$\therefore \quad (F_1 + F_2) = (F_3 + F_4 + F_5)$$

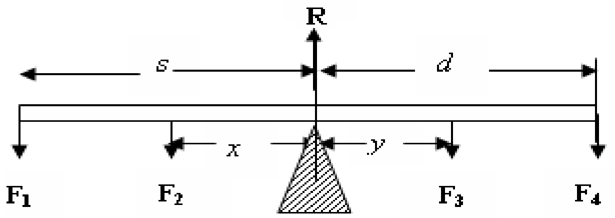


Figure 6.4

$$\left(\text{Sum of Clockwise moments}\right) = \left(\text{Sum of Anti-clock moments}\right)$$

$$\therefore \quad F_3y + F_4d = F_1s + F_2x$$

⇒ And Using condition (i) above we have:

$$R = F_1 + F_2 + F_3 + F_4$$

(b) Resultant Moment

When dealing with problems involving a number of moments acting on a body which is NOT in equilibrium, the following steps are taken:

(i) Draw the sketch diagram indicating all the forces and their respective distances from the pivot (fulcrum).

(ii) Give a positive sign to anticlockwise moments and a negative sign to clockwise moment(s).

(iii) Add the moments algebraically. The numerical value of the answer gives the magnitude of the resultant moment and the arithmetic sign of the answer gives the direction of the resultant moment.

Example 1 *Two forces 20 N and 50 N act on a body as shown in figure 6.5. If the 20 N is 0.1 m from the pivot and has clock wise effect while 50 N is 0.2 m from the pivot and has ant clockwise effect.*

(a) Sketch the diagram showing all the forces and their respective distances from the pivot.

(b) Calculate the resultant moment on the body.

Solution: (a)

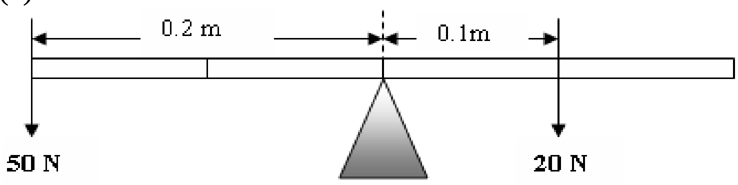


Figure 6.5

(b) Resultant moment $= {}^{-}(20 \times 0.1) + {}^{+}(50 \times 0.2)$

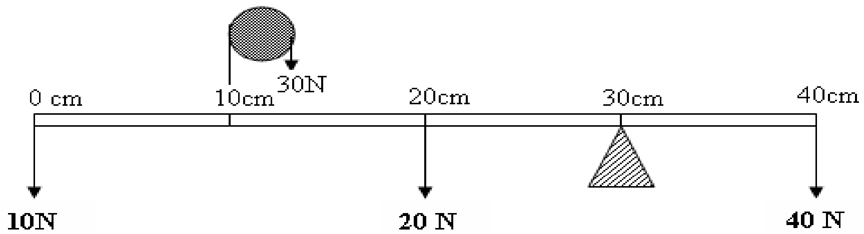
$= -2.0 \quad + \quad 10$

$= +8$ Nm

Hence the resultant moment is 5 Nm acting in anti-clockwise direction.

2. *Four forces, 10 N, 20 N, 40 N, act downward and 30 N acts upward on a body which is 40 cm long. The 10 N is hung at 0 cm mark, the 30 N acts at 10 cm mark, 20 N acts at 20 cm mark and 40 N acts at 40 cm mark. If the knife edge is placed at 30 cm mark, calculate the resultant moment on the body,*

The sketch of the diagram showing the four forces acting on a body.



Calculations

Resultant moment $= +10 \text{ x} \left(\frac{30 - 0}{100}\right) + 20 \text{ x} \left(\frac{30 - 20}{100}\right) - 30 \text{ x} \left(\frac{30 - 10}{100}\right) - 40\text{x} \left(\frac{40 - 0}{100}\right)$

$= +10 \text{ x } \frac{30}{100} \quad + \quad 20 \text{ x } \frac{10}{100} \quad - 30 \text{ x } \frac{20}{100} \quad - 40 \text{ x } \frac{10}{100}$

$= \frac{300}{100} + \frac{200}{100} + \frac{-600}{100} + \frac{-400}{100}$

$= +3 + 2 - 6 - 4$

$= +5 - 10$

= -5 Nm

Hence the resultant moment is 5 Nm acting in a clockwise direction.

6.2 Centre of Gravity

Every body may be regarded as being made up of a very large number of very tinny and equal particles (according to Dalton Atomic theory). Each of these particles is pulled towards the centre of the earth as shown in figure 6.6 below.

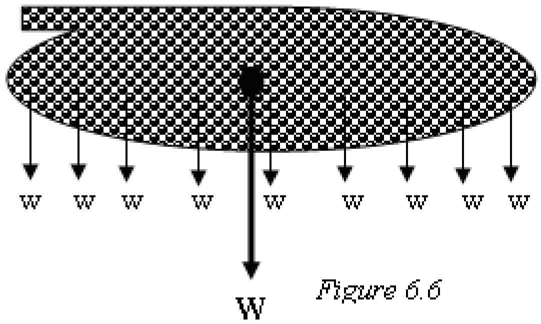


Figure 6.6

Thus, the earth's pull on the body consists of very large number of equal parallel forces. The resultant of the forces is equal to the total force of gravity on the body and it acts through a point **G** called the *centre of gravity*.

Definition: *The centre of gravity of a body is defined as the point of application of the resultant force due to the earth's attraction on it.*

Resultant force is a single force which has an effect of two or *more forces acting on a body.*

6.21 Methods to locate the Centre of gravity of a body

The method chosen to determine the centre of gravity of a body depends on the following factors:
- The nature and
- The shape of the body.

The shape may either be regular or irregular.

(a) A regular body

The centre of gravity of a regular body is found by using:
- Blancing and
- Intersection of diagonal method.

(i) Balancing method

The centre of gravity of a long uniform object such as a metre rule may be determined by balancing method.
In this method, the metre rule may be balanced on a knife edge or hang from a loop of thread and then adjusted until it balances horizontally.
The point at the knife edge is the centre of gravity.

Note:
- *A uniform metre rule or a uniform body is one in which the particles are uniformly distributed.*
- *The centre of gravity of a uniform body is always at its centre.*

(ii) Intersection of diagonals method

This method applies mostly to two dimensional figures e.g a rectangular lamina or cardboard.

Experiment: To determine the centre of gravity of a regular lamina

Procedure

Draw straight lines along the diagonals of the lamina as shown in figure 6.7 diagram below.

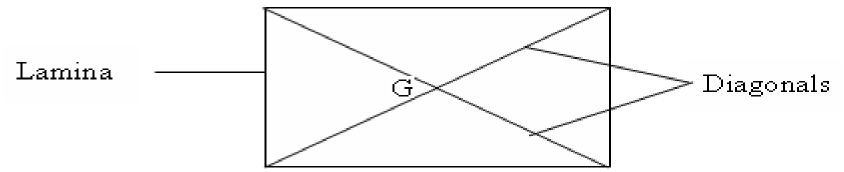


Figure 6.7

*The point of intersection **G** is the centre of gravity.*

(b) Irregular Body

The centre of gravity of irregular body is determined by using *plumb line method.*

Experiment 6.1 To determine the centre of gravity of irregular lamina

Apparatus

Retort stand/Clamp, a pin fixed to a rubber bang, plumb, thin string, paper punch machine

Procedure

a) Make three holes at well spaced intervals round the edge of the lamina and label them A, B and C.
b) Tie one end of the string to the plumb and tie the other end to form a loop so that it can easily be suspended from the support pin.
c) Using hole A, suspend the lamina and the plumb from the support pin and wait until the two come to a rest.
d) Mark two crosses with a thin pencil point on the plumb line one near hole A and the other below it.
e) Remove the plumb and the lamina.
f) Join the two crosses on a straight line.
g) Repeat procedures (c) to (f) for the remaining holes B and C.

Observation

All the three straight lines intersect at a common point G as shown in figure 6.7 below.

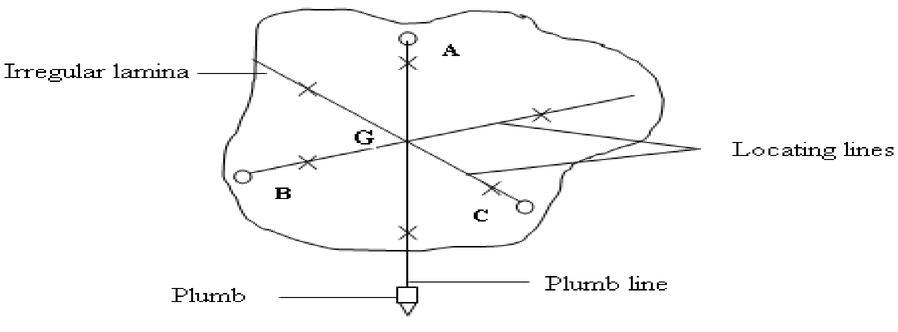


Figure 6.8

Conclusion: Since the centre of gravity lies on the straight lines drawn on the lamina, it must be situated at the point of *intersection G.*

Note: (i) *In the laboratory, the plumb can be replaced by a pendulum bob.*

(ii) *Not always that the centre of gravity of a body lies within the material, it may also be at a point in the air near by.*

The best examples are: - tripod and a laboratory stool.

6.23 Applications of the Principle of Moments

The principle of moment is applied in: the following.

(i) The determination of Mass or weight of a uniform metre rule.
(ii) The determination of Mass or weight of an object.

(a) Determination of the mass of a uniform metre rule using a standard (known) mass.

- Suspend a uniform metre rule using a loop of thread from a support.
- Adjust the metre rule until it balances horizontally.
- Read and record the value at the pivot, say 50 cm.
- Suspend a standard mass say 100 g by means of a thread at 0 cm mark.
- Readjust the position of the metre rule until it balances horizontally again.
- Read and record the new value at the point of pivot, say 30 cm.
- Measure and record the respective perpendicular distances from 100 g mass and M from the pivot.

Consider the diagram in figure 6.9 below

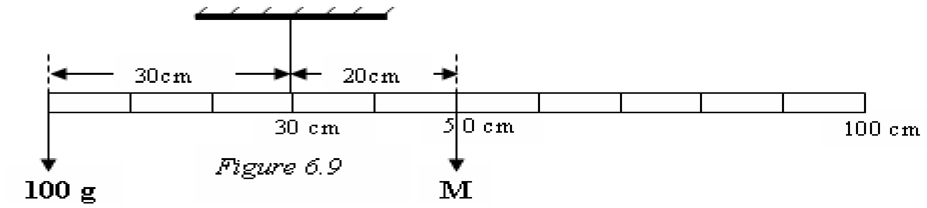


Figure 6.9

Applying the principle of moments

Clockwise moment = *Anti-Clockwise moments*

$M \times (50 - 30)\ cm = 100\ g \times (30 - 0)\ cm$

$M \times 20\ cm = 100\ g \times 30\ cm$

$$\frac{M \times 20\,cm}{20\,cm} = \frac{100\ g \times 30\,cm}{20\,cm}$$

M = 150 g

(b) Determination of the mass or weight of an object using a uniform metre rule and a standard mass or weight

Procedure

- Suspend a uniform metre rule using a loop of thread from a support.
- Adjust the metre rule until it balances horizontally.
- Read and record the cm mark at the pivot.
- By means of a thread, suspend a standard mass, say, 200 g and the object whose mass is to be determined on either sides of the pivot.
- Readjust their distances from the pivot until the metre rule once more balances horizontally.
- Measure and record their respective perpendicular distances from the pivot.

For calculations consider the diagram in figure 6.10 below

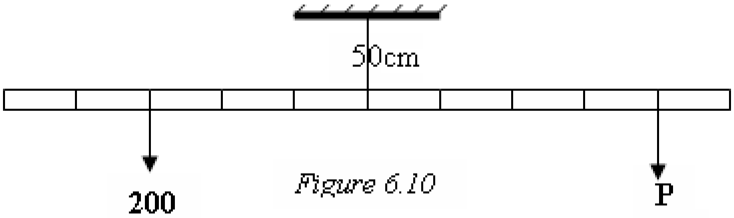


Figure 6.10

Applying the principle of moments

$$\text{Clockwise moment} = \text{Anti-Clockwise moments}$$

$$P \times (90 - 50)\ cm = 200\ g \times (50 - 20)\ cm$$

$$P \times 40\ cm = 200g \times 30\ cm$$

$$\frac{P \times 40\ cm}{40\ cm} = \frac{200\ g \times 30\ cm}{40\ cm}$$

$$\boldsymbol{P = 150\ g}$$

NB: *To get the weight (W) of the object, we use the formula:*

$$\boldsymbol{W = mg}$$

Where: $\boldsymbol{g}$ *= gravity and*
$\boldsymbol{m}$ *= mass of the object in kg.*

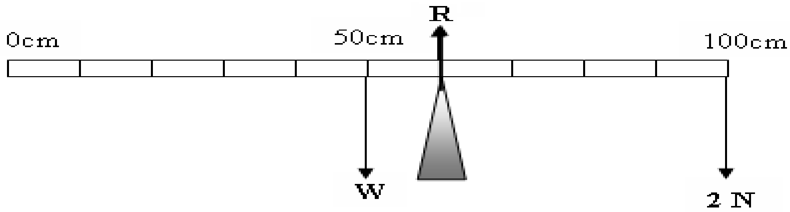
To get the weight once, we use a standard weight directly instead of a standard mass.

Worked Examples

Note: ***When solving problems involving the law of moments, identify the force(s) which has/have clockwise moment(s) and anti-clock wise moment(s). So that you substitute them correctly. Experience has shown that many students tend to forget this as such they put clockwise moments under anti-clock wise moments and vice versa.***

1. A uniform metre rule balances horizontally on a knife edge placed at 60 cm when 2 N weight is suspended at one end.
   (a) With a help of diagram show at which end of the metre rule the 2 N weight must be suspended.
   (b) Calculate: (i) the weight,
   (ii) the mass of the metre rule,
   (iii) the reaction, R, at the knife edge. (Take g = 10 ms^{-2})

Solution: (a)



Since the metre rule is uniform, its weight **W** acts through centre, i.e at 50cm mark, then the 2 N weight must be suspended on the other side of the pivot so as to balance it.

(b) (i) Applying the principle of moments

$$\begin{aligned}
\textit{Clockwise moment} &= \textit{Anti-Clockwise moments}\\
2\,N \times (100 - 60)\,cm &= W \times (60 - 50)\,cm\\
2\,N \times 40\,cm &= W \times 10\,cm\\
\frac{2\,N \times 40\,cm}{10\,cm} &= \frac{W \times 10\,cm}{10\,cm}\\
\mathbf{W} &= \mathbf{8\ N}
\end{aligned}$$

(ii) W = 8 N, g = 10 ms^{-2}, m = ?

Using

$$\begin{aligned}
W &= mg\\
8 &= m \times 10\\
m &= \frac{8}{10}\\
\mathbf{m} &= \mathbf{0.8\ kg}
\end{aligned}$$

(iii) Applying the condition for a body in equilibrium,

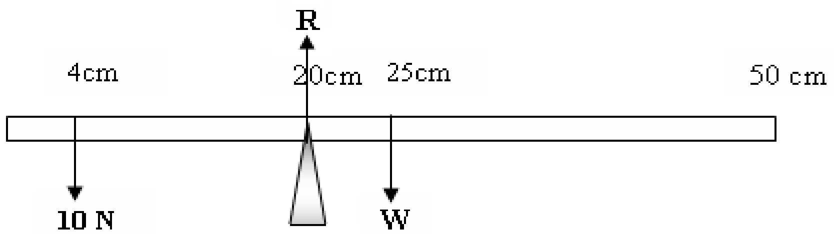
(The upward force) = (Sum of downward forces)

$$\begin{aligned}
R &= W + 2\,N\\
&= 8\,N + 2\,N\\
\therefore \mathbf{R} &= \mathbf{10\ N}
\end{aligned}$$

2. A uniform half-metre rule is freely pivoted at the 20 cm mark and it balances horizontally when a weight of 10 N is suspended at the 4 cm mark.
   (a) Draw a sketch diagram showing all the forces acting on the metre rule.
   (b) Calculate: (i) the weight of the metre rule.
   (ii) the reaction at the knife edge.

Solution

(a) A sketch diagram showing the forces acting on a half-metre rule



(b) (i) Applying the principle of moments

$$\begin{aligned}
\textit{Clockwise moment} &= \textit{Anti-Clockwise moments}\\
W \times (25 - 20)\,cm &= 10 \times (20 - 4)\,cm\\
W \times 5\,cm &= 10 \times 16\,cm\\
\frac{W \times 5\,cm}{5\,cm} &= \frac{10\,N \times 16\,cm}{5\,cm}\\
&= \frac{160}{5}\,N\\
\therefore \mathbf{W} &= \mathbf{32\ N}
\end{aligned}$$

(ii) Applying **(Upward force) = (Sum of downward forces)**

we have:

$$\begin{aligned}
R &= W + 10\,N\\
&= 32\,N + 10\,N\\
\therefore \mathbf{R} &= \mathbf{42\ N}
\end{aligned}$$

6.3 Stability and Equilibrium

(a) Stability

Stability, in physics and engineering, refers to the property of a body that causes it to return to its original position or motion as a result of the action of the restoring forces, or torques, once the body has been disturbed from a condition of equilibrium or steady motion.

In simple terms we can define stability as the difficulty in causing a body to fall or topple over.

(b) Equilibrium

***Equilibrium** refers to the state of a body where the net force acting on the body is zero. In the case of a stationary body, the large-scale property of the position of the body will remain unchanged over time.*

6.31 Types of Equilibrium

Mechanical equilibrium can be of three kinds:

(i) Stable Equilibrium,
(iii) Unstable Equilibrium and
(iv) Neutral Equilibrium

(a) Stable Equilibrium

Stable equilibrium is a state of a body in which on a slight displacement it returns to its original position.

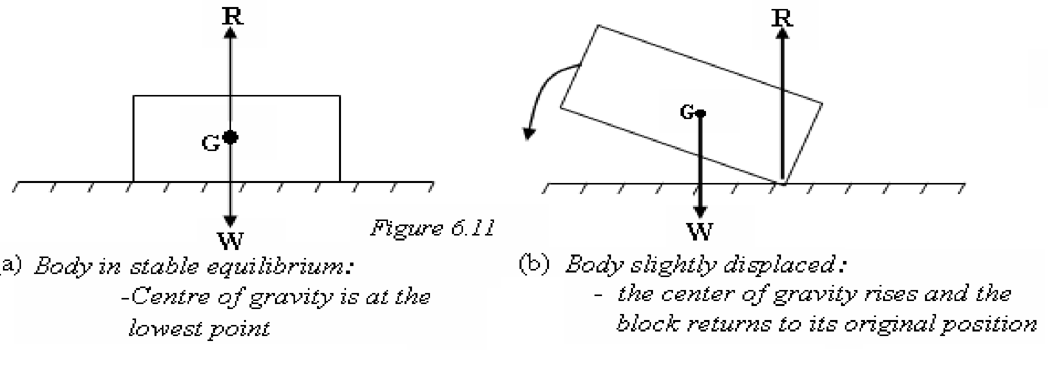
Characteristics of a body in stable equilibrium

A body in a stable equilibrium is characterized by or is associated with the following:

(i) The centre of gravity of the body is at the lowest position.
(ii) The base area is large.

Thus on a slight displacement of a body in stable equilibrium, the centre of gravity and is raised and the body returns to its original position after the displacement.

Consider a glass block resting on a table as shown in figure 6.11 below.



Explanation

The block of glass exerts force, W, (its weight) on the table and the table exerts an upward force called reaction force, R, equal to the weight of the block. Since the weight and the reaction are equal and opposite forces, the resultant force on the block is zero, so the block is stable (i.e. does not move).

When the block is tilted slightly, the two forces act as shown in figure 6.11 (b). The reaction, R, acts at the point of contact while the weight, W, still acts through G, the centre of gravity, but out side the point of contact. The result is a couple, two equal and opposite parallel forces, which tend to rotate the block in anti-clockwise direction and therefore, the block falls back to its original position.

(b) Unstable Equilibrium

Unstable equilibrium refers to a state of a body in which on a slight displacement it does not return to its original position.

Characteristics of a body in unstable equilibrium

A body in an unstable equilibrium is associated with the following:

(i) The centre of gravity of the body is at the highest position.

(ii) The base area is small.

Thus on a slight displacement of a body in unstable equilibrium lowers the centre of gravity and the body does not return to its original position after the displacement; it topples over.

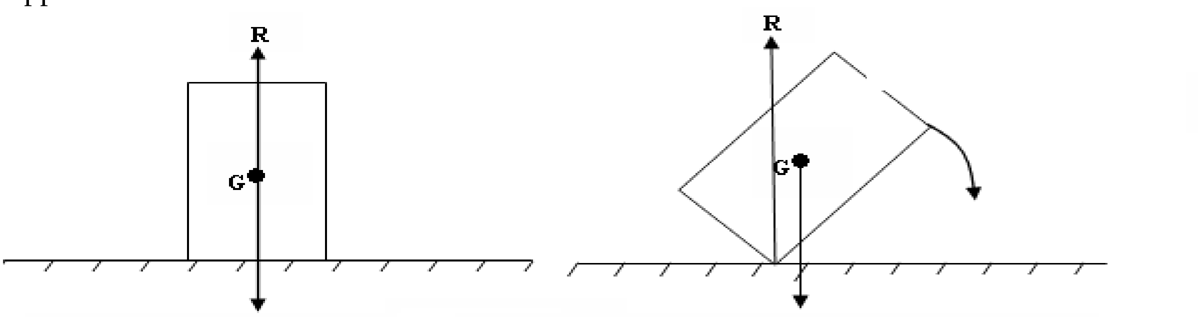


Figure 6.12

(a) *Body in unstable equilibrium:*
-Centre of gravity at highest point

(b) *On slight displacement:*
- *the centre of gravity is lowered and*
- *the block topples over.*

Explanation

In figure 6.12 (a), the block is in an unstable equilibrium. When it is tilted slightly, the two forces act as shown in figure 6.12 (b). The reaction, R, acts at the point of contact while the weight, W, still acts through G, the centre of gravity, but out side the point of contact. The result is a couple, two equal and opposite parallel forces which tend to rotate the block in *clockwise direction* and does not return to its original position, hence topples over.

(c) Neutral Equilibrium

This is a state of a body in which on slight displacement its centre of gravity is neither raised nor lowered.

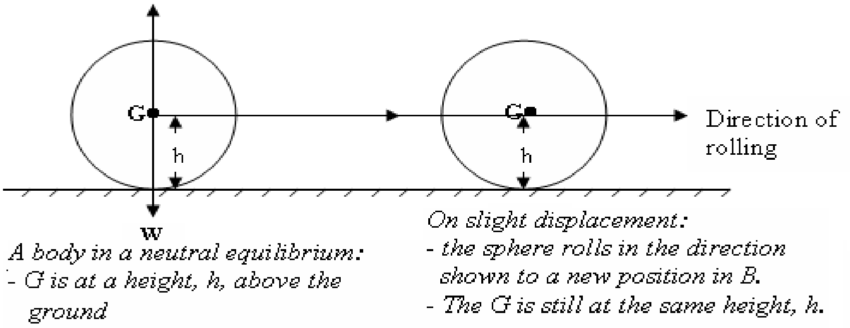
Characteristics of a body in neutral equilibrium

A body in a neutral equilibrium is associated with the following:

(i) The centre of gravity of the body is always at the same height and directly above the point of contact.

(ii) The area in contact is very small.

A slight displacement does not alter the position of the centre of gravity, thus the body is always at rest whichever position it is placed.
Example of a body in neutral equilibrium is any perfect sphere.



NB: *Using the characteristics of a body in stable equilibrium, a body be designed such that it is in stable equilibrium.*

6.32 Making a body Stable

In engineering, the stability of a structure is increased by:

(i) Making its base wide and
(ii) Keeping its centre of gravity as low as possible by putting more weight in the lower part than in the upper part.

Notable examples are seen in the following:

(i) Racing cars - which have both a low centre of gravity and a wide wheel base.
(ii) Modern Isuzu/Scania coach buses
- They have fairly wide wheel base and low centre of gravity which is achieved by packing load in boots.

Note: *The old system of loading buses on the racks of buses raises the centre of gravity of the bus thus making it unstable on the road.*

Self-Check 6.0

1. Figure 6.1 shows a uniform beam in equilibrium when a force R acts on it at one end. Find the weight W of the beam.

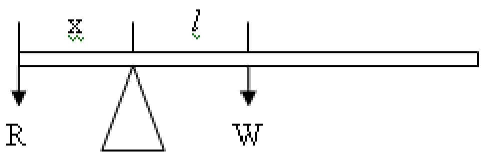
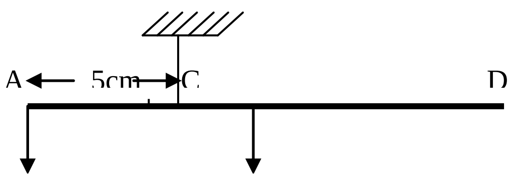


Figure 6.1

A. $\frac{x}{Rl}$ B. $\frac{Rl}{x}$
C. $\frac{l}{Rx}$ D. $\frac{Rx}{l}$

2. The diagram in the figure 6.2 shows a uniform half-meter rule suspended at point C. The mass of the rule is

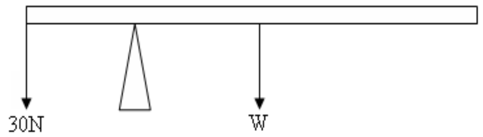


0.1kg *Fig. 6.2* M kg

A. 0.020 kg
B. 0.025 kg
C. 0.100 kg
D. 0.125 kg

3. A uniform wooden beam of weight W is pivoted at a distance $^1/_5$ of its length from the end A and kept in equilibrium by applying a force of 30 N as shown in figure 6.3. The force exerted by the pivot on the beam is

A. 50 B. 40 C. 30 D.20



4. A uniform rod 100cm long pivoted at the 90 cm mark, balances horizontally when a mass of 200 g is suspended at the 100cm mark as shown in the figure 6.3.
The mass of the rod is

A. 40 g B. 50 g C. 400 g D. 800 g

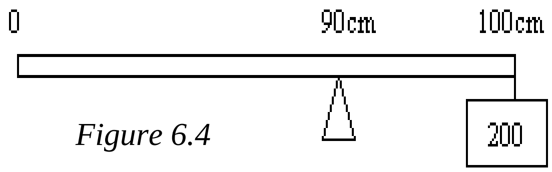


Figure 6.4

5. Which one of the following statements is true about two equal forces acting on a bar of length l , shown in the figure.

(i) The resultant force on the bar is zero.
(ii) The forces cause a rotational effect.
(iii) The forces act in opposite directions.
(iv) The forces produce different turning effects.

A. (i) only B. (i) and (ii) only C. (i),(ii) and (iii) only D. All

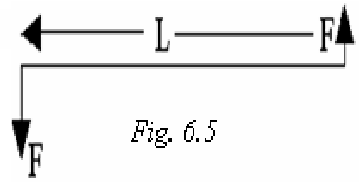


Fig. 6.5

6. The figure below shows a uniform metre rule of mass 0.1kg pivoted at the 80 cm mark. It balances horizontally when a mass P is hanging at the 95 cm mark. Find P.

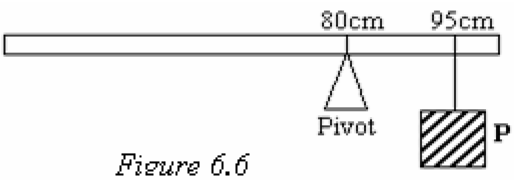


Figure 6.6

A. 0.08 kg B. 0.2Kg
C. 0.4 kg D. 1 kg

7. A uniform metre-rule is pivoted at its centre shown in the figure. If the rule is in equilibrium, find value of F.

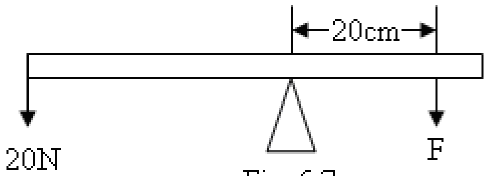


Fig. 6.7

A. 4 N B. 33.3 N
C. 50 N D. 100 N

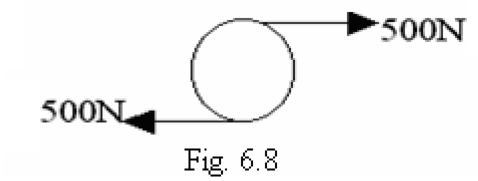
8. 

Fig. 6.8

The shaft in an engine is subjected to two parallel but opposite forces of 500N each as shown in the figure. The rotation is best stopped by applying
A. Two forces of 500 N acting at right angles to each other
B. A single force of 1000 N.
C. Two parallel but opposite forces of 500 N D. A single force of 250 N.

9. The above figure shows a crank of a bicycle pedal. The force a cyclist exerts on the pedal varies from a minimum to maximum. When does the cyclist exert maximum turning effort?

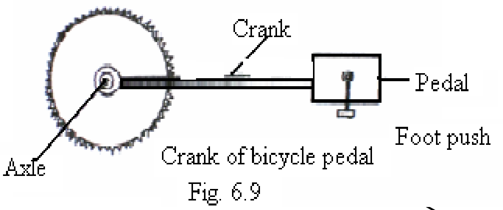


Fig. 6.9

A. crank makes 90^0 with the foot push
B. crank makes 0^0 with the foot push
C. cyclist is climbing a hill
D. cyclist is turning a corner

10. Find the weight, w, of a uniform metre rule if a force of 60 N at one end balances it as shown figure.

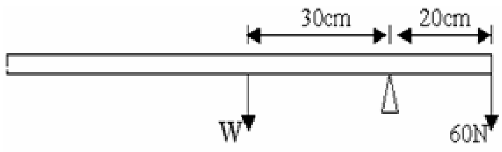


Figure 6.10

A. 24 N B. 40 N C. 90 N D. 100 N

SECTIONS B

11. (a) (i) Define moment of a force and state its SI unit.
(ii) State the principle of moments.
(iii) State the conditions for a body to be in equilibrium.

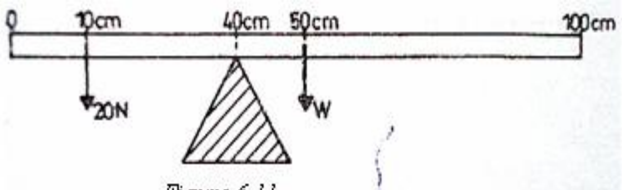


Figure 6.11

(b)A uniform meter ruler is pivoted at the 40 cm mark as shown in the fig. 6.11
The meter ruler is the equilibrium under its weight W and a 20 N force acting at the 10 cm mark. Calculate:
(i) The weight W of the metre rule.
(ii) The reaction at the knife edge.

12. (a) What is meant by *centre of gravity*?
(b) (i) Describe an experiment to determine the centre of gravity of an irregular lamina.
(ii) Describe how you would measure the mass of a metre rule using a known mass and a knife-edge only.
(c) If the metre is in equilibrium when weights of 10N, 2N and 5N are attached to it as shown in figure 6.12.

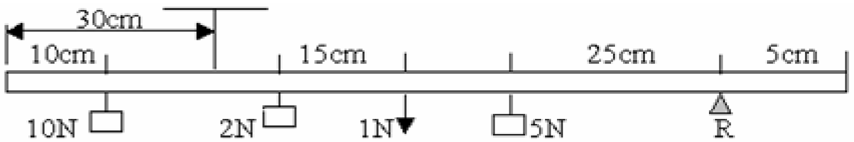


Figure 6.12

Calculate the: (i) Tension in the string.
(ii) Normal reaction, R, at the wedge.

14. (a) State (i) the types of equilibrium.

(ii) the characteristics associated with the types of equilibrium you have stated in 5 (a) above.

(b) Explain giving reasons whether it is advisable to load a bus on the rack. Describe how you would design a given structure to have a high stability.

(c) A uniform beam of weight 2.5 N is pivoted at its mid-point P, as shown in figure 6.14.

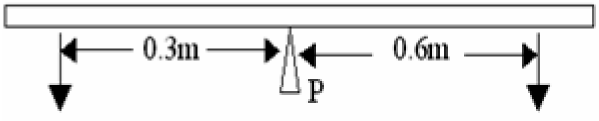


Figure 6.14

The beam remains in equilibrium when force R and S act on it. If R is 5N, find the:

(i) Value of S.

(ii) Reaction at the pivot

CHAPTER SEVEN
WORK, POWER AND ENERGY

LEARNING OBJECTIVES

By the end of this chapter, you should be able to:

1. (a) Define: -The following terms and state their SI units;
- Work and Power
(b) Describe an experiment to measure human power
(c) Solve numerical problems involving; Work done and Power

2. (a) Define: - the term Energy and state its S.I unit.
(b) State: (i) - the Law of Conservation of energy
(ii) - the forms and sources of energy

3. (a) Define: - Potential and Kinetic Energy
(b) Describe the Interchange between P.E and K.E.
(c) Solve numerical problems involving P.E and K.E

4. (a) State: - The components of Internal-Combustion Engine.
(b) Describe:- The mechanism of Four-stroke and Two-stroke Engines
(c) State: - The factors that limit the efficiency of Internal-Combustion engines
(d) Explain: - How to improve the efficiency of Internal-Combustion engines.

7.10 WORK

(a) Introduction

The word "work" in everyday life describes any activity which requires muscular or mental effort. But in Physics, work has a special meaning. In the scientific sense, work involves motion and work is done when a force changes the position or speed of an object.

Definition: Work *is the product of a force applied to a body and the displacement of the body in the direction of the applied force.*

Mathematically, it is expressed as:

Work done = Force (F) x Distance (s) moved in the direction of the force

Work done = Fs

Note: *1. Work is only said to be done when a force moves its point of application along*
the direction of its line of action.
2. If a force acts on a body and there is no motion, then there is no work done.
3. The distance must be in metres.

While work is done on a body, there is a transfer of energy to the body, and so work can be said to be energy in transit.

(b) Factors which determine the amount of work done

From the formula of work, we can see that the amount of work done depends on:

(i) The magnitude (size) of the force applied.
(ii) The distance moved.

The S.I unit of work is joule, J.
A joule is defined as the *work done when a force of one Newton (1N) moves through a distance of one metre* (1m).

Larger units are: - the kilojoule (kJ) and
- the megajoule (MJ)

1 kJ = 1 000 J (10^3 J)
1 MJ = 1 000 000 J(10^6 J)

Work is also done in moving against some opposing force such as gravity and any form of resistance to the motion of the force.

For example: (i) When a crane is lifting a heavy load, work is done against the force of gravity. Or when a person lifts a load to a given height.
(ii) When a nail being driven into a wooden block by hammering, work is done against the resistance of the wood.

Worked Examples

1. Calculate work done by an engine which exerts a force of 9000N over a distance of 6 m.

Solution: Force, F = 9000 N, Distance, s = 6 m, Work done = ?

Work done = Force x Distance
= F x s
= 9000 x 6

$\therefore$ **Work done = 54,000 J Or 54 kJ**

2. A man lifts a box of mass 3 kg vertical upwards through 2 m. If the, gravitational field, *g* is 10 m/s^2, calculate the work done by the man in lifting the box.

Solution: Mass of box = 3 kg, gravitational field, g = 10m/s^2 Force, F = mg, Distance, s = 2 m, Work done = ?

Work done = Force x Distance
= mg x s
= 3 x 10 x 2

$\therefore$ **Work done = 60 J**

3. In the diagram below, a force of 10 N acts on a block of weight 30 N placed on a rough table as shown in figure 7.1 below. Given that μ is 0.2 and the block moves through a distance of 3 m.

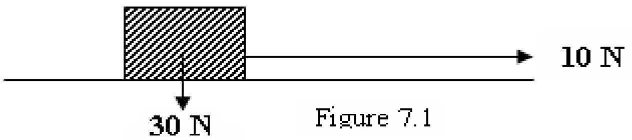


Figure 7.1

Calculate: (a) the useful work,
(b) the useless work; done if the block moves 3 m along the direction of the 10 N and the coefficient of friction is 0.2.

Solution: **Method I**

(a) Force applied, F = 10 N, s = 3 m, $\mu = 0.2$, Weight, W = R = 30 N

Useful work done = Resultant force x Distance moved

= (Force applied – Frictional force) x s

$= \left[10 - (\mu R)\right] \times 3$ Where: μR = Frictional force and

$= \left[10 - (0.2 \times 30)\right] \times 3$ R = *Weight of the object*

= (10 – 6) x 3

= 4 x 3 N

∴ **Useful work done = 12 J**

Method II

Step I*:* First calculate the frictional force

Frictional force $= \mu R$

= 0.2 x 30

= 6 N

Step II*:* Calculate the Resultant Force

Resultant Force = Force applied – Frictional force

= 10 – 6

= 4 N

Step III: Now calculate the useful work done from the formula:

Work done = Resultant force x Distance

= 4 x 3

∴ **Useful work done = 12 J**

7.2 POWER

Power is defined as the *rate of doing work*. Or

Power *is the rate of performing work or transferring energy.*

Power measures how quickly work is done.
Mathematically speaking, power is equal to the work done divided by the time interval over which the work is performed.

I.e. $$\text{Power} = \frac{Work\,done}{Time\ taken}$$

In the sense of power being defined as *rate of transfer of energy*, we can also mathematically express power as:

$$\text{Power} = \frac{Energy\ change}{Time\ taken}$$

S.I unit of power

The S.I unit of power is the **Watt** (W).

The watt is defined as *the rate of doing work at one joule per second*.

1 Watt = 1 Joule per second.

i.e 1 W = 1 J/s

Larger units of power are:

- The Kilowatt (kW) and
- Mega watt (MW)

1 kW = 1000W (10^3 W)

1MW = 1 000 000W (10^6 W)

1 MW = 1 000 kW

NB: 1. Engine power is sometimes measured in horse power (hp).
1 hp = 746 W $\approx$ ¾ kW

2. From Work done = Force x Distance

Power = $\frac{F \text{ x } s}{t}$ = F x $\frac{s}{t}$ But Velocity = $\frac{s}{t}$

$\therefore$ **Power = Force x Velocity**

Worked Examples

1. Calculate the power of a water pump which can fill a water tank 10 m height with 3000 kg of water in 20 s. (Assume g = 10 ms^{-2}).

Solution: m = 3000 kg, h = 10 m, g = 10 m^{-2}, P = ?
We can solve this problem by using any one of the methods below.

Method I

Step I: First calculate the work done.

Work done = Force x Distance
= mg x s
= 3000 x 10 x 10

$\therefore$ **Work done = 300,000 J or 3 x 10^5J**

Step II: Now calculate the power from the formula:

Power = $\frac{Work\,done}{Time\ taken}$

= $\frac{300{,}000}{20}$

$\therefore$ **Power = 15,000 W**

Method II Substitute the values in the data collected directly as shown below.

Power = $\frac{Work\,done}{Time\ taken}$

= $\frac{Force \text{ x } Distance}{Time\,taken}$

= $\frac{Mass \text{ x } Acceleration\,due\,to\ gravity \text{ x } Distance}{Time\,taken}$

= $\frac{mgs}{t}$

= $\frac{3000 \text{ x } 10 \text{ x } 10}{20}$

$\therefore$ **Power = 15,000 W**

2. A man lifts a box of mass 10 kg through a vertical height of 2 m in 4 seconds. Calculate the power he developed.

Solution: m = 10 kg, h = 2 m, t = 4 s, g = 10 m^{-2}, P = ?

Using the formula Power $= \frac{F \text{ x } s}{t}$

$= \frac{mgs}{t}$

$= \frac{10 \text{ x } 10 \text{ x } 2}{4}$

= 50 W

7.21 Measurement of Human Power

The rate of working of a person is at its highest when he or she is running up a hill or up stairs lifting his or her own weight. So human power can be measured by measuring the total vertical height of a stairway and the time taken to run up the vertical height.

Experiment

Apparatus: Stop watch, Weighing scale, a long flight of steps.

Procedure

- ❖ Run up a long flight of steps while someone times you using a stop watch.
- ❖ Make three attempts each time the person records the time taken.
- ❖ Weigh your self on weighing machine to get your mass.

Calculations

⇒ ***Calculating the Vertical height***

Measure the height of one step and count the number of steps you climbed.

Find the total vertical height from the formula:

Vertical height = Height of one step x Number of steps

⇒ ***Average time taken***

Find the average time taken from the formula;

$$\text{Average time taken} = \frac{\textit{Sum of time taken}}{\textit{Number of attempts}}$$

For the case of above, Average time taken, t = $\frac{t_1 + t_2 + t_3}{3}$

Where $t_1 + t_2 + t_3$ *are the times taken in seconds in the three attempts.*

⇒ **The Power**

The power can then be calculated using any one of the following methods.

Method I

Step I: *First calculate work done*:

$$\textit{Work done by the boy} = \textit{Force overcome} \text{ x } \textit{Vertical distance climbed}$$
$$= mg\,h$$

Where h = *Height of 1 step* x *Number of steps in metres*

$$\therefore \textit{Work done by the boy} = mg \text{ x } \textit{Height of 1 step} \text{ x } \textit{Number of steps}$$

Step II: Now calculate the power developed from the formula:

$$\textbf{Power} = \frac{\textit{Work done}}{\textit{Time taken}}$$

$$= \frac{mg \text{ x } h}{t} \qquad h = \text{as defined in step I above.}$$

$$\therefore \textbf{Power} = \left(\frac{mg}{t}\right) \text{x}$$

$$\textit{Height of one step} \text{ x } \textit{Number of steps}$$

Method II

Simply calculate the power from the formula

$$\text{Power} = \left(\frac{mg}{t}\right) \text{x}$$

$$\textit{Height of one step} \text{ x } \textit{Number of steps}$$

Example

1. A boy of mass 60 kg runs up a flight of 60 steps in 10 seconds. If the height of one step is 20 cm, calculate the power he developed.

Solution

When collecting the data, always remember that the h must be in metres.

m = 60 kg, t = 10 s, height of 1 step = 20 cm = $\frac{20}{100}$ = 0.2 m

Method I

Step I: First calculate work done:

$$\textit{Work done by the boy} = \textit{Force overcome} \text{ x } \textit{Vertical distance climbed}$$
$$= mgh$$
$$= mg \text{ x } \textit{Height of 1 step} \text{ x } \textit{Number of steps}$$
$$= 60 \text{ x } 0.2 \text{ x } 60$$

$$\therefore \textbf{Work done by the boy} = \textbf{720 J}$$

Step II: Now calculate the power developed from the formula:

$$\textbf{Power} = \frac{\textit{Work done}}{\textit{Time taken}}$$

$$= \frac{720}{10}$$

$$\therefore \textbf{Power} = \textbf{72 W}$$

Method II
Simply calculate the power from the formula

$$\text{Power} = \frac{mg}{t} \text{ x } \textit{Height of one} \text{ x } \textit{Number of steps}$$

$$= \frac{60 \text{ x } 10}{10} \text{ x } 0.2 \text{ x } 60$$

$$\therefore \textbf{Power} = \textbf{72 W}$$

7.3 ENERGY

Energy is defined as, *capacity of matter to perform work*. Or *Energy is the ability to work*.

(a) The S.I unit of energy
Like work, the SI unit of energy is joule, J.

Energy exists in various forms, including

- Mechanical, thermal, chemical, electrical, radiant, and atomic.

All forms of energy are interconvertable by appropriate processes. In the process of transformation either kinetic or potential energy may be lost or gained, but the sum total of the two remains always the same. This is in accordance with the *Law of Conservation of Energy*.

(b) The Law of Conservation of Energy

The law states that: *Energy can neither be created nor destroyed, but only changes from one form to another thus the sum total always remains the same.*

7.31 Forms of energy

Energy exists in various forms. The various forms are shown in the table below.

Form of energy		Example
Mechanical energy - Potential (gravitational) - Potential (elastic) - Kinetic energy)	Is the type of energy possessed by a body by a reason of its position at rest or on motion.	- Energy stored in a book resting on a table. - A stretched spring or catapult - A shot fired from a gun.
Chemical energy	The type of energy released during a chemical reaction.	- Energy stored in fuels and energy giving foods that becomes active when oxidized by oxygen. - Energy stored in battery cells.

Nuclear (atomic) energy	Is the energy released by nuclei of heavy atoms of disintegrating radioactive elements.	- Energy stored in the nucleus of an atom
Electrical energy	The type of energy produced by electric cells, generators etc.	- Energy produced in electric appliances such as cooker, filament lamp etc.
Radiant energy (heat and light)		- Infra-red rays.
Sound energy	Type of energy produced by vibrating objects.	- Vibrational energy.

Table 7.1

7.32 Sources of Energy

Sources of energy are the raw materials for production of energy. They can be classified into two main categories namely:

(i) Renewable energy sources
(ii) Non-renewable energy sources.

(a) Renewable energy sources

These are energy sources which cannot be exhausted.

Examples of renewable sources of energy include:

- Solar energy - energy tapped from the sun using solar panels.
- Hydroelectric energy - electric energy produced from falling water which rotates turbines connected to generator which intern produce electricity.
- Wind energy - electric energy produced from moving air which rotates turbines connected to generator which intern produce electricity.

(b) Non-renewable energy sources.

These are energy sources which once used cannot be replaced.

Examples of non-renewable sources of energy include:

- Fossil fuels (oil, coal and natural gas. Fossil fuels are formed from remains of plants and animals which have accumulated over millions of years).
- Nuclear energy.

7.33 Mechanical Energy

In mechanics, energy is divided into two kinds, namely:

- Potential energy and
- Kinetic energy.

(a) Potential energy (p.e)

Potential energy is the form of energy possessed by a body as a result of its position at rest.

For example, a body lifted to a height, *h*, above the surface of the earth is said to possess p.e.

When something is lifted vertically upwards, work is done against the gravitational force acting on the body (i.e its weight) and this work is stored in the body as gravitational potential energy.
Another example of Potential energy is the elastic potential energy stored in a stretched spring or catapult.

Formula of Potential Energy
Suppose a body of mass ***m*** kg is raised to a height of ***h*** metres at a place where the acceleration due to gravity is ***g*** m/s^2.

Then Force overcome (Weight), $F = mg$ ………………….…… (1)
Work done on the body $= F \times s$ …………………. (2)

Substituting equation (1) in equation (2) i.e replacing F in equation (2) with mg in equation (1) we have,
Work done on the body $= mg \times s$
But the work done = gravitational potential energy (p.e) and s = h,
$\therefore$ ***P.E*** ***= mgh***

Recall that: *m* = mass of the body in kg.
g = acceleration due to gravity (m/s^2).
h = height in metres.

Example

1. A box of mass 5 kg is raised to a height of 2 metres above the ground. Calculate the potential energy stored in the stone (take g = 10 ms^{-2})

Solution: Mass of box = 5 kg, gravitational field, g = $10m/s^2$ Height, h = 2 m,

Applying P.e = mgh
= 5 x 10 x 2
$\therefore$ **P.E = 100 J or 0.1 kJ**

2. A man has raised a load of 25 kg on a platform 160 cm vertically above the ground. If the value of gravity is $10m/s^2$, calculate the potential energy gained by the box when it is on the platform.

Solution: Mass of stone = 25 kg, gravitational field, g = $10m/s^2$
Height, h = 160 cm $= \frac{160}{100}$ = 1.6 m, P.E = ?
P.e = mgh
= 25 x 10 x 1.6
$\therefore$ **P.E = 400 J or 0.4 kJ**

(b) Kinetic energy (k.e)
Kinetic energy is the energy possessed by a moving body.

Examples of kinetic energy include:
- Moving bullet, Moving car, etc.

A body possessing k.e does work by overcoming resistance force when it strikes something.

Formula of Kinetic energy (k.e)
Kinetic energy can be calculated from the formula;

K.E $= \frac{1}{2}mv^2$ Where m = mass (kg), v = velocity (m/s) .

Note that: *Kinetic energy is directly proportional to the speed or velocity of a body. Therefore, the faster the body moves, the more the kinetic energy it has.*

The derivation of the above formula, K.E = ½mv² (NB: NOT important at 'O' Level)

Consider a body of mass, ***m,*** moving with an initial velocity, ***u,*** from rest and is acted upon by a force, ***F***. The force gives the body an acceleration, ***a,*** and acquires a final velocity, ***v,*** after covering a distance, ***s*** in metres.

These quantities are related by the equation of linear motion (See chapter 11).

$$v^2 = u^2 + 2as$$

Since u = 0 then $v^2 = 2as \Rightarrow a = \frac{v^2}{2s}$ …………………… (1)

Now *Work done* = *Force* x *Distance*
= *F* x *s*

But *F* = *ma* (Newton's second law of motion)

$\therefore$ Substituting for F, we have:
Work done = *mas* …………………………….(2)

Substituting equation (1) in equation (2), i.e. substituting for **a** in equation (2), we obtain

$$\text{Work done} = \frac{mv^2 s}{2s}$$
$$= \frac{1}{2}mv^2$$

By the law of conservation of energy, the work done by the force ***F*** in pushing the body through the distance ***s*** will all be converted into kinetic energy of the body.

$$\mathbf{K.E = \tfrac{1}{2}mv^2}$$

Worked Examples

1. Calculate the k.e of a bullet of mass 0.05 kg moving with velocity of 500 m/s.
Solution: m = 0.05 kg, v = 500 m/s, k.e = ?

Kinetic Energy $= \frac{1}{2}m\,v^2 = \frac{1}{2} \times 0.05 \times 500^2$ = **6,250 J or 6.25 kJ**

2. A 10 g bullet traveling at 400 m/s penetrates 20 cm into a wooden block. Calculate the average force exerted by the bullet.

Solution: $m = 10\text{ g} = \frac{10}{1000}$ kg, $v = 400$ m/s, distance = 20 cm = $\frac{20}{100}$ m, k.e = ?

Note: *This question seem to be difficult and quite different.*

Hint: *The work done in penetrating the block is related to the average force by the formula:*

Work Done = Fs, so find the work done first and then use the above formula to find F.

Kinetic Energy $= \frac{1}{2} \times m v^2 = \frac{1}{2} \times 10 \times 400^2 = \frac{1 \times 10 \times 400 \times 400}{2 \times 1000} = 5 \times 40 \times 4 = \mathbf{800\ J}$

This kinetic energy is converted into work in penetrating the wooden block

Applying Work done = Force x Distance

$$800 = F \times \frac{20}{100}$$

$$20\,F = 800 \times 100$$

$$F = \frac{800 \times 100}{20}$$

$$\therefore \mathbf{F = 4\,000\ N}$$

7.34 The Law of Conservation of Energy

The law states that: *Energy can neither be created nor destroyed, but only changes from one form to another.*

Interchange of Energy between P.e and K.e

Energy is interchangeable between p.e and k.e depending on the body's present state. This interchange between p.e and k.e is seen in the following:

(i) Simple pendulum.

(ii) Falling object e.g. stone.

(a) Simple pendulum

In a swinging pendulum bob, the energy of the bob can be either p.e or k.e or both. This is explained in the diagram below.

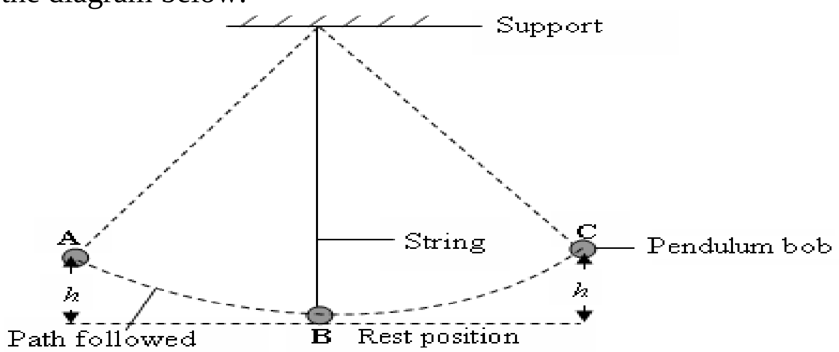


Figure 7.2

Facts about a swinging pendulum

- At the extreme ends (A and C) of the swing, the energy is all potential energy and maximum since h is maximum at A and C.
- When passing through the rest position (i.e B), it is all kinetic energy and maximum. This is because at B, v = maximum and h = 0. And since h = 0, P.e = 0.
    - While at the intermediate points (i.e between AB and BC) the energy is partly kinetic and partly potential.

(b) Falling object e.g stone

Consider a piece of stone raised to a certain height above the ground level and let to fall.

At the maximum height, it possesses potential energy and no kinetic energy.

As the stone falls, its velocity increases. Since kinetic energy is directly proportional to the square of the velocity, then the k.e of the stone increases at the expense of the p.e (i.e at any particular moment, the stone possesses both p.e and k.e).

Ignoring the energy losses due to the air resistance, then the loss in p.e is equal to the gain in k.e in accordance with the *Law of Conservation of Energy*.

Consider the diagram below

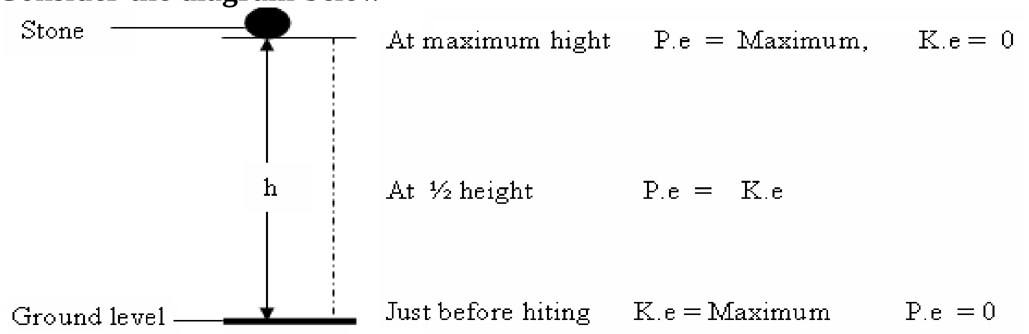


Figure 7.3

Note that: *Potential energy decreases from maximum to zero and the kinetic energy increases to maximum. This is because as the h value decreases to zero, the velocity increases to maximum.*

(c) Energy Transformations

By means of suitable mechanisms and apparatus, energy can be transformed from one form to another. This is shown in the table 7.2 below.

Activity	Energy Transformation
1. A boy running up a stair case	Chemical energy in the muscles is converted to K.E and then to p.e. (C.E $\rightarrow$ K.E $\rightarrow$ P.E)
2. Running water at a hydroelectric power station (water turning turbine which finally drives a generator)	P.E is converted to K.E and then electrical energy. (P.E $\rightarrow$ K.E $\rightarrow$ E.E)
3. A stone dropped from rest at a certain height until it hits the ground.	P.E is converted K.E then to heat and sound energy. (P.E $\rightarrow$ K.E $\rightarrow$ H.E $\rightarrow$ S.E)
4. A moving car	Chemical energy due to the burning of fuel in the engine is converted to heat energy which is converted by pistons to kinetic energy. (C.E $\rightarrow$ H.E $\rightarrow$ K.E)
5. A coal fired engine drives a dynamo which lights a bulb.	Chemical energy is converted to heat energy, kinetic energy, electrical energy and lastly to light energy. (C.E $\rightarrow$ H.E $\rightarrow$ K.E $\rightarrow$ E.E $\rightarrow$ L.E)
6. A torch bulb flashing.	Chemical energy is converted to electrical energy, light energy and heat energy. (C.E $\rightarrow$ E.E $\rightarrow$ L.E $\rightarrow$ H.E)

Figure 7.2

7.4 Engines

An engine is a machine for converting energy into motion or mechanical work. The energy is usually supplied in the form of a chemical fuel, such as oil or gasoline, steam, or electricity, and the mechanical work is most commonly delivered in the form of rotary motion of a shaft.

(a) Classification of Engines

Engines are usually classified according to the following:

(i) The form of energy they utilize, as:
- *Steam*, *compressed air*, and *gasoline*;

(ii) The type of motion of their principal parts, as:
- *Reciprocating* and *rotary*;

(iii) The place where the exchange from chemical to heat energy takes place, as:
- ***Internal-combustion*** and *external combustion*;

(iv) The method by which the engine is cooled, as:
- *Air-cooled* or *Water-cooled*;

(v) The position of the cylinders of the engine, as:
- *V*, *in-line*, and *radial*;

(vi) The number of strokes of the piston for a complete cycle, as:
- *Two-stroke* and *Four-stroke*;

(vii) The type of cycle, as:
- *Otto* (*in ordinary gasoline engines*) and *diesel*; and

(viii) The use for which the engine is intended, as:
- *Automobile* and
- *Airplane engines*.

NB: *Engines are often called motors, although the term motor is sometimes restricted to engines that transform electrical energy into mechanical energy. Other specialized engines are the windmill, gas turbine, steam turbine, and rocket and jet engines.*

(b) Internal Combustion Engine (Heat Engines)

A heat engine is a machine which changes heat energy (obtained by burning fuel) to kinetic energy. The common heat engines are:

(i) Petrol Engine and
(ii) Diesel Engine

In internal combustion engine e.g. petrol engine or diesel engine, fuel is burnt in the cylinder where the energy change occurs.

(c) The Components of Internal-Combustion Engine

The basic components of an internal-combustion engine are:

(i) *The engine block,* - made of cast iron or aluminum alloy and houses the cylinders, pistons, and crankshaft.

(ii) *Cylinder head* - This is the upper part of the engine. It is bolted to the top of the block and seals the tops of the cylinders.

(iii) *Cylinders* - This refers to the cylindrical space in which the piston reciprocates (i.e. moves freely up and down).

(iv) *Pistons* -These are cylindrical metals fitted with rings for tight fitting. They move up and down thus compressing the mixture of air and fuel against the cylinder head prior to ignition.The top of the piston forms the floor of the combustion chamber.

(v) *Valves* An engine valve is a metal shaft with a disk at one end fitted to block

the opening. And the other end of the shaft is mechanically linked to camshaft.

There are two valves:

- *Inlet valve* - Controls the entry of fuel vapour in to the combustion engine.
- *Outlet valve* - control the exit of exhaust gases out of the combustion chamber.

(vi) *Crankshaft* This is the part of the engine which transforms the reciprocating motion of the piston into rotary motion.
It rotates at speeds ranging from about 600 to thousands of revolutions per minute (rpm), depending on how much fuel is delivered to the cylinders thus allowing the up and down movement of the piston.

(vii) *Camshaft* This a round rod with odd-shaped lobes located inside the engine block or in the cylinder head.

7.41 Petrol (Otto-cycle) engine (Four-stroke engine)

The ordinary Otto-cycle engine is a four-stroke engine; that is, in a complete power cycle, its pistons make four strokes, two toward the head (closed head) of the cylinder and two away from the head. In a gasoline engine a volatile mixture of fuel and air is ignited within a cylinder causing a sudden expenditure of gases. The expanding gases push down on a piston which turns the crankshaft.

A stroke is one movement of the piston either up or down. Most cars use four stroke cycle. The piston moves four strokes and the repeats the action continuously.

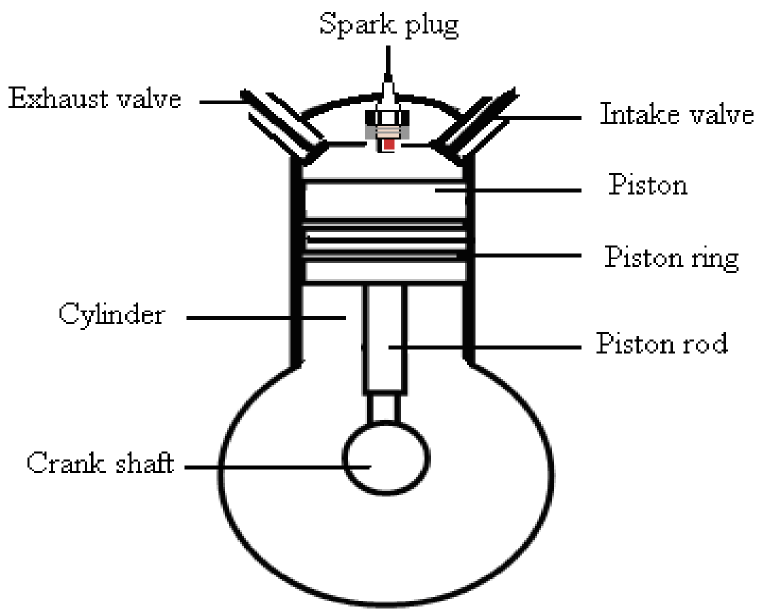


Figure 7.5

Mechanism of a four stroke engine

The mechanism of the four stroke engine is divided into:

- Intake stroke, Compression stroke, Power stroke and Exhaust stroke.

What happen in every stroke are illustrated in the diagrams below.

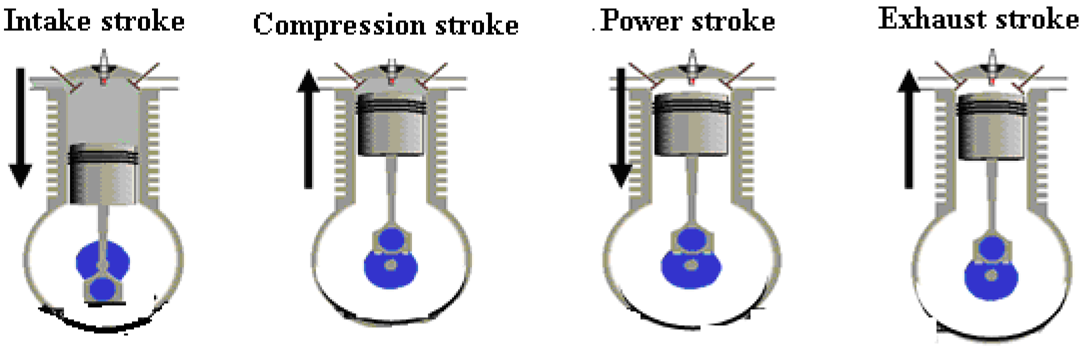


Figure 7.6

1. **Intake**
During in take (the first stroke of the cycle), the piston moves down (i.e. away from the cylinder head), the intake valve opens. A quantity of a fuel and air mixture is drawn into the combustion chamber.

2. **Compression stroke**
During the compression stroke, the valve closes, piston moves up and the fuel-air mixture is compressed.

3. **Power stroke (**the moment when the piston reaches the end of this stroke)
In the power stroke, both valves close and the volume of the combustion chamber is at a minimum, the spark plug produces electric spark, the fuel mixture ignites and burns. The expanding gaseous products exert pressure on the piston and force it down.

4. **Exhaust stroke**
During the final stroke (exhaust stroke), the exhaust valve opens and the piston moves up, driving the exhaust gases out of the combustion chamber and leaving the cylinder ready to repeat the cycle.

7.42 Diesel Engine

A Diesel Engine is a type of internal-combustion engine in which heat caused by air compression ignites the fuel.

The essential parts of diesel engines are similar to gasoline internal-combustion engines. But they differ in the following ways.

- They do not use spark plugs for igniting the air-fuel mixture.
- Diesel engines compress air inside the cylinders with greater force than a gasoline engine does, producing temperatures hot enough to ignite the diesel fuel on contact.
- Combustion takes place at constant volume rather than at constant pressure.
- During the in take stroke, is only drawn.

Like petrol engines, most diesel engines are four-stroke engines but they operate differently than the four-stroke Otto-cycle engines.

Mechanism of Four stroke Diesel Engines

1. **Intake stroke** During the intake stroke, the inlet valve opens; air is drawn into the combustion chamber.

2. **Compression stroke** On the second, or compression, stroke the air is compressed to a small fraction of its former volume and is heated to approximately 440° C by this compression.

3. **Power stroke** At the end of the compression stroke, vaporized fuel is injected into the combustion chamber and burns instantly because of the high temperature of the air in the chamber. This combustion forces the piston down.

4. **Exhaust stroke** During the final stroke (exhaust stroke), the exhaust valve opens and the piston moves up, driving the exhaust gases out of the combustion chamber and leaving the cylinder ready to repeat the cycle.

Facts about Diesel Engines

- ❖ Diesels engines are, in general, slow-speed engines with crankshaft speeds of 100 to 750 revolutions per minute (rpm) as compared to 2500 to 5000 rpm for typical Otto-cycle engines.
- ❖ Some types of diesel engine, however, have speeds up to 2000 rpm. Because they use compression ratios of 14 or more to 1, they are generally more heavily built than petrol engines, but this disadvantage is counterbalanced by their greater efficiency and the fact that they can be operated on less expensive fuel oils.
- ❖ They are commonly used in large trucks or buses and machinery.

Advantages of diesel engines over petrol engines

(i) Diesel engines are more efficient than petrol engines

(ii) They consume less fuel and therefore, they are less expensive to operate than gasoline-powered engines, partly because diesel fuel costs less.

(iii) Diesel engines emit fewer waste products than petrol engines.

Disadvantage of diesel engines

Diesel engines produce sooty and smelly smoke.

7.43 Two-Stroke Engines

Mechanism of two-stroke diesel engine

- ❖ In the two-stroke cycle, the fuel mixture or air is introduced through the intake port when the piston is fully withdrawn from the cylinder.
- ❖ The compression stroke follows, and the charge is ignited when the piston reaches the end of this stroke.
- ❖ The piston then moves outward on the power stroke, uncovering the exhaust port and permitting the gases to escape from the combustion chamber.

NB:
- *The power of a two-stroke engine is usually double that of a four-stroke engine of comparable size.*
- *The general principle of the two-stroke engine is to shorten the periods in which fuel is introduced to the combustion chamber and in which the spent gases are exhausted to a small fraction of the duration of a stroke instead of allowing each of these operations to occupy a full stroke.*

7.44 Carburetor and Fuel-Injection System

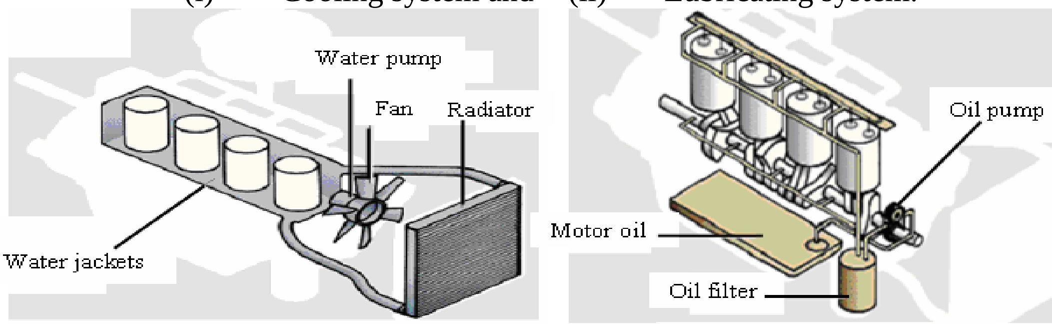
(a) **Carburetor** A carburetor is a device that mixes fuel and air for burning in an internal-combustion engine. It atomizes (converts into a vapor of tiny droplets) liquid gasoline. An air-flow carries the atomized gasoline to the engine's cylinders, where the gas is ignited.

(b) **Fuel-Injection System**
A fuel-injection system is a system of delivering fuel to an internal-combustion engine. In a fuel-injection system, electronically controlled fuel injectors spray measured amounts of fuel into each of the engine's cylinders where the fuel is burned to power the engine.
The fuel-injection system replaces the carburetor in most new vehicles. Its advantage over carburetor is that it provides a more efficient fuel delivery system.

(c) **Factors limiting the efficiency heat engines**
The efficiency of a modern Otto-cycle engine is limited by a number of factors. These include energy losses: (i) By cooling and (ii) Friction.

(d) **Improving the efficiency of heat engines**
The efficiency of heat engines is increased by equipping all engines with:

(i) Cooling system and (ii) Lubricating system.



(a) Cooling system *Figure 7.7* (b) Lubricating system

(a) **Cooling System**
Cooling in engines is achieved by circulating water. A water pump circulates engine coolant, a mixture of water and antifreeze, through the non-moving parts of the engine to absorb heat. The coolant routes through tubes in the radiator, where heat passes through the tubes into the metal fins. A fan blows air through the fins to increase the rate of cooling. In addition to this, the radiator is painted black in order to increase the rate of cooling since 'black colour' is a good emitter of heat energy.

(b) **Lubricating System**
In the lubricating system, a pump circulates motor oil, the main lubricant in an auto mobile engine is called *galleries.* It is circulated to all the moving parts of the engine. The lubricating system reduces the friction produced by the engine's moving parts, which rub against each other thousands of times per minute.

NB: *Before the oil circulates, it passes through an oil filter which strains particles from the oil.*

Self-Check 7.0

1. A crane raises a mass of 500 kg vertically upwards at a speed of 10 ms^{-1}. Find the power developed

A. 5.0×10^0 B. 5.0×10^1 C. 5.0×10^2 D. 5.0×10^4

2. A girl whose mass is 50 kg runs up a staircase 25 m high in 4 s. Find the power she develops.

A. $\frac{50 \text{ x } 4}{25}W$ B. $\frac{50 \text{ x } 10}{25 \text{ x } 4}W$ C. $\frac{50 \text{ x } 25}{4}W$ D. $\frac{50 \text{ x } 10 \text{ x } 25}{4}W$

3. A train traveling at a constant speed of 20 m/s overcomes a resistive force of 8 kN. The power of the train is

A. (8 x 20) W B. (8x10x20) W C. (8 x 100 x 20) W D. (8 x 1000 x 20) W

4. A pump is rated at 400W. How many kilograms of water can it raise in one hour through a height of 72m?

A. 0.8kg B. 5.6kg C. 33.3kg D. 2000kg

5. A boy carrying a load of 6 kg runs upstairs. If the work that the boy does is 300 J, find the height of the stairs.

A. 3m B. 5m C. 6m D. 10m

6. Tony can pull a box 2m in 5 sec. Ever (Tony's sister) can pull the same box in 10 sec. Assuming both apply the same force, what is the ratio of Tony's power to the sister's power = ?

A. 1 B. 2 C. ½ D. 4

7. An engine exerts a force of 2000N at a speed of 15ms^{-1}. Find the power developed by the engine in kW.

A. 30 000 B. 3 000 C. 300 D. 30

8. A constant force of 5N acts on a body and moves it through a distance of 20m in 10 seconds. Calculate its power.

A. 2.5W B. 10W C. 40W D. 100W

9. A mouse of mass 0.03 kg climbs through a distance of 2 m up a wall in 4 s. The power expended in watts is

A. 0.03 x 2 x 4 x 10 B. $\frac{0.03 \text{ x } 4 \, x \, 2}{10}$ C. $\frac{0.03 \text{ x } 4 \, x \, 10}{2}$ D. $\frac{0.03 \text{ x } 10 \text{ x } 2}{4}$

10. A bullet of mass 0.02kg is fired with a speed of 40m s^{-1}. Calculate its kinetic energy.

A. 0.4 J. B. 0.8 J. C. 16 J. D. 32 J.

11. Which of the following statements is true about an electric motor? It changes

A. Kinetic energy to electric energy B. Electrical energy to light energy
C. Electrical energy to kinetic energy D. Chemical energy to electrical energy

12. A body pulls a block of wood with a force of 30N through a distance of 300m in 2 minutes. Find the power he develops, if he pulls the block at a constant speed.

A. $\frac{30 \text{ x } 300}{2}$ B. $\frac{30 \text{ x } 300}{2 \text{ x } 60}$ C. $\frac{30 \text{ x } 2 \text{ x} 300}{300}$ D. $\frac{300}{2 \text{ x } 60 \text{ x } 30}$

13. A ball of 1kg bounces off the ground to a height of 2m after falling from a height of 5m, find the energy lost.

A. 5 J B. 20 J C. 30 J D. 50 J

14. A man weighing 800N climbs a vertical distance of 15m in 30s. What is the average power out put?

A. 80/3 W **B**. 800/15 W **C**. 400 W **D**. 5 kW

15. In which action(s) below is there a work done?

I. Pushing a wall without moving it. II. Taking a book from a table to a higher shelf.

III. Walking on a bridge for 50 m

A. I only B. II only C. III only D. II and III only

16. A bullet of mass 5g is fired at a speed of $400ms^{-1}$. How much energy does it have?

A. ½ x 5 x 10^2 x 400J B. ½ x 5 x 10^3 x 400J

C. ½ x 5 x 10^{-3} x 400 x 400J D. ½ x 5 x 10^2 x 400 x 400J

17. Which of the following forms mechanical energy?

A. Electrical energy and kinetic energy B. Potential energy and nuclear energy

C. Nuclear energy and kinetic energy D. Potential energy and kinetic energy

18. An object, of mass 2kg, dropped from the top of a building hits the ground with kinetic energy of 900J. The height of the building is

A. 30m B. 45m C.90m D. 180m

19. A mass attached to the end of a string moves up and down to maximum and minimum points X and Y as shown in figure 7.1 below. When the mass is at X the

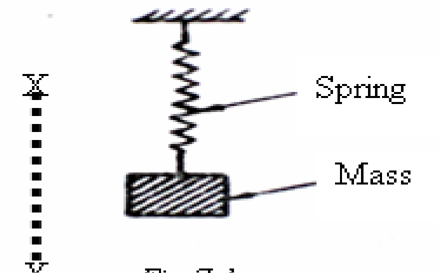


Fig 7.1

A. kinetic energy is maximum, potential energy is minimum
B. kinetic energy is zero, potential is maximum
C. kinetic energy is equal to potential energy
D. kinetic energy and potential energy are both zero

20. An electric motor of power 500 watts lifts an object of 100 kg. How high can the object be raised in 20 sec?

A. 40m **B**. 30m **C**. 20m **D**. 10m

21. A motor can pull a 400 kg box up to a height of 10m in 4 sec. What is the power of the motor in kW?

A. 10 **B**. 20 **C**. 30 **D**. 40

22. The diagram in the figure 3 shows an oscillation pendulum lob. Which of the following statements is true about its motion?

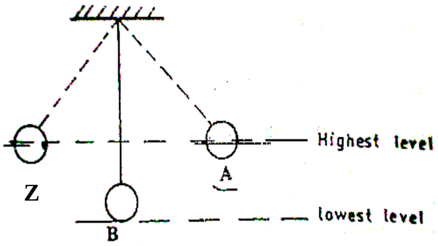
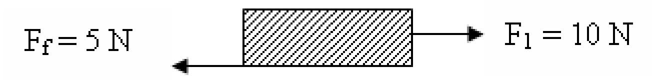


Figure 7.2

A. the K.E at B is equal to the K.E at A
B. the K.E at B is less than the P.E at A
C. the K.E at B is equal to the P.E at A.
D. the K.E at B is greater than the P.E at Z.

23. A toy car is pulled with a force of 10 N for 5m. If the friction force between the block and the surface is 5N, what is the net work done on the toy car?



A. 50 J B. 100 J C. 200 J D. 25 J

24. The energy changes that take place when a stone falls freely from rest to the ground can be orderly arranged as:
A. Kinetic energy → Potential energy → Sound energy → Heat.
B. Sound energy → Potential energy → Kinetic energy → Heat.
C. Potential energy → Sound energy → Kinetic energy → Heat.
D. Potential energy → Kinetic energy → Heat energy → Sound.

25. Ali and Veli move identical boxes equal distances in a horizontal direction. Since Ali is a weak child, the time needed for him to carry his box is two times longer than for Veli. Which of the following is true for Ali and Veli.
A. Ali does less work than Veli B. Veli does less work than Ali.
C. Each does the same work. D. Neither Ali nor Veli do any work

SECTION B

26. (a) Define the following terms.
(i) Work. (ii) Power.
(b) State and define the SI units of the terms you have defined above.
(c) A crane lifts a load of 3500 N through a vertical height of 5 m in 5 second.
Calculate: (i) the work done.
(ii) the power developed by the crane.

27. (a) Define the term energy and state the SI unit for measuring it.
(b) Distinguish between potential energy and kinetic energy.
(c) A block of mass 2 kg falls freely from rest through a distance of 3m. Find the kinetic energy of the block.

28. (a) Define a joule.
(b) Describe briefly how you can measure your power.
(c) A boy of mass 45 kg runs up a flight of 60 steps in 5 seconds. If each step is 12 cm.
Calculate: (i) the work done against gravity by the boy.
(ii) the power developed by the boy.

29. (a) (i) State the types of heat engines you know.
(ii) Describe the mechanism of operation of a four stroke petrol engine.
(b) (i) What are the factors that affect the efficiency of an engine?
(ii) State how the factors you have stated in (c) above are minimized in a heat engine.

CHAPTER EIGHT
MACHINES

LEARNING OBJECTIVES

***By the end of this chapter, you should be able to*:**

1. *Define:* - *Machine,*
   - *Mechanical Advantage (M.A),*
   - *Velocity Ratio (V.R),*
   - *Efficiency (η).*
2. *State:* - *The types of machines,*
   - *Classes of levers,*
   - *Practical uses of machines.*
3. *Carry out calculation on:*
   - *M.A, V.R and η for the various types of machines.*
4. *Sketch graphs of:* - *M.A against load and*
   - *η against load.*
5. *Explain:* - *The shapes of the graphs and*
   - *Why the graph for efficiency against load levels below 100%.*

8.1 Machine

Definition: *A machine is a device that makes work easier for man*.

The “machine” is a symbol of modern life. In machines an effort (force) is applied to move a load. The effort can be: - Muscular effort from man or
- Force derived from an engine.

Principle of Simple Machines

The principle used in simple machines is to produce a big force over a small distance by using a small force over large distance. The force which we apply to the machine is known as effort (E) and the force which we have to overcome is known as load (L).

8.11 Types of machines

There are two types of machines namely:

- Simple machines and
- Complex machines.

(a) Simple Machines

These are devices that work with one movement and change the size and direction of forces. Examples of simple machines are simple tools with one or two parts. Although simple machines provide the advantage of using less force (effort) and thus making the work easier, they do not reduce the amount of work.

(b) Kinds of Simple Machines

There are six kinds of simple machines. They include the following:

- Lever, Pulley, Wheel and Axle, Inclined plane, Gears, Screws.

Note: *The screw and the wedge are modified forms of inclined plane. So we can say that basically there are four classes of simple machines.*

8.12 **Levers:** A lever is a rigid bar which is free to move about a fixed point known as the fulcrum or pivot. It is divided into classes.

Classes of Lever

There are there are three classes of levers, namely:

(i) the first class lever
(ii) the second class lever and
(iii) the third class lever.

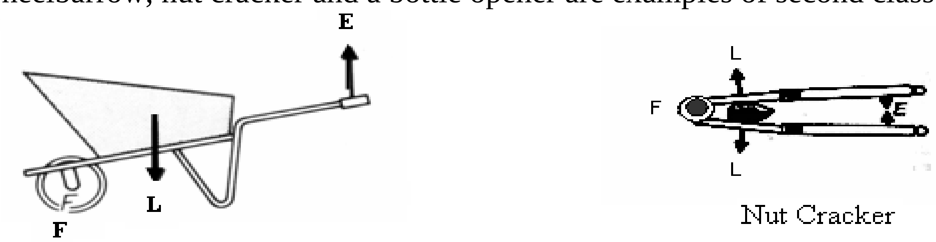
(a) First Class Lever

A first class lever is the type of lever in which the fulcrum is in between the effort and the load. Examples of first class levers are:

Pair of scissors   Figure 8.1   Pair of Pliers

(b) Second Class Lever

A second class lever is a lever in which the load is between the effort and the fulcrum. A wheelbarrow, nut cracker and a bottle opener are examples of second class levers.



Wheel barrow   Figure 8.2   Nut Cracker

(c) Third Class Lever

A third class lever is a lever in which the effort is between the load and fulcrum.

Examples of third class levers are:

A pair of tong   Figure 8.3   Table knife

Memory Aid

1. **E**dward **F**eels **L**onely **E**dward **F**eels **L**onely

First Class Lever

Second Class

Third Class Lever

2. Use the word **PLE** to remember the classes of lever. When each letter is in the middle beginning from P to E, the order of the classes starts from 1^{st} to 3^{rd}. See summary in the table below.

Position of letter	Class of lever
L ***P*** E	**1^{st}**
P ***L*** E	**2^{nd}**
P ***E*** L	**3^{rd}**

***PLE** stands for Pivot, Load and effort respectively.*

8.13 Mechanical Advantage (M.A)

The relation-ship between the load (L) and the effort (E) acting on a simple machine is called *mechanical advantage*. In ideal conditions (condition where there is no loss in energy due to friction) mechanical advantage is defined as: - *the ratio of the load to the effort.*

$$\text{Mechanical Advantage (M.A)} = \frac{Load\ (L)}{Effort\ (E)}$$

$$\boldsymbol{M.A} = \frac{L}{E}$$

NB: *Mechanical advantage has no unit. This is because it is the ratio of the same quantity i.e. force.*

Examples

1. If a lever can be used to overcome a load of 50 N by applying an effort of 10 N. Find the M.A of the ever.

Solution: L = 50 N, E = 10 N, M.A = ?

$$M.A = \frac{L}{E} = \frac{50}{10} = \mathbf{5}$$

2. (a) A lever is used to overcome a load of 2000 N. If the M.A 2, calculate the effort applied.
   (b) If the same lever is used to overcome a load by applying an effort of 50 N, determine the maximum load that can be overcome.

Solution:

(a) L = 2000 N, E = ?, M.A = 2

$$M.A = \frac{L}{E}$$

$$2 = \frac{2000}{E}$$

$$2E = 2000$$

$$E = \frac{2000}{2}$$

$$\mathbf{E = 1000\ N}$$

(b) L = ?, E = 50 N, M.A = 2

$$M.A = \frac{L}{E}$$

$$2 = \frac{L}{50}$$

$$L = 2 \times 50$$

$$\mathbf{E = 100\ N}$$

8.2 Pulleys

A pulley is a wheel with a grooved rim over which passes a rope or string. The effort is applied to one end of the rope and the disk of the pulley rotates as the rope moves over it.

There may be several pulleys in a frame work called a *block.*

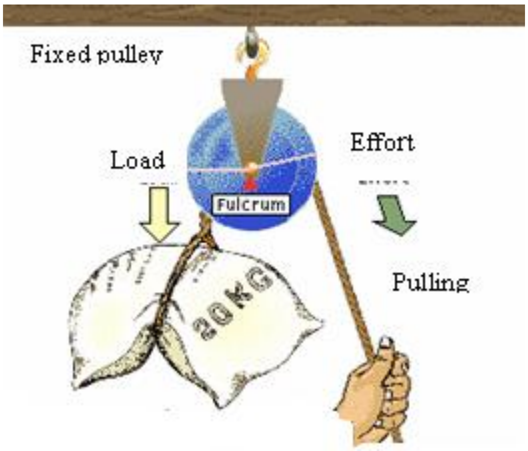
Types of Pulleys

There are several types of pulleys in use. But the most common ones are:

- Single fixed pulley.
- Single movable pulley and
- Block and Tackle system.

Pulleys reduce the effort to lift an object by increasing the distance over which the effort is applied.

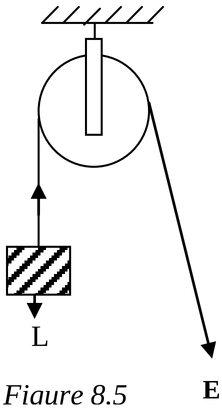
8.21 A Single Fixed Pulley



A single fixed pulley is a type of pulley fixed to a support usually up as shown in figure 8.4. A rope passes over the groove of the pulley. One end of the rope is attached to the load and the effort is applied at the other end.

Common examples of a single pulley can be found at the top of a flagpole and in construction sites. Pulling down on the rope causes the flag (load) to move upward. Thus, the pulley changes the direction of the force applied to the rope.

(a) Facts about the Single fixed pulley



Fiaure 8.5

- ❖ A single fixed pulley is like first class lever because its fulcrum is in between the load and the effort.
- ❖ The force arms of the pulley are equal to each other, therefore: E = L in an ideal machine. However, in practice the effort applied to overcome the load must be greater than the load. This is due to friction in the moving parts of the pulley.
- ❖ The tension is the same throughout the rope.
- ❖ If we pull the rope down by *x* m, the load rises up by the same distance, *x* m, and gains a potential energy.
- ❖ It diagram is always drawn as simple as in figure 8.5.

(b) To Show that for an ideal condition E = L (i.e. Neglecting friction)

Neglecting friction, in the system we have:

Work in put = Work out put

Work done by effort = Work done by load

Effort x Effort distance = Load x Load distance

$$E \text{ x } x\,\text{m} = L \text{ x } x\,\text{m}$$

$$E = \frac{L \text{ x } x\,m}{x\,m}$$

$$\mathbf{E = L}$$

(c) Mechanical Advantage (M.A) of a Single Fixed Pulley

In a single fixed pulley, the tension is the same throughout the rope. Neglecting the weight of the string and the frictional forces in the pulley, we have;

$$\text{Load} = \text{Effort}$$

$$\text{M.A} = \frac{Load}{Effort}$$

$$= \frac{L}{E} \qquad \text{But } L = E$$

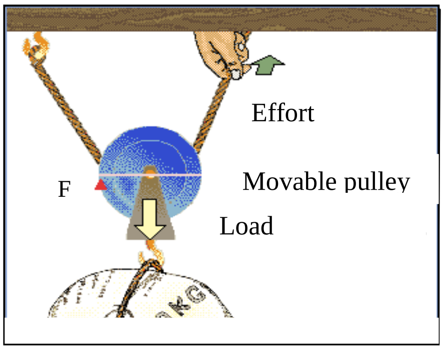
$$\therefore \quad \textbf{M.A} = \textbf{1}$$

NB: *(i)* *M.A = 1 means that there is no gain in either force or distance in a fixed pulley, therefore, no mechanical advantage.*

(ii) *In reality, the actual MA is slightly less than 1 because of the friction of the rope against the pulley and the friction between the pulley and the axle on which it turns.*

(iii) *The primary benefit of a single pulley is to change the direction of the force or to move a load to a point (such as the top of a flagpole) that cannot be reached by the user.*

8.22 A Single Movable Pulley



Fiaure 8.6

A Single movable pulley is a pulley which moves along a rope with the load attached to it. One end of the rope is tied to a fixed support and passes over the pulley and the other end where the effort is applied goes back up to the user.

(a) Facts about a single movable fixed pulley

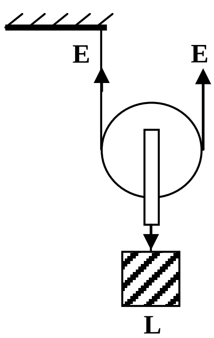


Figure 8.7

- ❖ A single movable pulley reduces the amount of effort needed to lift the load. i.e. half the load is necessary to lift a given load.
- ❖ If we pull the rope up by x m, the load rises by half the effort distance (i.e. ½ x m).
- ❖ The tension in the string is equal to the effort applied, so that the total upward pull is twice the effort i.e. 2E.
- ❖ For simplicity, its diagram is drawn as in figure 8.7

(b) Mechanical Advantage of Single movable pulley

Let load of **L** N be supported by the pulley and that the weight of the pulley and the string plus the frictional force in pulley are negligible.

Since the load is supported by two sections of the string, the effort applied is equal to **2E**.

Load = L N and Effort = 2E N But Effort = Load

In equilibrium: Down ward force = Upward force

$$L = 2E$$

$$\frac{L}{E} = 2 \qquad \text{But } \frac{L}{E} = \text{M.A}$$

Alternatively From $L = 2E$

$$E = \frac{L}{2}$$

Substituting for L and E in $\text{M.A} = \frac{L}{E}$ we have;

$$\text{M.A} = L \div \frac{L}{2}$$

$$= L \times \frac{2}{L}$$

$$\therefore \quad \textbf{M.A} = \textbf{2}$$

NB: *(i) The MA of a movable pulley (or a system of pulleys with a movable part)equals the number of strands of rope coming from the movable part (the load being lifted).*

(ii) Since a pulley system with an MA of 2 increases the force by a factor of 2, the pulley system must also double the distance the effort travels. Therefore, in order to raise a load a given distance, the user must pull and take in twice as much rope.

Worked Example

1. An effort E is used to raise a load of 200 N. If the effort moves through a distance of 4 m, calculate: (i) the effort (ii) the M.A and (iii) the load distance.

Solution: E = ?, M.A = ?, Effort distance = ?, Load distance = 4 m, L = 200 N.

(i) In equilibrium;

Down ward force = Upward force

$$L = 2E$$

$$E = \frac{L}{2} = \frac{200}{2} = \textbf{100 N}$$

(ii) Mechanical Advantage,

$$\text{M.A} = \frac{\textit{Load}}{\textit{Effort}}$$

$$= \frac{200}{100}$$

$$\therefore \quad \textbf{M.A} = \textbf{2}$$

(iii) Neglecting friction, in the system we have:

Work in put = Work out put

Work done by effort = Work done by load

Effort x Effort distance = Load x Load distance

$E \times E.d = L \times Ld$

$100 \times 4 = 200 \times Ld$

$$Ld = \frac{100 \times 4}{200}$$

$$\therefore \quad \mathbf{Ld = 2\ m}$$

8.3 Block and Tackle System

A block and tackle system is a pulley system consisting of both fixed and movable pulleys.

In this system there are two blocks each block contains one or more pulley(s), depending on the M.A required. A single rope is used and passes over the groove of each pulley. The frame work of pulleys is called *block* and the rope passing over each pulley is called the *tackle*.

(a) Practical applications where Block and Tackle system is used

They are commonly used to raise or move load in:

- Sailing ships.
- Cranes,
- Lifts and
- Brake downs.

Diagrams Showing Block and Tackle Systems

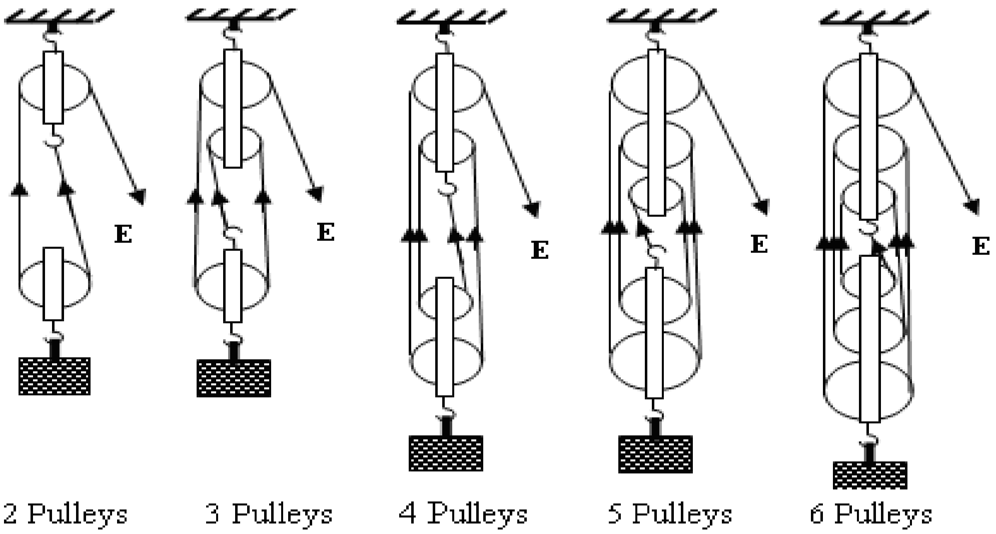


Fig. 8.8

Note: (i) *For an odd number of pulleys in the system, the fixed upper block contains more by one and the string starts from the lower block.*

(ii) The number of strings supporting the lower movable block is equal to the number of pulleys in the system.

(i) M.A of A Block And Tackle System

In block and tackle system an effort **E**, is applied and creates a tension, **T**, in the string. The movable lower block is acted upon by ***n*** sections of the string pulling it upwards and the load, **L,** pulling downwards.

For equilibrium in a perfect machine,

Downward force = Upward force

Load = Total upward tension

L = nT But T = E

L = n E

Substituting for L in M.A $= \frac{L}{E}$ we obtain:

$$\text{M.A} = \frac{nE}{E}$$

$$\therefore \textbf{M.A} = \textbf{n}$$

Where: ***n*** = the number of strings supporting the lower block and is also
= the total number of pulleys in the system.

NB: (i) The more the number of pulleys in the system, the higher the M.A and easier to do work.

(ii) In practice, however, the M.A is always less than the number of string supporting the lower block or less than the number of pulleys in the system.

Reason: Extra effort is required to overcome:
(i) Friction in the moving parts of the pulleys
(ii) The weight of the movable pulley and the string.

(c) Velocity Ratio (V.R) or Speed Ratio

Definition: *The velocity ratio of a machine is defined as: the ratio of the distance moved by the effort to the distance move by the load.*

$$\text{Velocity Ratio (V.R)} = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

$$\text{V.R} = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

In any system of block and tackle, to raise the load by a distance of x m, each string supporting the load shortens by x m. The effort is therefore applied through a total distance of ***nx*** metres.

From
$$\text{V.R} = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

$$\text{V.R} = \frac{nx}{x}$$

$$\textbf{V.R} = \textbf{n}$$

∴ the V.R of a block and tackle system* = *the number of strings supporting the lower block.
= *the number of pulleys in the system*

(d) Calculations for Perfect pulley systems

Steps in problem solving
Before solving any problem, ask your self the following questions.

- What is asked in the question?
- What information is given to help you to solve the problem?
- What are the equations to solve the problem?

These questions can only be answered when you collect the data.

Example 1
A force of 10 N is required to raise a load, L, using a smooth (frictionless) and weightless block and tackle system of four pulleys. Calculate:

(a) (i) Load,
(ii) M.A and
(b) Effort distance if the load rises by 2 m.

Solution: E = 10 N, M.A = ?, Effort dist = ?, Load dist = 2 m, L = ?, No. of pulleys = 4

(a) (i) In equilibrium;

Down ward force = Upward force

$$L = 4E$$
$$= 4 \times 10$$
$$\therefore \mathbf{L = 40\ N}$$

(ii) Mechanical Advantage,

$$M.A = \frac{Load}{Effort}$$
$$= \frac{40}{10}$$
$$\therefore \mathbf{M.A = 4}$$

(iii) Neglecting friction, in the system we have:

Work in put = Work out put
Work done by effort = Work done by load
Effort x Effort distance = Load x Load distance

$$E \times E.d = L \times L.d$$
$$10 \times E.d = 40 \times 2$$
$$E.d = \frac{40 \times 2}{10}$$
$$\therefore \mathbf{E.d = 8\ m}$$

8.31 The Principle of Work

The Principle of Work states that:
The amount of work out put is always less than the amount of work input.

(a) Efficiency (η) of a Machine

Definition: *The efficiency of a machine is the ratio of work output to work input.*

Efficiency is always expressed in percentage form i.e.

$$\textbf{\textit{Efficiency}} = \frac{\textit{Work output}}{\textit{Work input}} \times 100$$

NB: *Work output = Work done by the load*
Work input = Work done by the effort

(b) Relation between M.A, V.R and η

From Efficiency $= \dfrac{Work\ output}{Work\ input}$ x 100

Efficiency $= \dfrac{Load\ x\ distance\ moved\ by\ load}{Effort\ x\ distance\ moved\ by\ effort}$ x 100

Efficiency $= \dfrac{Load}{Effort}$ x $\dfrac{Ddistance\ moved\ by\ load}{Distance\ moved\ by\ effort}$ x 100

$= M.A \text{ x } \dfrac{1}{V.R}$ x 100

$$\eta = \frac{M.A}{V.A} \textbf{ x } 100$$

NB: (a) *This equation is useful only for solving problems. It is not a fundamental definition for efficiency and should* ***NOT*** *be used as such.*

(b) *Efficiency of a machine is always less than 100.*
Reasons: (i) Because of frictional forces between the moving parts of a machine.
(ii) For the case of pulleys, other reasons are:
- The weight of the string and
- the weight of the lower movable block.

(c) *Efficiency of a machıne can be ıncreased by:*
(i) Lubrıcatıng the movıng parts of the machıneç
(ii) For the case of pulleys, by makıng the strıng and the block plus the pulley(s) as lıght as possıble.

8.32 Graphical Relations between M.A and Load and Efficiency and Load

When we plot M.A or η against load, the following graphs are obtained.

The graph of M.A = n against Load **The graph of η against Load**

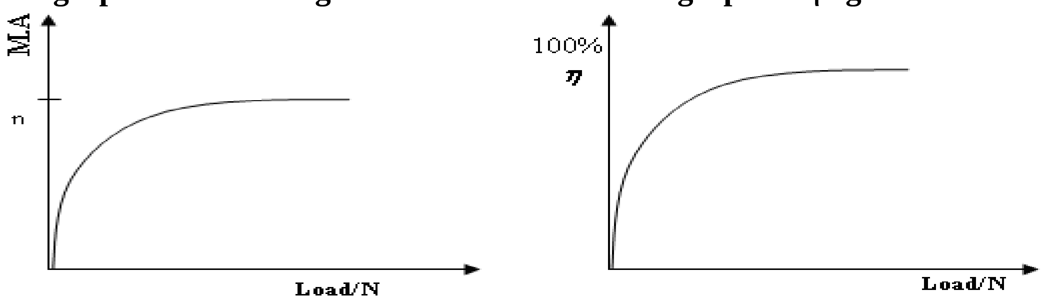


Figure 8.9

Explanation of the Shapes of the graphs

In both graphs, a small increase in load causes high increase in *M.A* and η. On further increase on the load, the graphs begin to level as M.A and η approach their maximum values. At a certain value of load, the M.A and η reach their maximum values and then remain constant, hence, the graphs level.

NB: *For figure (a), the graph levels at the value of the M.A. While for figure (b), the graph levels below 100% for imperfect machines and levels at 100% for perfect machines.*

Worked Examples

1. (a) Define the following terms

   (i) Mechanical advantage

   (ii) Velocity ratio

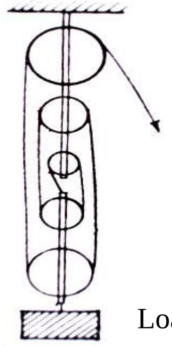


Figure 8.10

(b) The diagram in the figure shows a pulley system used to raise a load.

(i) What is the velocity ratio of the system?

(ii) Find how far the load is raised if the effort moves down by 4m.

(iii) Calculate the effort required to raise a load of 800N, if the mechanical advantage of the system is 4.

(iv) Calculate the efficiency of the system.

(c) Explain what happens to the efficiency of the system in (b) above if the load is much:

   (i) less than 800N.

   (ii) more than 800N.

(d) Draw a sketch graph to show how mechanical advantage of the system in (b) varies with the load.

(e) Give two practical applications where pulley systems are used.

Solution: **(a)** **(i)** *Mechanical advantage is the ratio of load to effort.*

(ii) *Velocity ratio is the ratio of effort distance to load distance.*

(b) **(i)** V.R = 5

(ii) Effort distance = 4 m, Load distance = ?, V.R = 5

$$\text{V.R} = \frac{Effort\ \ distance}{Load\ \ distance}$$

$$5 = \frac{4}{Load\ \ distance}$$

$$\text{Load distance} = \frac{4}{5}$$

$\therefore$ **Load distance = 0.8 m**

(iii) E = ?, L = 800N, M.A = 4.

$$\text{M.A} = \frac{Load}{Effort}$$

$$4 = \frac{800}{E}$$

$$\text{E} = \frac{800}{4}$$

$\therefore$ **E = 200 N**

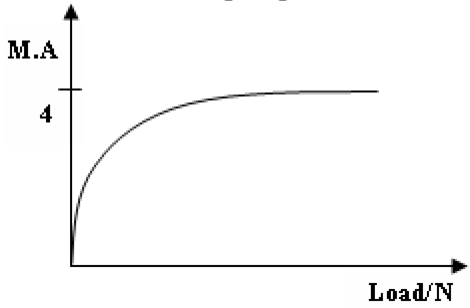
(iv) η = ?, M.A = 4, V.R = 5

$$\eta = \frac{M.A}{V.R}\ x\ 100 = \frac{4}{5}\ x\ 100\ = \mathbf{80\%}$$

(c) **(i)** The efficiency is less than 80%.

(ii) The efficiency remains constant at 80%.

(d) The graph of mechanical advantage against load.



(e) They are used in: - Cranes, brake downs, ships, docks - for lifting load.

8.4 Inclined Plane

An inclined plane refers to a type of machine in which a plane is inclined to an angle to the horizontal such that one end is higher than the other. It is used to raise heavy load by pulling/pushing it along the sloping surface. This is much easier than lifting the load through the vertical height, h, since the weight of the load acts vertically downwards.

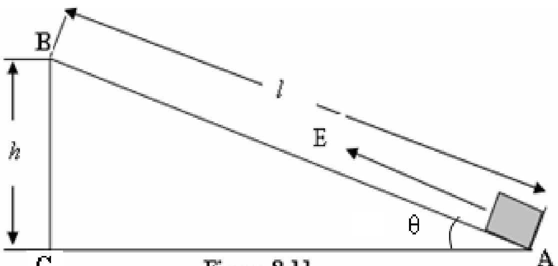


Figure 8.11

In order to raise the load through a vertical height *h*, the effort, *E,* is applied through a longer distance, *l,* equal to the length of the plane and the load is raised through a vertical height, *h*.

Examples of inclined plane include:
- Sloping roads and
- Stair case.

(a) M.A of an Inclined Plane

The M.A of an inclined plane may be obtained by assuming the machine to be perfect (i.e. neglecting the work done against friction).

Thus, Work output = Work input

Load x Distance moved by load = Effort x Distance moved by effort

$$\frac{Load}{Effort} = \frac{Distance\ moved\ by\ Effort}{Distance\ moved\ by\ load}$$

$$M.A = \frac{l}{h} \quad \text{But in practice M.A} < \frac{l}{h}$$

(b) The V.R of the an Inclined plane

$$\text{Velocity Ratio} = \frac{Distance\ moved\ by\ effort}{Distance\ moved\ by\ load}$$

$$\text{V.R} = \frac{Length\ of\ the\ plane, l}{Height\ of\ the\ plane, h}$$

$$\boldsymbol{V.R} = \frac{l}{h}$$

(c) The V.R of an Inclined plane in terms of angle of inclination, θ, (i.e ∠BAC)

$$\text{Velocity Ratio} = \frac{Effort\ \ distance}{Load\ \ distance}$$

$$V.R = \frac{AB}{BC} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 1$$

Using trigonometry, AB and BC are expressed in terms of Sin θ and Tan θ and then substituted in equation 1.

From $\quad \sin\theta = \frac{BC}{AB}$

$$AB = \frac{BC}{Sin\theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 2$$

And from $\quad \tan\theta = \frac{BC}{AC}$

$$BC = \tan\theta \times AC \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 3$$

Substituting equations (2) and (3) in equation (1) we have:

$$V.R = \frac{BC}{Sin\theta} \div Tan\theta.AC$$

$$= \frac{BC}{Sin\theta} \times \frac{1}{Tan\theta.AC}$$

$$\boldsymbol{V.R} = \frac{BC}{Sin\theta.Tan\theta\ AC}$$

NB: *The derivation of the formula is not necessary at this level. So you should not get scared. Other wise you will understand it when you cover trigonometry in mathematics in S.2..*

Example 1

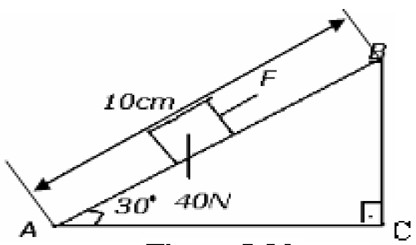


Figure 8.12

A load of 40N is pulled steadily from A to B along an inclined plane by a force F as shown in the figure 8.12. Find the velocity ratio of the system

A 1.0 B 1.2 C. 2.0 D. 4.0

Answer **= C**

How to work it: AC = 10 cm, BC = ?, θ = 30°

BC = Tan θ x AC
= Tan 30° x 10
= 0.5774 x 10
BC = 5.774

Using the formula $\quad V.R = \frac{BC}{Sin\theta.Tan\theta\ AC}$ we have:

$$V.R = \frac{5.774}{Sin30° \ x\ Tan30° \ x\ 10} = \frac{5.774}{0.5\ x\ 0.5774\ \ x\ 10} = \frac{5.774}{2.887} = \mathbf{2}$$

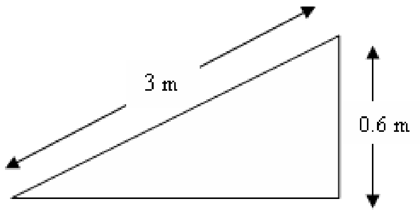
Example 3

A wooden plank 3 m long is used to raise a load of 1200 N through a vertical height of 60 cm. If the frictional force between the load and the plane is 40 N, calculate:

(a) The effort required.

(b) The mechanical advantage.

Solution: **(a)**



Data: L = 1200 N, E = ?, l = 3 m, h = 60 cm = 0.6 m, Frictional force = 40 N.

The effort required can be got by applying the principle of work.

Work input = Work output + Useless work done

Work done by effort = Work done load + Work by frictional force

Effort x Effort dist. = Load x Load dist. + Frictional force x Effort distance

$$E \times 3 = 1200 \times 0.6 + 40 \times 3$$

$$E \times 3 = 720 + 120$$

$$3E = 840$$

$$E = \frac{840}{3}$$

$$\therefore \mathbf{E} = \mathbf{280\ N}$$

(b) **M.A** $= \frac{L}{E} = \frac{1200}{280} = \mathbf{5}$

8.5 Wheel and Axle

A wheel and axle is a type of simple machine made up of a wheel and axle tightly attached to each other so that they turn together about an axis. The effort is applied to the larger wheel and the load is raised by a string attached to the axle as shown in figure8.13 below.

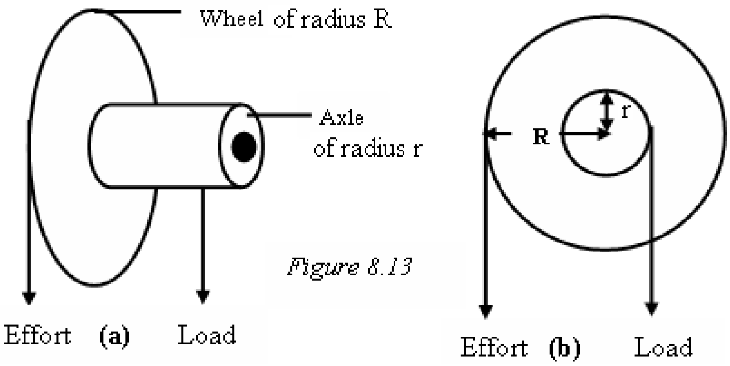


Figure 8.13

For one complete turn, the load and the effort move through distances equal to the circumferences of the wheel and the axle respectively.

(a) Facts about the Wheel and Axle

- ❖ A small force applied over a large distance raises a large force over a small distance.
- ❖ A wheel and axle is simply a first class lever which move in a circle; having the fulcrum at the centre. See figure 8.13 (b).

(b) The V.R of Wheel and Axle

$$\text{Velocity Ratio} = \frac{Dis\tan ce\ moved\ by\ effort}{Dis\tan ce\ moved\ by\ loadt}$$

$$= \frac{Circumfren ce\ of\ the\ wheel}{Circumfren ce\ of\ the\ axle}$$

$$= \frac{2\pi R}{2\pi r}$$

$$\boldsymbol{V.R} = \frac{R}{r}$$

Where: *R and r are the radius of the wheel and the axle respectively.*
The larger the radius of the wheel, the higher the V.R and easier work done..

(c) The M.A of Wheel and Axle

The M.A of wheel and axle is obtained by taking moments of the load and effort about the axis of rotation.

Applying the principle of moments, figure 8.13 (b) we have:

Clockwise moment = Ant-Clockwise moment

Load x Radius of axle = Effort x Radius of the wheel

$$\frac{Load}{Effort} = \frac{Radius\ of\ wheel}{Radius\ of\ axle}$$

$$M.A = \frac{R}{r}$$

But in practice M.A < $\frac{R}{r}$ due to frictional forces in the machine.

Example

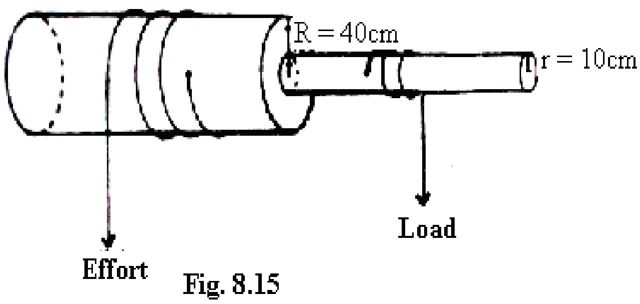


Fig. 8.15

The above figure shows a wheel and axle system. When an effort of 300N is applied, a load of 900N is raised through a distance of 1.0 m. Calculate;

(a) The velocity ratio.

(b) The efficiency of the system.

Solution: **(a)** R = 40 cm, r = 10 cm, E = 300 N, L = 900 N

$$\text{V.R} = \frac{R}{r} = \frac{40}{10} = \mathbf{4}$$

(b) $$\eta = \frac{M.A}{V.R} \times 100 = \frac{L}{E} \times \frac{1}{V.R} \times 100 = \frac{900}{300} \times \frac{1}{4} \times 100 = \frac{3}{4} \times 100 = \mathbf{75\ \%}$$

8.6 Gears

A gear is a device which consists of a set of toothed wheels. Gears change the direction and the speed of rotation. They are similar to wheel and axle. In gear wheels, the effort and the load are applied to the shafts connected to the gears.

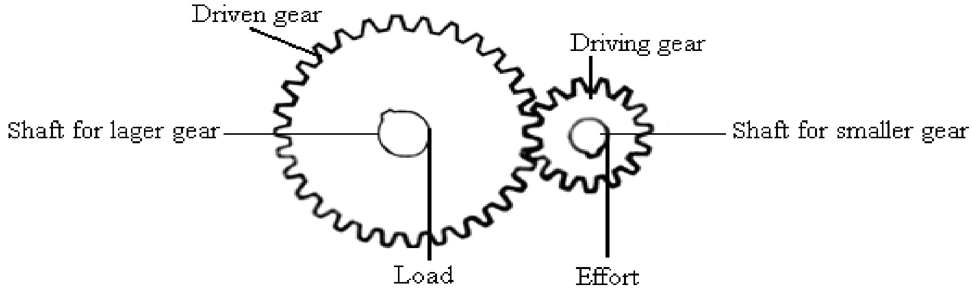


Figure 8.16

(a) Types of gears

There are four types of gear systems, namely:

- Spur gears,
- Worm gear,
- Rack and Pinion and
- Bevel gears

Figure 8.17 below shows the common types of gear systems in use.

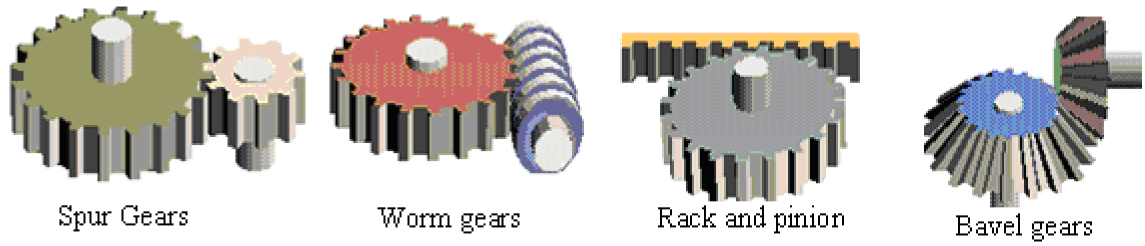


Figure 8.17

(b) Facts about gears

- ❖ When a gear is engaged, the order of the rotation and the speed of the gears change.
- ❖ The number of rotation of gear wheels depends on the ratio of the number of teeth and radii of the wheels.
- ❖ The direction of driven gear is opposite to that of the driving gear.
- ❖ There are several gear systems as shown in figure 8.17 above. But we always consider the spur gears.

(c) The Velocity Ratio of Gears

Whichever the driving and the driven wheel is the V.R is given by the formula:

$$\textbf{\textit{Velocity Ratio}} = \frac{\textit{Number of teeth in the driven wheel}}{\textit{Number of teeth in the driving wheel}}$$

The velocity ratio of gears depends on which gear wheel is the effort applied.
Torque may be applied to the smaller gear in order to increase the torque and decrease the rate of rotation of the larger gear. Or
Torque may be applied to the larger gear in order to decrease the torque and increase the rate of rotation of the smaller gear.

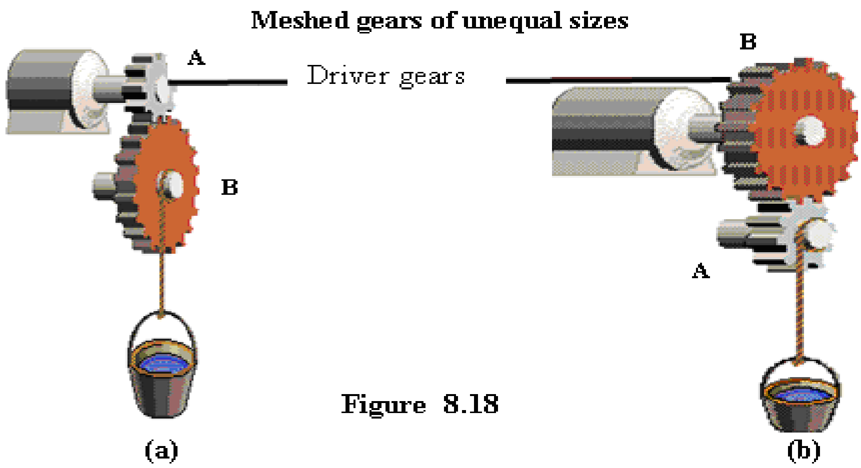


Figure 8.18

Example

1. Two gear wheels A and B with 20 and 10 teeth respectively lock into each other. They are fastened on axles of equal diameters such that a weight of 100 N attached to a string wound around one axle raises a load of 160 N attached to a string wound around the other axle. Calculate: (a) (i) the velocity ratio.
(ii) the efficiency of the system, for figure 8.18 (a).
(b) (i) the velocity ratio.
(ii) the efficiency of the system, for figure 8.18 (b).

Solution: No. of teeth of driving wheel, A = 10, No. of teeth of driven wheel, B = 20,

E = 100N, L = 160 N

(a) **(i)** V.R $= \dfrac{\textit{Number of teeth in the driven wheel}}{\textit{Number of teeth in the driving wheel}} = \dfrac{10}{20}$ **= 0.5**

(ii) $\eta = \dfrac{M.A}{V.R} \text{ x } 100 = \dfrac{L}{E} \text{ x } \dfrac{1}{V.R} \text{ x } 100 = \dfrac{160}{100} \text{ x } \dfrac{1}{0.5} \text{ x } 100 = \dfrac{600}{0.5}$ **= 320 %**

(b) **(i)** V.R $= \dfrac{\textit{Number of teeth in the driven wheel}}{\textit{Number of teeth in the driving wheel}} = \dfrac{20}{10}$ **= 2**

(ii) $\eta = \dfrac{M.A}{V.R} \text{ x } 100 = \dfrac{L}{E} \text{ x } \dfrac{1}{V.R} \text{ x } 100 = \dfrac{160}{100} \text{ x } \dfrac{1}{2} \text{ x } 100$ **= 80 %**

8.7 The Screw

The screw is a slope turning around a pole. It is like spiral stair case. Like a load moving up or down an inclined plane under the effect of an effort, a screw goes in or comes out of a bolt or wood. It is an essential feature of machines such as the vice and the car screw jack. The distance between any two successive threads on a screw is called *pitch*. Normally the vice and the car jack have handles to which effort, E, is applied. When we turn the screw, it changes the direction of the effort. For a complete turn, the load, L, moves a distance equal to the pitch.

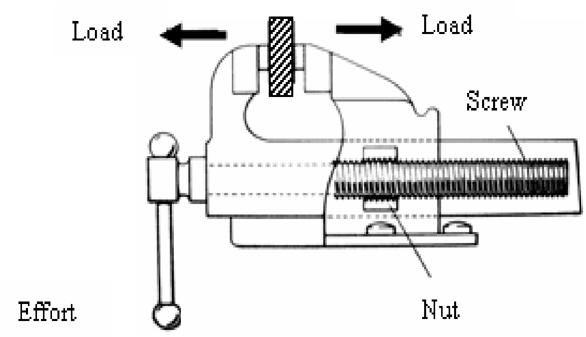


Figure 8.19 Vice

(a) M.A of the screw

The M.A is obtained by assuming the system to perfect i.e. ignoring frictional forces.

Thus, Work done by effort, E = Work done by load, L

Effort x Distance moved by effort = Load x Distance moved by the load

$$\text{Effort} \times \frac{\text{Circumfrence of the}}{\text{circle traced by effort}} = \text{Load} \times \text{Screw pitch}$$

$$\text{Effort} \times 2\pi l = \text{Load} \times \text{Screw pitch}$$

$$\frac{Load}{Effort} = \frac{2\pi l}{Screw\ pitch}$$

$$\boldsymbol{M.A} = \frac{2\pi l}{Screw\ pitch}$$

(b) The V.R of screw

The V.R of the screw is given by the formula:

$$\text{Velocity Ratio (V.R)} = \frac{Distance\ moved\ by\ effort}{Distance\ moved\ by\ load}$$

$$\boldsymbol{V.R} = \frac{2\pi l}{Screw\ pitch}$$

Note that: *Since the screw is assumed to be perfect M.A = V.R*

Worked Examples

1. (a) In a screw jack the length of handle is 24 cm and the screw pitch is 2 mm. If it is used to raise a car of mass 2 000 kg, calculate:
   (i) the effort required.
   (ii) the V.R.
   (iii) the M.A.

   (b) Give reason for the value of M.A you have obtained in (a) (iii) above.
   (Take g = 10 m/s^2, π = 3.14)

Solution: **(a)** $l = 24\text{ cm} = \frac{24}{100} = 0.24\text{ m}$, pitch = 2 mm = $\frac{2}{1000}$ = 0.002 m,

L = 2000 kg = 2000 x 10 = 20000 N, E = ?

(i)

$$\text{Effort} \times 2\pi l = \text{Load} \times \text{Screw pitch}$$

$$\text{Effort} = \frac{Load \times Screw\ pitch}{2\pi l}$$

$$= \frac{20000 \times 0.002}{2 \times 3.14 \times 0.24}$$

Effort = 26.54 N

(ii)

$$\text{Velocity Ratio} = \frac{Distance\ moved\ by\ effort}{Distance\ moved\ by\ load}$$

$$\text{V.R} = \frac{2\pi l}{Screw\ pitch}$$

$$= \frac{2 \text{ x } 3.14 \text{ x } 0.24}{0.002}$$

V.R = 753.6

(iii) M.A $= \frac{L}{E}$ **Or** M.A $= \frac{2\pi l}{Screw\ pitch}$

$$= \frac{20000}{26.54} \qquad = \frac{2 \text{ x } 3.14 \text{ x } 0.24}{0.002}$$

M.A = 753.6 **M.A = 753.6**

(b) M.A = V.R. This is because the screw jack is assumed to be perfect i.e. frictionless.

NB: *In practice the effort must be much greater than 26.54 N in order to overcome friction.*

8.8 The Wedge

The wedge is a kind of simple machine which is an inclined plane having either one or two sloping sides. With a wedge, the sloping surface is pushed through the material which is held still.

Examples of wedges are:
A knife, axe, chisel, needle, nail, razor blade and other cutting tools.

Uses of wedges

Wedges are used to:
- cut materials,
- raise heavy object off the ground,
- split wood etc.

Self-Check 8.0

1. Which one of the following are true for simple machines?

I. If there is a gain in force, there is a loss in distance.

II. The mechanical advantage is the ratio of the load to the effort.

III. There is a gain in work.

A. I-II B. II-III C. I-III D. I-II-III

2. Which one of the following below is an example of a 3^{rd} class lever?

A. tweezers B. equal –arm balance
C. see- saw D. fixed pulley

3. The system that uses fixed and movable pulleys is called

A. windlass B. inclined plane C. block and tackle D. lever

4. Which one of the following describes the type of pulley, and the lever that the pulley epresents?

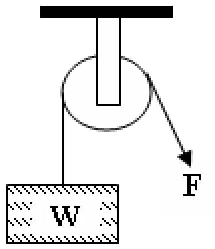


Fig. 8.20

A. Fixed pulley – I[st] class
B. Fixed pulley – III[rd] class
C. Movable pulley – II[nd] class
D. Movable pulley – III[rd] class.

5. In a windlass, which changes decrease the force needed to pull up the load
I. increasing the radius of the wheel.
II. decreasing the radius of the wheel
III. decreasing the radius of the axle.
A. I-II B. I-III C. II only D. III only

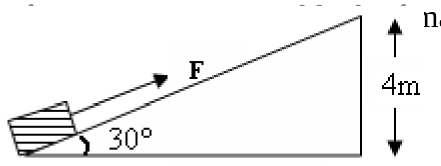
. 

Figure 8.21

nass 2 kg to push it on an inclined plane. If F = 5 N, what is the le
A. 4 m B. 8 m
C. 12 m D. 16 m

7. A screw of length 5 cm enters totally into a wooden block after 25 turns. What is the pitch of the screw?
A. 0.1 cm B. 0.2 cm C. 0.4 cm D. 0.5 cm

8. The maximum efficiency that can be obtained with four pulleys and a mechanical advantage of 3 is?
A. 100% **B. 75%** **C. 12%** **D. 1.33%**

9. Calculate the effort when a load of 72 N is raised using a block and tackle system of 5 pulleys and efficiency 80%.
A. 11.52N B. 18N C. 57.6N D. 288N

10. The bock and tackle system in the figure has an efficiency of 80%. The load, which can be lifted by an effort of 10N, is

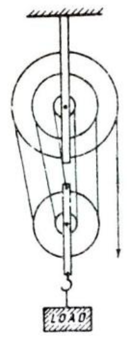


Figure 8.22

A. 4N
B. 8N
C. 40N
D. 50N

11. A machine which is 80% efficient is run by an engine with an out put of 40 watts. The time taken to raise a load of 1500 N through 0.15 m may be
A. 4.5s B. 5.6 s C. 7.0s D. 28.1s

12. Which of the following statements is true of a wedge used as a simple machine?
A. Very small force is required to lift a big load. B. Work done is always so much.
C. Effort on the wedge is applied vertically. D. There is no frictional force.

13. Calculate the effort when a load of 72 N is raised using a block system of 5 pulleys and efficiency 80%.

A. 11.52N B. 18N C. 57.6N D. 288N

14. Figure 8.23 shows a light, smooth pulley used to lift a load of 16 N with an effort E. The mechanical advantage of the system is

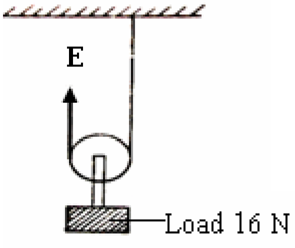


Figure 8.23

A. 128

B. 2

C. 1

D. $^1/_2$

15. Which one of the following is not a correct formula for calculating velocity ratio?

A. $V.R = \frac{l}{h}$ B. $V.R = \frac{Load\ distance}{Effort\ distance}$

C. $V.R = \frac{R}{r}$ D. $V.R = \frac{Number\ of\ teeth\ in\ driven\ wheel}{Number\ of\ teeth\ in\ the\ driving\ wheel}$

SECTION B

16. (a) What is meant by a first class lever?
(b) Give two examples of first class lever.
(c) By means of a lever, an effort of 50 N moves a load of 200 N through a distance of 3m. If the effort moves a distance of 16 m, calculate:
(i) the mechanical advantage
(ii) the efficiency of the system.

17. (a) What is meant by efficiency of a machine?
(b) Draw a single string pulley system of velocity ratio 3.
(c) State one reason why the efficiency of a machine is always less than 100%.

18. (a) Define efficiency of a machine.

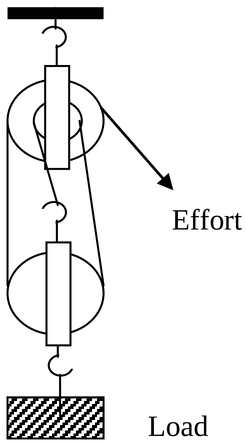


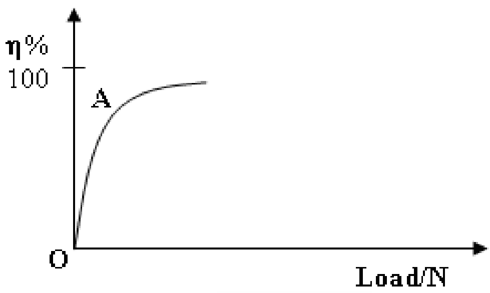
Figure 8.24

(b) The diagram in figure 8.24 above represents a pulley system in which an effort E is applied to raise the load L.
(i) Copy the diagram and indicate the forces acting on the string
(ii) What is the velocity ratio of the system?
(iii) How far will the load move if the effort moves by 2.4m.
(iv) What effort would just raise a load of 960 N, if the mechanical advantage is 2.4.
(v) Use your results above to calculate the efficiency of the pulley system.

(c) (i) Draw a sketch graph to show how the mechanical advantage of the pulley system in (b) varies with the load.
(ii) Explain the features of the sketch in (c) (i).

(d) Give two practical examples where pulley systems are used.

19. (a) Draw a labeled diagram to illustrate the lever principle as applied to a wheelbarrow.



(b) The graph in the figure shows the variation of the efficiency of a pulley system with load

Explain why:
(i) part OA of the graph is almost a straight line
(ii) from A, the graph curves and finally levels off before reaching 100%.

20. The figure 8.25 shows a wheel and axle system. When an effort of 300 N is applied, a load of 900 N is raised through a distance of 1.0 m.

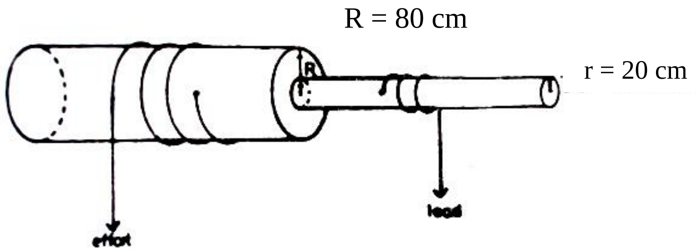


Figure 8.25

Calculate;
(a) the velocity ratio
(b) the efficiency of the system

21. (a) State what is meant by each of the following terms as applied to simple machines:
(i) Mechanical advantage (ii) Efficiency

(b) (i) Give two reasons why the efficiency of any simple machine is always less than 100%.
(ii) Give two ways in which the efficiency of a machine can be increased.

(c) The figure shows a load of 10 N raised slowly by a simple frictionless pulley system.

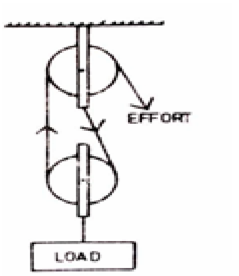


Figure 8.26

(i) What is the velocity ratio of the system?

(ii) Calculate the effort required to lift the load if the mass of the pulley is 0.2 kg.
(iii) If the load is raised through a distance of 5 m in 5 s, calculate the efficiency of the system.

22. Two gear wheels A and B with 80 and 20 teeth respectively lock into each other. They are fastened on axles of equal diameters such that a weight of 150 N attached to a string wound around one axle raises a load of 450 N attached to a string wound around the other axle. Calculate:
(a) (i) the velocity ratio.
(ii) the efficiency of the system, if the A drives B.
(b) (i) the velocity ratio.
(ii) the efficiency of the system, if the B drives A.

23. A smooth wooden board 3.6 m long is used to raise a box of 600 N to a height of 1.2 m. Calculate: (a) The effort required (b) The M.A of the system.

CHAPTER NINE
PRESSURE

LEARNING OBJECTIVES

By the end of this chapter, you should be able to:

1. (a) Define: - Pressure and state its S.I unit and other units;
(b) State: - The conditions for Pressure in liquids.
(c) Describe: - An experiment to show that liquid pressure depends on depth.
- How to measure liquid and gas pressure using manometer.
(d) State: - The applications of liquid pressure.
(e) Describe: - How to measure Relative Density of liquids using manometer.

2. Solve problems involving pressure:
- Exerted by solids on a surface and
- In liquids.

3. (a) State: (i) - Pascal's Principle (The law) of equal transmission of pressure in liquids.
(ii) - Applications of the law of equal transmission of pressure.
(b) Describe: - The structures and the mechanisms of:
- The Hydraulic Press and
- The Hydraulic Brake.
(c) Solve problems involving pressure, force and area in the Hydraulic Press.

4. (a) Define: - Atmospheric Pressure.
(b) Describe: (i) - Simple experiment to show the effect of atmospheric pressure.
(ii) - How to measure atmospheric pressure using Mercury Barometer.
(c) State: - The applications of atmospheric pressure.
(d) Describe: - The structures and the mechanisms of:
- The Lift pump and
- The force pump.

9.1 Pressure

Pressure is defined as: *force acting normally (perpendicularly) per unit area*.

Mathematically, it is expressed as: Pressure, $P = \frac{Force\ (F)}{Area\ (A)}$

$\therefore \quad P = \frac{F}{A}$ *Where area A is in m^2*

S.I Unit of Pressure

The SI unit of pressure is Newton per square metre (Nm^{-2} or N/m^2). It is a derived unit i.e. it is derived from the SI units of the quantities used to define pressure.

From the definition; Pressure, $P = \frac{Force\,(F)}{Area\,(A)}$

The SI unit of pressure $= \frac{SI\ unit\ of\ force\ (N)}{SI\ unit\ of\ area\ (m^2)} = \frac{N}{m^2}$ (N/m^2) or Nm^{-2}

Other units of pressure are:

(i) Pascal (Pa) is also SI unit of pressure.
1 Pa = 1 Nm^{-2} Therefore, Pascal is also the S.I unit of pressure

(ii) Atmosphere (atm).

(iii) Millimeter of mercury (mmHg)

1 atm = 101325 Pa or Nm^{-2}

Also 1 atm = 760 mmHg *When measured at a temperature of 0 ℃ at sea level.*

The larger units of pressure are:

- kiloPascal (kPa) 1 kPa = 1 000 Nm^{-2}
- Mega Pascal (MPa) 1 MPa = 1 000 000 Nm^{-2}

Note that from the formula $P = \frac{F}{A}$, it follows that:

(i) At constant area, $P \propto F$. i.e. at constant area, the pressure is directly proportional to force (pressure increases with increase in force).

(ii) At constant force, $P \propto \frac{1}{A}$, i.e. pressure is inversely proportional to area (pressure increases with decreasing area and decreases with increasing area).

Hence; - Pressure is greatest when area A is smallest and
- Pressure is least when area A is greatest.

Worked Examples

1. A force of 10,000 N is applied to an area of 2 m^2. Calculate the pressure exerted on the area.

Solution: F = 10,000 N, A = 2 m^2, P = ?

$$P = \frac{F}{A}$$

$$= \frac{10000}{2}$$

$$= \mathbf{5\ 000\ Nm^{-2}}$$

2. A glass block of mass 60 g exerts a pressure of 1000 Nm^{-2} on a table top. Determine the area of contact between the glass block and the table top. (Take g = 10 ms^2)

Solution: m = 60 g = $\frac{60}{1000}$ kg, F = ? g = 10 ms^{-2}, P = 1000 Nm^{-2}, A = ?

Hint: *Since the force is not given directly, we first find the force F using the formula:*

$$F = mg = \frac{60}{1000} \times 10 = 0.6 \text{ N}$$

Now force F and pressure P are known. We can then use the formula for finding pressure to calculate the area A.

$$P = \frac{F}{A}$$

$$1000 = \frac{0.6}{A}$$

$$A = \frac{0.6}{1000}$$

$$= \mathbf{0.0006\ m^2 \quad or \quad 6 \times 10^{-4} m^2}$$

3. A block of wood of mass 1200 g measures by 30 cm by 6 cm by 5 cm. Calculate;

(a) The greatest pressure.

(b) The least pressure exerted by the wood on a flat surface. (Take g = 10 ms^{-2})

Solution: $m = 1200\text{ g} = \frac{1200}{1000} = 1.2\text{ kg}$, $F = W = mg = 1.2 \times 10 = 12\text{ N}$

$$l = 30\ cm = \frac{30}{100} = 0.3m,\ w = 6\ cm = \frac{6}{100} = 0.06\ m,\ h = \frac{5}{100} = 0.05\ m$$

First calculate the three possible surface area of the cuboid.

The three possible areas are: $A_{(Largest)} = l \times w = 0.30 \times 0.06 = 0.018\ m^2$

$A_{(Medium)} = l \times h = 0.30 \times 0.05 = 0.015\ m^2$,

$A_{(Smallest)} = w \times h = 0.06 \times 0.05 = 0.003\ m^2$

(i) *The greatest pressure:*

Since pressure in inversely proportional to area in contact, the greatest area acts on the smallest area. So from the three areas calculated above, we select the smallest.

$$P_{(Greatest)} = \frac{F}{A_{Smallest}} = \frac{12}{0.003} = \mathbf{4\ 000\ Nm^{-2}}$$

(ii) *The least pressure:*

The least pressure acts on the largest area. So we select the largest area from the three possible areas.

$$P_{(least)} = \frac{F}{A_{Largest}} = \frac{12}{0.0183} = \mathbf{666.67\ Nm^{-2}}$$

Self-Check 9.0

1. A metal cube of density 3 000 kg/m^3 is 4 m high and stands on a square base. What is the pressure exerted by the weight of the block on the surface on which it stands?
A. 120 000 N/m^2 B. 7500 N/m^2 C. 60 000 N/m^2 D. 15 000 N/m^2

2. The rectangular block has a mass of 3 600 kg and base area of the base is 9 m^2. What is the pressure exerted on the ground?
A. 3000 N/m^2 B. 9000 N/m^2 C. 4000 N/m^2 D. 4500 N/m^2

3. A block of concrete weighs 900 N and its base is a square of side of 3 m. What pressure does the block exert on the ground?
A. 50 N/m^2 B. 100 N/m^2 C. 200 N/m^2 D. 300 N/m^2

4. A man of mass 80 kg wears snow shoes with a total area of 0.5m^2. What is the pressure exerted by him on the ground?
A. 1 200 Pa B. 1400 Pa C. 1 600 Pa D. 1 800 Pa

5. A book weighing 18 N has 0.06 m^2 surface area and lies on a table. Calculate the pressure exerted by the book on the table.
A. 1.08 Pa B. 0.3 Pa C. 0.3 kPa D. 0.003 Pa

9.2 Pressure in fluids

A liquid in a container exerts pressure at the bottom of the container. The pressure exerted has the following properties:

The pressure in liquids:

(i) Is independent of the base area.
(ii) Increases with depth below its surface.
(iii) Increases with the density of liquid.
(iv) At the same point (depth), the pressure is the same and acts equally in all directions.

Experiment 9.0 **To show that pressure in liquids depends on the depth**

Apparatus: A tall can (tin), a small nail, running water from tap.

Procedure:

- Make three holes A, B and C of the same diameter along a vertical line on one side of the can using a small nail.
- Fill the tall can with water from tap up to a level above hole A as shown in the diagram below.
- Observe the jets of water from the holes.

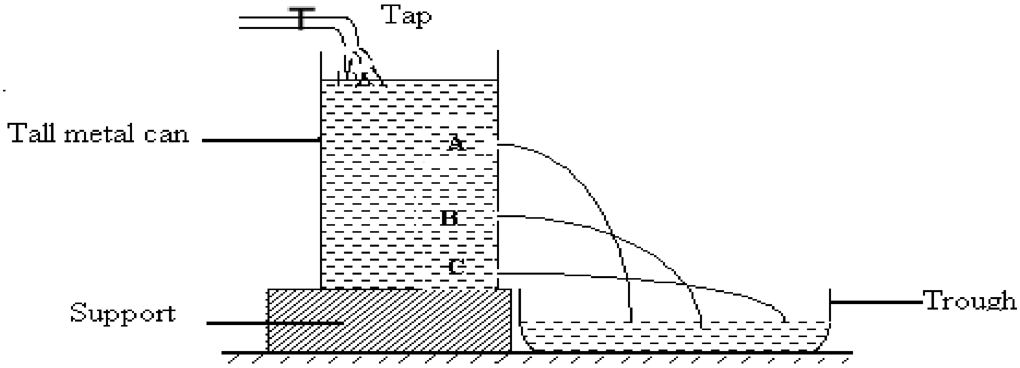


Figure 9.1

Observation

- Water jets through the holes with different speed.
- The lower hole, C throws water farthest and fastest followed by B and lastly hole A.
- Water leaves the holes at right angles to the wall of the can. This shows that the pressure is perpendicular to the wall of the can.

Conclusion: Pressure due to water at hole C is the greatest and at hole A is the least. Therefore, pressure in liquids depends on depth.

(a) To show that pressure in liquids is independent of base area but depends on the depth and density of the liquid

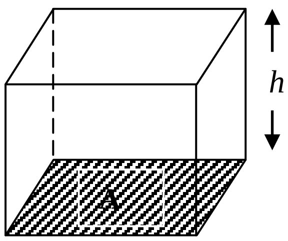


Figure 9.2

Consider a liquid of density mass *m kg* and density, ρ kg/m^3 filled in a container of base area A to a depth, *h* (metres) as shown in the diagram below.

The liquid exerts pressure, *P*, on the bottom of the container as a result of its weight.

The volume of the liquid = Volume of the container

= Base area x Height

= A

Mass of the liquid = Volume x Density

$= Ah \times \rho$

Weight of the liquid = Mass x Acceleration due to gravity

= mg *But* $m = Ah\rho$

$\therefore$ Weight of the liquid $= A\rho hg$

From Pressure $= \dfrac{Force}{Area}$ we have;

$$= \frac{Ah\rho g}{A}$$

$\therefore$ Pressure in liquids is given by the formula, $\boldsymbol{P = h\rho g}$

Note that the final result does not have A. This shows that:
Pressure in liquid is independent of the base area but depends on the depth of the liquid.

That is pressure is directly proportional to the depth, *hρ* of the liquid, since *g is a* constant.

NB: *The same result will be obtained what ever shape of a container is used.*

(b) Pressure and liquid density
If the pressure is measured at the same depth below the surface of different liquids we find that: *The pressure is proportional to the density of the liquid.*

(c) Liquid levels (A liquid finds its own level)
When a liquid is poured into a set of connected tubes of various shapes the liquid flows round the tubes until all the liquid surfaces are the same level, fig. 9.21. We say that a liquid finds its own level.

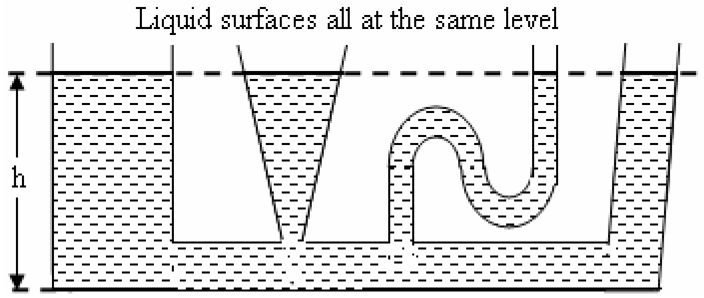


Fig. 9.21

NB: ***See the photograph on this book cover.***

(d) Calculation of Liquid Pressure
The pressure exerted by a liquid can be calculated using the formula:

$$P = h\rho g$$

Where: P = Pressure, h = height/depth in *metres*, ρ = density of liquid in kg/m^3, g = gravity = $10 m/s^2$.

Worked Examples

1. What is the pressure on the bottom of a vessel when it is filled with a liquid of density of 800 kg/m^3 to a depth of 2 m. ($g = 10 m/s^2$).

Solution:: $\rho = 800 kg/m^3$, $h = 2 m$, $g = 10 m/s^2$, $P = ?$

$$P = h\rho g$$
$$= 2 \times 800 \times 10$$
$$\mathbf{= 16\,000\ Nm^{-2}}$$

2. The pressure at the bottom of a column of mercury is 50 Nm^{-2}. Calculate the height of the mercury column. (Density of mercury = 13600 kg/m^3 and $g = 10 m/s^2$).

Solution: $\rho = 13\,600 kg/m^3$, $g = 10 m/s^2$, $P = 50 Nm^{-2}$, $h = ?$

$$P = h\rho g$$
$$50 = 13\,600 \times 10 \times h$$
$$h = \frac{50}{13\,600 \times 10}$$
$$\mathbf{= 3.68\ m}$$

9.21 Measurement of Pressure in Fluids

Fluid pressure is measured using an instrument called *manometer*.

(a) The U-Tube Manometer

A manometer consists of a U-shaped tube with one end connected either directly to the container containing a gas or to a thistle funnel whose base (bottom) is covered with a thin membrane and the other end remains open to the atmosphere. The manometer is filled with a liquid, such as water, oil, or mercury, the difference in the liquid surface levels, *h*, in the two manometer arms indicates the pressure difference from local atmospheric conditions.

Diagrams showing U-tube manometer

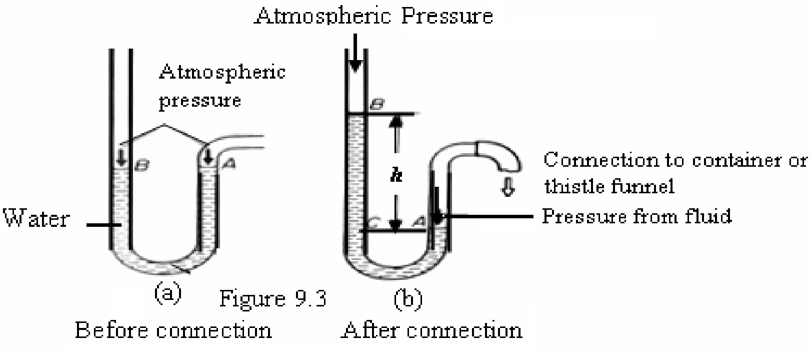


Figure 9.3

9.22 Measurement of Gas Pressure

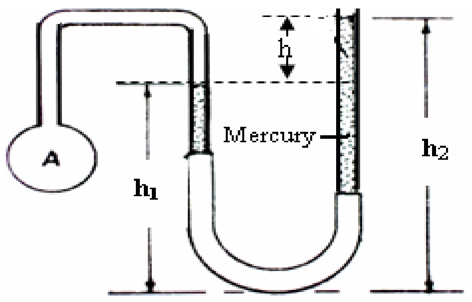


Figure 9.4

One arm of the manometer is connected to the container in which the gas whose pressure to be measured is kept. The difference in the levels of the two arms of the manometer gives the pressure of the gas.

If the actual pressure is required, then it is calculated from the formula:

$$\textit{Gas pressure} = \begin{pmatrix}\text{Atmospheric} \\ \text{Pressure(PA)}\end{pmatrix} + \begin{pmatrix}\text{Pressure due to} \\ \text{liquid column}\end{pmatrix}$$

$$\textbf{\textit{Gas pressure}} = \boldsymbol{P_A + h\rho g}$$

Note that: *For the case of the above diagram,* $h = h_2 - h1$

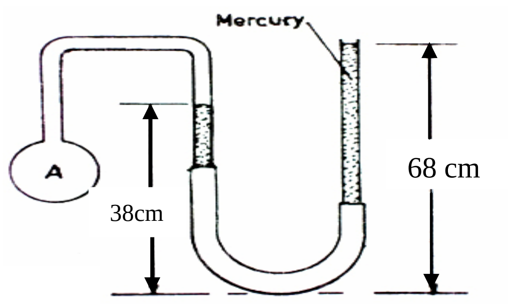


Figure 9.5

Worked Examples

1. (a) In figure 9.5, a fixed mass of dry air is trapped in bulb A. If the atmospheric pressure is 76cm of mercury.

   Calculate the total pressure of the air in A, in:

   (i) mmHg

   (ii) Pa (atm. pressure = 101325 Pa, ρ of mercury = 1.36×10^4kg m^{-3}, g = 10 ms^{-2})

Solution: Atmospheric Pressure = 76 cmHg = 76 x 10 = 760 mmHg

h_1 = 38 cm = 38 x 10 = 380 mm, h_2 = 68 cm = 68 x 10 = 680 mm

(i) Pressure due to mercury column, h = $h_2 - h_1$= 680 – 380 = 300 mmHg

Gas pressure = P_A + Pressure due to liquid column
= 760 + 300
= 1060 mmHg

(ii) Pressure, P, due to mercury column = hρg
= 0.3 x 13600 x 10
= 40,800 Pa

Gas pressure = P_A + Pressure due to liquid column
= 101,325 + 40,800
= 142,125 Pa

2. (a) Figure 9.6 below shows a gas trapped by a mercury column in a J-tube. If the atmospheric pressure is 1.0×10^5 Pa and the density of mercury is 1.36×10^4 kg m^{-3}, find the pressure at which the gas is. (g = 10 m^{-2})

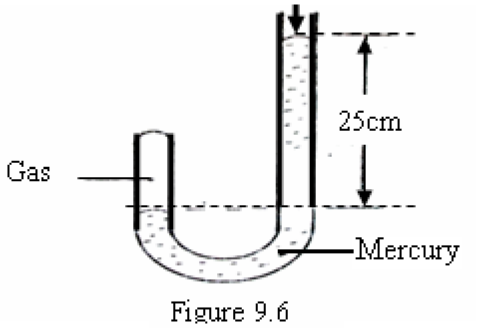


Figure 9.6

(b) What would happen if the closed end of the J-tube was opened?

(c) Would it have been better to use water instead of mercury in the J-tube? Give a reason for your answer.

Solution: Mercury column = 25 cm = 0.25 m, ρ of mercury = 1.36×10^4 kg m^{-3}, g = 10 ms^{-2}, Atm. pressure = 1×10^5 Pa

(a) Pressure, P, due to mercury column = hρg = 0.25 x 13600 x 10 = 34,000 Pa

Gas pressure = P_A + Pressure due to liquid column
= 100,000 + 34,000
= 134.000 Pa

(b) *- The gas trapped would escape. This would reduce the pressure inside the J tube.*
- The mercury column on the right arm would fall while the one on the left arm would rise until the two levels become the same level.

(c) - No. **Reason:** *Water is less dense than mercury. Therefore, to balance the same pressure, it would require a large volume of water which in tern would need a manometer with a very long arm to hold the higher water column.*

(b) Measurement of Pressure in a liquid

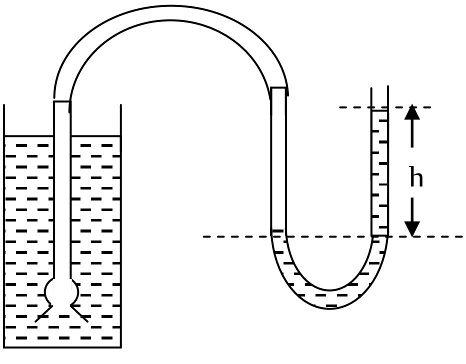


Figure 9.7

In measuring the liquid pressure, one arm of the manometer is connected to a thistle funnel covered with a thin membrane.
The thistle funnel is immersed into the liquid to the point where the pressure is to be measured.
The difference in the levels of the liquid in the two limbs of the manometer gives the pressure at that point. Since the pressure in a liquid is proportional to the depth below the surface, pressures can be measured in terms of the height of a liquid.

The diagram above shows how liquid pressure at different points in a liquid can be measured using a manometer.

Note:

- Manometers can be used to measure pressure both above and below atmospheric pressure.
- Mercury is used as manometer liquid unless the pressure being measured is close to the atmospheric pressure, in which case a liquid of lower density (e.g oil or water) is more suitable.
- The pressure registered by the manometer, $h\rho g$, is known as the *gauge pressure*. Gauge pressure is the *excess pressure* i.e is the amount by which the pressure measured in a fluid exceeds the atmospheric pressure.
- It is often convenient to express the gauge pressure simply in terms of the liquid column, *h*, of the liquid used in the manometer only.
- In this case the units generally used are:
  - mmH_2O and mmHg, read as millimeters of water and
  - millimeters of mercury respectively.

  Where H_2O is a chemical formula of water and Hg is the chemical symbol of mercury.
- The actual pressure, Pa + $h\rho g$, is called the *absolute pressure*?

Actual Pressure = Atmospheric pressure + Pressure due to liquid column
= Pa + $h\rho g$

9.23 Pressure due to flow of liquid in a pipe

Liquid flowing in a pipe has kinetic energy due to its speed, potential energy due to its position, and pressure energy because it is under pressure. The sum of these energies is a constant.

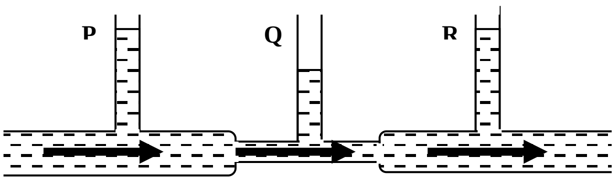


Figure 9.8

Consider water flowing in a pipe a constriction at one point as shown in the diagram.

Facts about the flow of the liquid

- Pressure of water changes with its rate of flow in the pipe.
- The speed of water is much greater at the constriction; as a result its kinetic energy is greater.
- Its pressure energy is less since its potential energy is constant.
- Pressure is highest in tubes P and R and lowest in Q (i.e. at the constriction).

9.24 Application of Liquid Pressure

Liquid pressure is applied in:

(i) Measurement of relative density of liquids
(ii) Water supplies

(a) Measurement of Relative Density of Liquids

(i) Determination of Relative Density of liquids that are Miscible with water

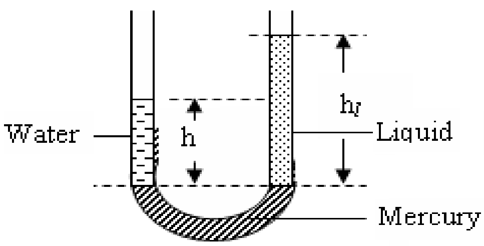


Figure 9.9

The Relative density of a liquid that is miscible with water is determined by balancing a column of small volume of the liquid with water on mercury surface in a manometer until the mercury levels in the two limbs of the manometer are at the same level as shown in figure 9.8 below.

Facts in calculating the Relative Density from the above diagram

(i) The pressure acting on the two surfaces of the mercury is the same.

(ii) The pressure due to the liquid column is equal to the pressure due to the water column

Calculation: Let pressure, P_l , due to the liquid column $= h_l\rho_l g$ *and*
Pressure, P_w , due to the water column $= h_w\rho_w g$

$$\text{But} \quad P_l = P_w$$

$$\therefore \quad h_l\rho_l \cancel{g} = h_w\rho_w \cancel{g}$$

$$h_l\rho_l = h_w\rho_w$$

$$\frac{\rho_l}{\rho_w} = \frac{h_w}{h_l} \qquad \text{But } \frac{\rho_l}{\rho_w} = \text{R.D (See chapter 3)}$$

$$\therefore \quad \text{R.D of the liquid} = \frac{h_w}{h_l}$$

$$\text{I.e R.D} = \frac{\textit{Height of water column}}{\textit{Height of liquid column}}$$

(ii) Determination of Relative Density of liquids that are Immiscible with water E.g. Carbon tetrachloride

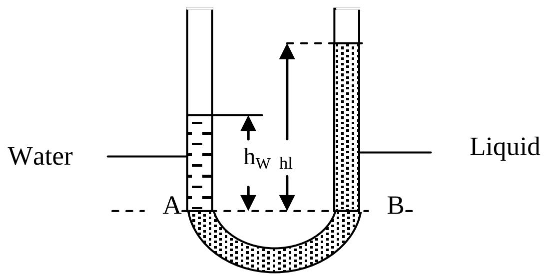


Figure 9.10

The Relative density of liquids that are immiscible with water is determined by balancing the columns of small volume of the liquid and water in a manometer as shown in figure 9.9 below.
By placing a ruler at the position AB, the liquid columns h_l and h_w are measured.

Points to note:
- Point A is called the interface of the two liquids.
- Points A and B are at the same levels so they are at the same pressure.
- The liquid column, h_l, balances the column of water, h_w.

Calculation: Pressure due to liquid column = Pressure due to water column

$$h_l\rho_l g = h_w\rho_w g$$

$$h_l\rho_l = h_w\rho_w$$

$$\frac{\rho_l}{\rho_w} = \frac{h_w}{h_l} \qquad \text{But } \frac{\rho_l}{\rho_w} = \text{R.D}$$

$$\therefore \quad \textbf{\textit{R.D of the liquid}} = \frac{h_w}{h_l}$$

Worked Example

1. An open U-tube contains columns of water and kerosene over mercury as shown in the figure 9.12 below. Calculate the relative density of kerosene.

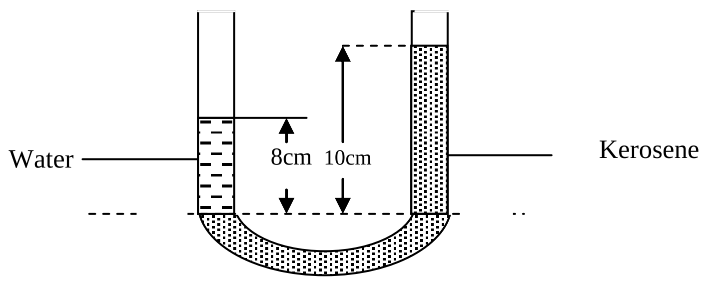


Figure 9.10

Solution: Height, h_w, of water column = 8 cm, Height, h_k, of kerosene column = 10 cm.

$$\text{Relative Density, R.D} = \frac{Height\ of\ water\ column}{Height\ of\ liquid\ column} = \frac{h_k}{h_w} = \frac{8}{10} = \mathbf{0.8}$$

(b) Water Supply

Facts about Water Supply

There must be:

- A water pump - situated near the water source to pump water to;
- A large reservoir - situated on top of a hill or mountain side to create a pressure difference between it and the taps to be supplied in the neighborhood.

In water supply, water is pumped against gravity using a water pump (generator) to a reservoir or water tower high up on a hill or a mountain side. Due to the pressure difference that exists between the water in the reservoir and the taps to be supplied, the in the reservoir is supplied to houses, factories and other buildings.

Note: *In some large blocks or flats there is/are large water tank(s) on the roof or on a high support. The main reservoir supplies water to this/these tank(s) which then supplies water to the taps in the building.*

9.25 (a) The Law of Equal Transmission of Liquid Pressure (Pascal's Principle)

The law states that:

Pressure applied to a liquid is transmitted equally throughout the liquid in all directions.

Experiment To show that pressure applied to a liquid is transmitted equally throughout the liquid in all directions

Apparatus

A round bottom flask with holes of same size on its surface, a tight fitting plunger and water.

Procedure

- Fill a round bottom flask with holes on the round surface with water.
- Insert a plunger and
- Push the plunger towards the bottom of the flask in the direction shown in figure 9.11.

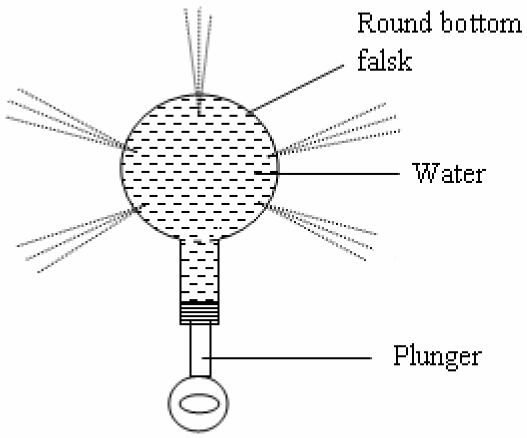


Figure 9.11

(b) Application of the Law of Equal Transmission of Hydraulic Pressure

The principle of equal transmission of pressure in liquids is applied in the following machines:
(i) The Hydraulic Press (Braham's Press).
(ii) The Hydraulic brake and
(iii) The Hydraulic jack.

9.26 The Hydraulic Press

The hydraulic press is a machine which uses the principle of equal transmission of pressure in fluids. It works on the principle that the effort required to move something is the product of the force and the distance through which the object is moved.

(a) The structure of the Hydraulic press

It consists of:
- Two pistons (of different diameters) that move up and down in;
- Two cylinders (pump cylinder and ram cylinder) of different diameters connected by a pipe.
- Incompressible fluid,
- A reservoir and
- Two valves (inlet valve and return valve).

The diagram showing the cross section of a Hydraulic Press

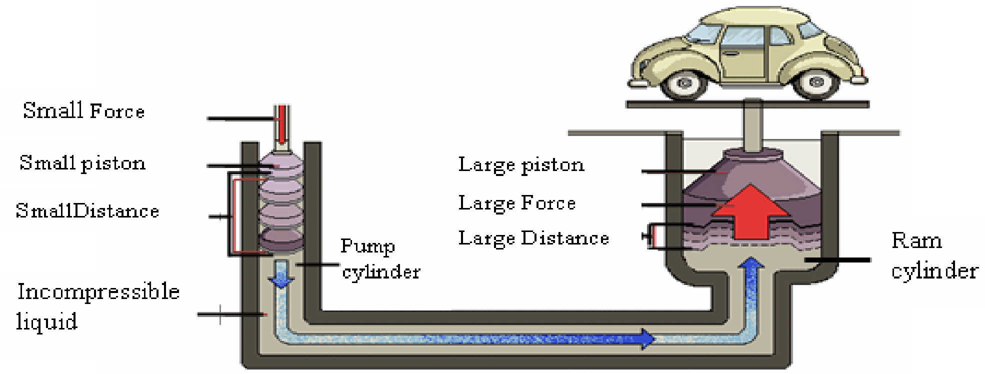


Figure 9.12

(b) How it works

Effort is applied at the smaller piston and moves it through a large distance in the pump cylinder. The hydraulic pressure acting on the liquid surface in the *pump cylinder* is transmitted equally through the liquid to the larger piston in the *ram cylinder* and a large force is applied over a small distance. A heavy load is raised by the force produced by the pressure acting on the ram piston.

Note: *In this way, a small hand pump may be used to lift an automobile. In order to fill the large cylinder under the car with fluid, however, the small pump must be operated many, many times.*
When the return valve is opened, the fluid flows back to the reservoir.
For simplicity, the above cross section is drawn excluding the reservoir.

Problems involving Hydraulic Press are solved by using the following formulae.

(i) Using force: $\frac{f}{a} = \frac{F}{A}$ Or $\frac{f}{F} = \frac{a}{A}$

(ii) Using mass: $\frac{m}{a} = \frac{M}{A}$ Or $\frac{m}{M} = \frac{a}{A}$

Derivation of the above formulae

Let:
- the area of the piston in the pump cylinder = a
- the area of the piston in the ram cylinder = A
- the mass placed at the pump cylinder = m
- the mass placed at the ram cylinder = M
- the acceleration due to gravity = g

Force at the pump piston, f = mg

Force at the pump piston, F = Mg

Pressure, P1 exerted at the pump cylinder $= \dfrac{Force}{Area\ of\ pump\ piston} = \dfrac{f}{a} = \dfrac{mg}{a}$

Pressure, P2 exerted at the ram cylinder $= \dfrac{Force}{Area\ of\ ram\ piston} = \dfrac{F}{A} = \dfrac{\mathrm{M}g}{A}$

By the principle of equal transmission of pressure in fluids;
Pressure exerted at the pump cylinder = Pressure exerted at the ram cylinder

$$P_1 = P_2 \qquad \text{Or} \qquad \frac{m}{\mathrm{M}} = \frac{a}{A}$$

$$\frac{mg}{a} = \frac{\mathrm{M}g}{A} \qquad \text{Using forces we have:} \qquad \frac{f}{a} = \frac{F}{A}$$

$$\therefore \quad \frac{m}{a} = \frac{\mathrm{M}}{A} \qquad\qquad \frac{f}{F} = \frac{a}{A}$$

NB: Using the above formulae,

(i) *The force at the pump piston or ram piston can be calculated when the areas of the pistons are given.*

(ii) *The area of the pump piston or ram piston can be calculated when the forces or masses are given.*

Worked Examples

1. The area of the large piston of a hydraulic press is 10 m^2 and that of the smaller one is 0.25 m^2. A force of 100 N is applied on the smaller piston. Calculate the force produced at the larger piston.

Solution

Data: a = 0.25 m^2, A = 10 m^2, f = 100 N, F = ?

From $\dfrac{f}{F} = \dfrac{a}{A}$

$$F = \frac{f \text{ x } A}{a}$$

$$= \frac{100 \text{ x } 10}{0.25}$$

$$= \mathbf{4000\ N}$$

2. The area of a smaller piston of a hydraulic pres is 0.5 m^2. If an effort of 250 N is applied on the smaller piston and raises a load of 20000 N, calculate the area of the large piston.

Solution: a = 0.5 m^2, A = ?, f = 250 N, F = 20000 N

From $\frac{f}{F} = \frac{a}{A}$

$$A = \frac{a \text{ x } F}{f} = \frac{0.5 \text{ x } 20000}{250} = \mathbf{40\ m^2}$$

3. In a hydraulic press a force of 400 N is applied to a piston of area 0.1m^2. The area of the other piston is 4m^2. Calculate:

(i) The pressure transmitted through the liquid.

(ii) The force on the other piston?

Solution: f = 400 N, F = ?, a = 0.1 m^2, A = 4 m^2

(i) Pressure $= \frac{F}{A}$

$$= \frac{400}{0.1}$$

$$= \mathbf{4000\ N\ m^{-2}}$$

(ii)

$$\frac{f}{a} = \frac{F}{A}$$

$$\frac{400}{0.1} = \frac{F}{4}$$

$$F = \frac{400 \text{ x } 4}{0.1}$$

$$= \frac{1600}{0.1}$$

$$\therefore \mathbf{F = 16\ 000\ N}$$

(c) The Hydraulic brake

(i) Structure of Hydraulic Brake

A hydraulic brake system consists of the following features.

(i) *Master cylinder* Contains two pistons and is connected to wheel cylinders by steel tubes called brake lines.

(ii) *Wheel cylinders They* are found in the wheels and also contain two pistons.

(iii) *Return spring* Returns the brake shoe back to its position hence forcing the brake fluid back to the master cylinder.

(iv) *Brake line* Steel tubes that connect the master cylinder to the wheel cylinders and is filled with the brake fluid.

(v) *Brake fluid* Special incompressible non-freezing liquid.

(vi) *Brake lining* A thin, replaceable strip of material attached to a brake shoe. It makes contact with the brake drum thus stopping the rotation of the wheel.

(vii) *Brake shoes* These are two curved blocks to which the brake linings are attached and they press against the brake drum thus slowing down the rotation of the wheel.

(viii) *Brake drum* A metal cylinder attached to the wheel of a vehicle. They slow down the rotation of the wheel when the brake is applied.

(ix) *Brake pedal* *L*ever operated by the foot that powers the mechanism of brake *system.*

A diagram showing Hydraulic Brake system

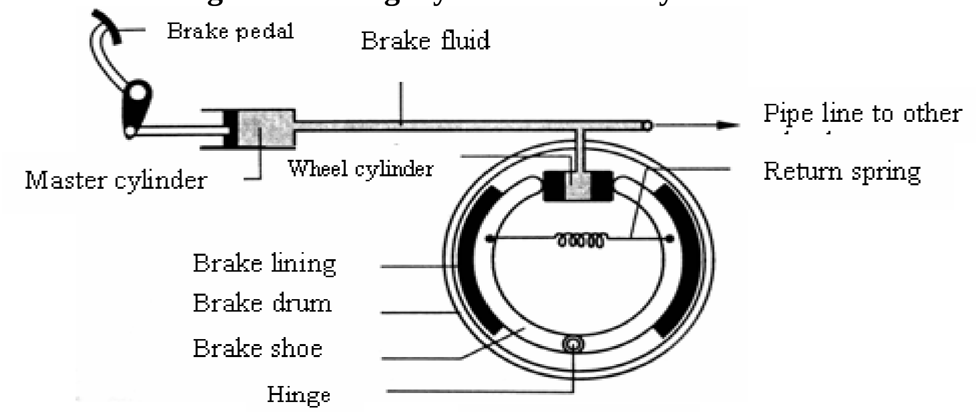


Figure 9.13

(ii) How the Hydraulic Brake System works

When the driver steps on a brake pedal, the pedal pushes a piston inside the master cylinder. This squeezes the fluid inside the master cylinder, creating hydraulic pressure which is transmitted through the brake lines to additional pistons inside each wheel cylinder in each brake. The pistons in the brake cylinder push the hinged brake shoes. The shoes pivot outward pressing the brake linings attached to them against the brake drum. The contact between the brake lining and the brake drum stops the rotation of the wheels and retards the motion of the car and the car slows down.

NB: *(i)* *When the pedal is released, the return springs causes the brake fluid to return to the master cylinder.*

(ii) *Some braking systems use disc system in which the force resulting from the hydraulic pressure pushes the brake pads in the caliper against the rotor, thus slowing the rotation of the wheel.*

9.3 ATMOSPHERIC PRESSURE

Atmosphere *refers to mixture of gases surrounding any celestial object that has a gravitational field strong enough to prevent the gases from escaping; especially the gaseous envelope of Earth.*

In reference to the earth, *atmosphere refers to the air that surrounds the earth*.
The atmosphere exerts pressure called *atmospheric pressure* on the earth surface and the objects found in the pool of the air surrounding the earth. The atmospheric pressure is due to the weight of the air.

Variation of Atmospheric Pressure with altitude

Pressure in fluids depends on the depth or height of the fluid column, the atmospheric pressure therefore, depends on the altitude (height above sea level). As a result of this, at low altitude (high air column), pressure is greatest while at higher attitude (low air column), like mountain peaks, pressure is lower than at low altitude.

Effect of atmospheric pressure on boiling

Boiling takes place when the vapour pressure from a boiling liquid equals the atmospheric pressure. Due to the variation in atmospheric pressure, a liquid will boil at different boiling points. At higher altitudes where atmospheric pressure is low, water boils at low boiling point. Therefore, at higher altitudes, cooking takes longer than at low altitudes where the atmospheric pressure is high.

The large forces which can be produced by atmospheric pressure may be demonstrated by means of a metal can fitted with an airtight stopper.

(a) Crushing can experiment

Apparatus: A metal can with a lid, water and source of heat

Procedure:
- Fill a metal can with some water.
- Heat the water until steam drives out the air from the can.
- Coke the can tightly.
- Remove the can and simultaneously put off the flame.
- Pour cold water on to the can.

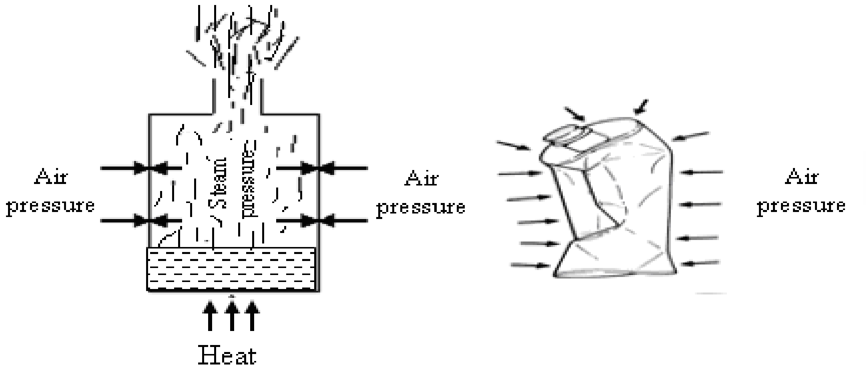


Fig. 9.14 Crashing can experiment

Observation: The metal can crash inwards.

Explanation

The cold water poured caused the steam to condense, producing water and water vapour at a very low pressure. Consequently, the excess atmospheric pressure outside the can causes it to collapse inwards.

(b) Measurement of atmospheric pressure

The atmospheric pressure is measured by using the following instruments

(i) Barometer (Mercury/water barometer)

(ii) Bourdon gauge

A simple Barometer

Barometer is an instrument used for measuring atmospheric pressure.

Types of Barometer

Barometers are named according to the type of liquid used in it. The common liquids used are water and mercury. As a result of this we have:

(i) Water barometer - Not convenient to be used.

(ii) Mercury barometer - The most convenient to be used.

Mercury barometer

Mercury is 13.6 times as heavy as water, and the column of mercury sustained by normal atmospheric pressure is only about 760 mm (0.76) m high. It is more convenient to use mercury barometer than water barometer which sustains extremely high column of water.

An ordinary mercury barometer consists of:

- A glass tube about 840 mm (0.84 m) high, closed at one end.
- A glass beaker or glass trough and
- A pool of mercury.

How to measure Atmospheric pressure using mercury barometer

- Fill the glass tube with mercury up to brim.
- Cover the open end of the tube with a finger or small glass plate.
- Invert the tube and place it vertically with its end well below the surface of the mercury in the beaker.
- Remove your finger.

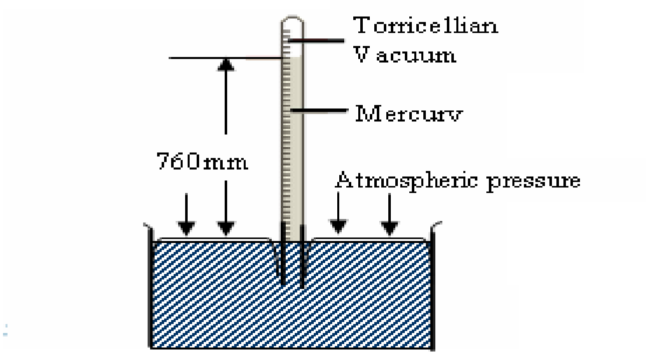


Figure 9.17 Simple mercury barometer

Observation

The mercury level in the tube drops until for some time and then remains constant.

The constant level is when the pressure due to the mercury column balances the atmospheric pressure acting on the mercury surface in the beaker.

Read and record the two levels of the mercury (i.e. top in the tube and down in the trough).

The mercury column is given by the formula:

Mercury column, h = Upper reading – Lower reading

At sea level, the mercury column = 760 mm (76 cm or 0.76 m).

Therefore, the atmospheric pressure is recorded as: *760 mmHg* at sea level.

NB:

(i) The falling of the mercury level leaves almost a perfect vacuum called Torricellian vacuum. After an Italian scientist who lived in Pisa.

(ii) When the mercury level is read with a form of gradated scale, known as a vernier attachment, and suitable corrections are made for:

- Altitude and latitude (because of the change of gravity), and

- Temperature (because of the expansion or contraction of the mercury).

(iii) The vertical height of the mercury column remains constant even when the tube is tilted, unless the top of the tube is less than 760 mm above the level in the beaker, in which case the mercury completely fills the tube.

Variation of the Mercury column with altitude

When the set up is taken to different places with different atmospheric pressure, the variation in the atmospheric pressure causes the liquid in the tube to rise or fall by small amounts, rarely below 737 mm (0.737 m) or above 775 mm (0.775 m) at sea level.

Bourdon gauge

Bourdon gauge, (named after the French inventor Eugène Bourdon), is an instrument used to measure higher pressures e.g. steam pressure and water pressure.

It consists of a hollow metal tube with an oval cross section, bent in the shape of a hook or question mark. One end of the tube is closed, the other open and is connected to the measurement region. If pressure (above local atmospheric pressure) is applied, the oval cross section becomes circular, and at the same time the tube will straighten out slightly. The resulting motion of the closed end which is proportional to the pressure, can then be measured via a pointer or needle connected to the end through a suitable linkage.

NB: *Gauges used for recording rapidly fluctuating pressures commonly employ piezoelectric or electrostatic sensing elements that can provide an instantaneous response.*

Other instruments for measuring atmospheric pressure are:

- The Fortin Barometer.
- Aneroid barometer and
- Altimeter.

Application of atmospheric pressure

Atmospheric pressure is applied in:

(i) The lift pump (the common pump) and
(ii) The force pump.

(a) The lift Pump

The lift pump consists of:

- A handle - which helps to move the piston up and down.
- A piston - which reciprocates (moves back and forth) in a cylinder.
- A pipe - which helps water to flow from under ground.
- Two one-way valves - which regulate the flow of water into and out of the cylinder.

Diagram showing parts of the lift pump

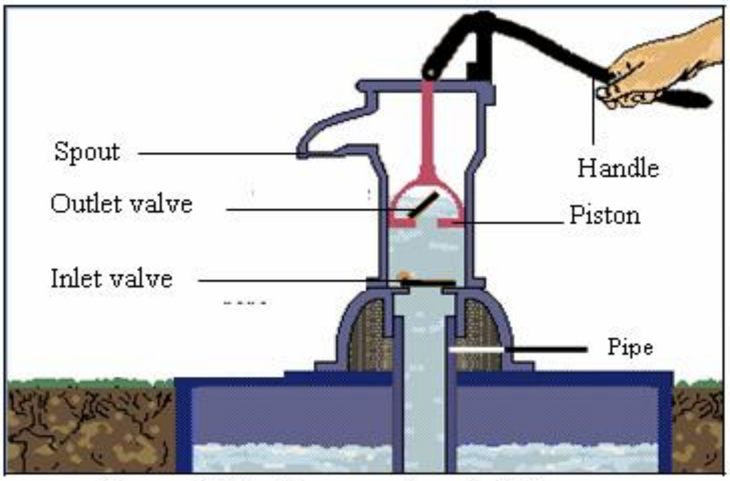
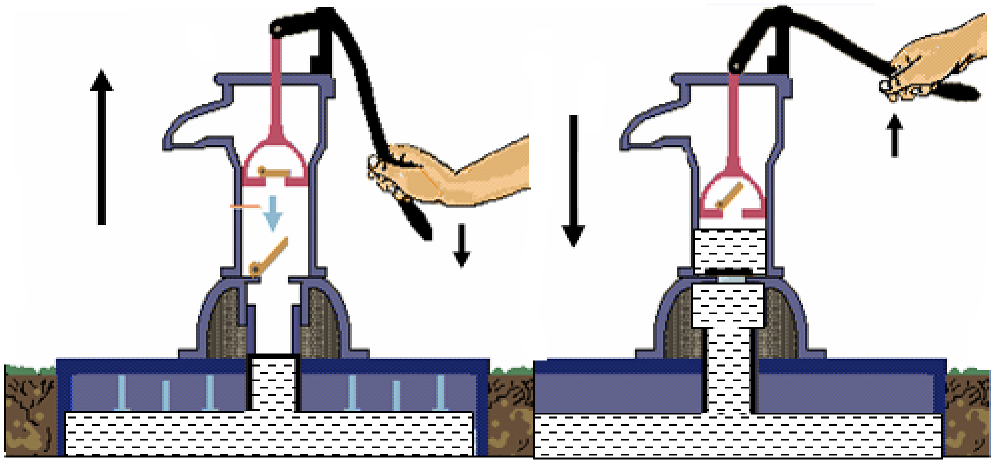


Figure 9.16 Cross section of a Lift pump

A lift pump is reciprocating pump which draws fluid (water) in at one end and discharges or expels it at the other end as described in the mechanism below.



(a) Upstroke Figure 9.17 (b) Down stroke

Points to note:

- *A stroke refers to the up and down movement.*
- *The up and down movement considered is of the piston* ***not*** *of the handle.*
- *The handle and the piston move antagonistically. I.e. when the piston moves up , the handle moves down and vice versa.*

Mechanism of Lift Pump (How a Reciprocating Pump Works)

Up-stroke

During the up stroke, the piston moves up reducing the air pressure in the cylinder. A partial vacuum is formed beneath the piston making the outside air having greater pressure than the inside. Atmospheric pressure then pushes water up the pipe. The rising water opens the inlet valve, and the water passes to the space above the inlet valve.

Down-stroke

During down stroke, the handle is pulled up, the piston moves down, the outlet valve opens and the inlet valve closes, preventing water from falling back down the pipe.

In the next repeated strokes, the water resting on top of the piston pours out of the spout. At the same time, more water is drawn up through the inlet valve.

Limitation of the Lift pump

Water is pushed up into the pump by the pressure of the atmosphere on the water in the well. This limits the distance through which water can be raised using a lift pump to about 8 meters.

The force pump The force pump barrel contains a tightly fitting piston and two valves as shown in figure 9.18 below.

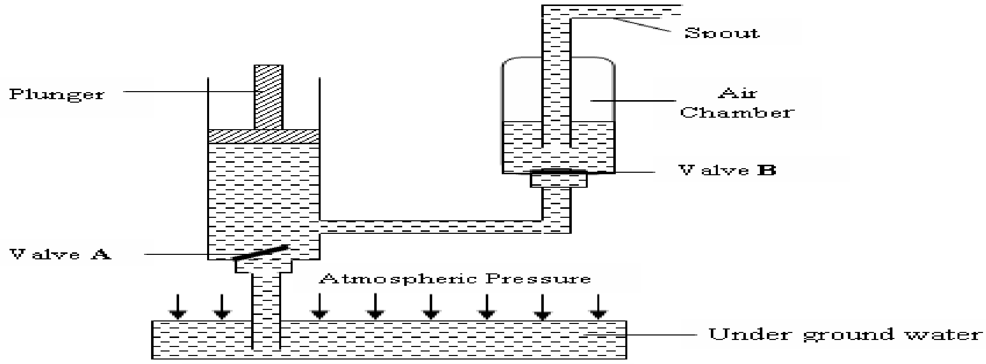


Figure 9.19 The force pump

Mechanism of the Force Pump (How the Force pump Works)

Up stroke

During up stroke, the piston moves up reducing the air pressure in the cylinder. Valve A, opens while valve B, remains closed then atmospheric pressure forces water into the barrel.

Down stroke

During down stroke, the water in the barrel is compressed; valve A closes, B opens and water is forced along the spout into the air chamber compressing the air in the chamber. During the next up stroke, Valve A, opens and valve B, remains closed. The compressed air forces water out of the air chamber along the spout. The result is a continuous delivery of water through the spout.

Other applications of atmospheric pressure include the following.

(i) The syringe.
(ii) The drinking straw.
(iii) The bicycle pump and
(iv) Siphon.

Self-Check 9.1

1. A tank is filled with water to a depth of 3 m. What is the pressure at the bottom of the tank due to the water alone? (d_{water} = 1000kg/m^3)
   A. 30 kPa B. 15 kPa C. 3 000 Pa D. 1 500 Pa

2. A diver is searching for treasure at a depth of 40m below the surface. What is the pressure exerted on the diver. (P_{atm} = 100 000 N/m^2; d_{water} = 100 kg/m^3)
   A. 400 000 N/m^2 B. 200 000 N/m^2 C. 500 000 N/m^2 D. 50 000 N/m^2

3. What is the pressure at the bottom of a 3 km deep oil well filled with oil of density 860kg/m^2?
   A. 5 400 kPa B. 1 080 kPa C. 10 800 kPa D. 25 800 kPa

4. What is the total pressure on a fish swimming in the sea at a depth of 2 m below the surface? (d_{sea} = 1 125 kg/m^3, P_{atm} = 101 300 Pa)
   A. 22.5 kPa B. 123.8 kPa C. 32.6 kPa D. 132.5kPa

5. The piston of a hydraulic automobile lift has 0.48 m^2 area. What pressure is required to lift a car of mass 1 200 kg?
   A. 25 kPa B. 12 kPa C. 30 kPa D. 60 kPa

6. In the hydraulic lift, the area of the smaller piston is one fourth that of the larger piston. If 40 N force is applied on the smaller piston, what is the force on the bigger piston?
A. 640 N B. 160 N C. 10 N D. 320 N

7. In a hydraulic press a force of 40 N is applied to a piston of area $0.1m^2$. The area of the other piston is $4m^2$. What is the pressure transmitted through the liquid and the force on the other piston?
A. 800 N/m^2; 3 200 N B. 400 N/m^2; 1 600 N
C. 400 N/m^2; 400 N D. 800 N/m^2; 1 600 N

8. A hydraulic jack is made with a small piston 12 cm^2 that is used to move a large piston 108 cm^2. If a man can exert a force of 270 N on the small piston, how heavy a load can he lift with the jack?
A. 1 215 N B. 2 430 N C. 4 860 N D. 3 645 N

9. A hydraulic press has a large piston with a cross-sectional area of 250 cm^2 and a small piston with a cross-sectional area of 1.25 cm^2. What is the force on the large piston when a force of 1 250 N is applied to the small piston?
A. 375 000 N B. 125 000 N C. 500 000 N D. 250 000 N

10. In a liquid, pressure is
A. Transmitted in a specific direction B. Transmitted in all direction.
C. Decreased with depth D. Decreased with density

11. Pressure in a liquid is independent of the
A. density of the liquid
B. depth below the surface of the liquid
C. pressure exerted on the surface of the liquid above
D. cross-sectional area and the shape of the vessel containing the liquid

12. A rectangular block of metal weighs 3N and measures (2 x 3 x 4) cm^3. What is the greatest pressure it can exert on a horizontal surface.
A. $5.0 \times 10^3\ Nm^{-2}$ B. $3.75 \times 10^3\ Nm^{-2}$ C. $2.5 \times 10^3\ Nm^{-2}$ D. $7.5 \times 10^{-1} Nm^{-2}$

13. The mass of a cuboid of dimensions 4 m x 2 m x 3 m is 48 kg. The minimum pressure it can exert is
A. 20 Nm^{-2} B. 40 Nm^{-2} C. 60 Nm^{-2} D. 80 Nm^{-2}

14. In a hydraulic machine
A. an object displaces its own weight of fluid
B. the pressure transmitted in the fluid is the same in all directions
C. the volume of fluid compressed is proportional to the applied force
D. an object experiences an upthrust equal to the weight of fluid displaced

15. Which one of the following is true about a manometer?
(i) It uses mercury because mercury is a good conductor of heat.
(ii) It is used for measuring gas pressures.
(iii) The maximum height of mercury it can support is 760mm.
A. (i) and (ii) only B. (i) and (iii) only C. (ii) only D. (ii) and (iii) only.

16. What is 730mm Hg in Nm^2?
A. $\frac{13600 \times 1000 \times 10}{730}$ B. $\frac{13600 \times 730 x 10}{1000}$ C. $\frac{13600 \times 730}{1000 \times 10}$ D. $\frac{13600 \times 10}{1000 \times 730}$

17. A metal cylinder contains a liquid of density 1100 kg/m^3. The area of the base of the cylinder is 0.005 m^2 and the height of liquid is 5m. Calculate the force exerted by the liquid on the base of the cylinder.
A. 27.5 N B. 55 N C. 220 N D. 275 N

18. A uniform tube with a narrowed middle part has three identical manometers attached to it as in the figure 9.21 below. If a steady flow of a liquid is maintained in the direction indicated by the arrows, the height of the liquid will be

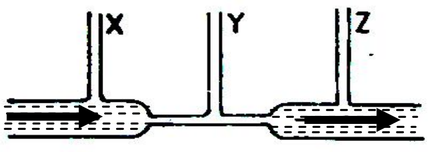


Figure 9.21

A. greatest in X and Y B. greatest in Y
C. greatest in Z D. equal in X, Y and Z

19. Which one of the following are true about a hydraulic brake?
(i) It uses water.
(ii) The brake pedal is connected to the master cylinder.
(iii) The return spring returns the brake drum in position.
(iv) The return spring returns the brake shoe in position.
A. (i) (ii) and (iii) B. (ii) (iii) and (iv) C. (ii) and (iv) D. (iii) and (iv) only.

20. Which one of the following statements is false? The pressure in a liquid
A. at any one point in a liquid would not change even when more liquid is added.
B. at any one point depends only on the depth and density.
C. at any one point acts equally in all directions.
D. increases with depth.

21. When the handle, H, of the force pump shown in figure 9.22 below is moved upwards, the valves at

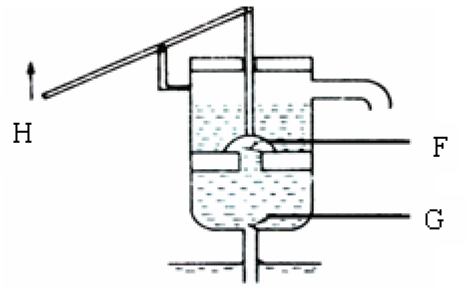


Figure 9.22

A. F and G will both close
B. F and G will both open
C. F will close, and G will open
D. F will open and G will close

22. A rectangular block of dimensions 4 cm x 2 cm x 1cm exerts a maximum pressure of 200 Nm^{-2} when resting on a table. Calculate the mass of the block
A. 4 g B. 16 g C. 40 g D. 400 g

23. Calculate the increase in pressure which a diver experience when he descends 30m in sea water of density 1.2×10^3
A. 3.0×10^2 Nm^{-2} B. 1.2×10^4 Nm^{-2} C. 3.6×10^4 Nm^{-2} D. 3.6×10^5 Nm^{-2}

24. In a hydraulic press, the area of the piston on which the effort is applied is made smaller in order to
A. facilitate the movement of the piston downwards
B. transmit a force as large as possible to the load
C. transmit pressure equally through out the liquid
D. obtain a pressure as large as possible.

25. Which of the following is true about pressure in liquids? It
A. increases with the surface area of the liquid B. is directly proportional to the depth
C. depends on the shape of the container D. is the same at equal depths in all liquids

SECTION B

26. **(a) (i)** Define pressure and state its units.

(ii) With the aid of a diagram, describe how you would show that the pressure of a liquid is in dependent of cross-sectional area and shape of a container.

(b) Two manometers P and Q contain a liquid X, and water respectively at the same level. They are then connected to a thistle funnel covered with a rubber membrane as shown in figure 9.23. When the thistle funnel is lowered into a beaker containing a dilute acid of density 1200 kgm^{-3}, the heights h_1 and h_2 are 15 cm and 12 cm respectively.

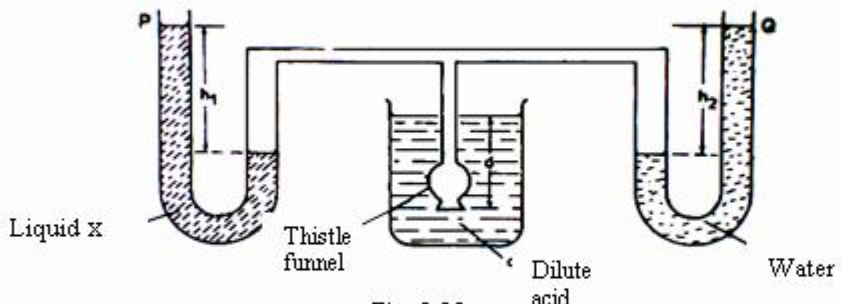


Fig. 9.23

Find the: (i) Ratio of the density of liquid X to that of water,

(ii) Depth d of the thistle funnel below the surface of the dilute acid.

(iii) Explain why a ship floats in water although it is made mainly of metal.

27. **(a)** (i) State the principle of transmission of pressure in fluids.

(ii) Give one assumption on which the principle is based.

(iii) State two application of the principle.

(iv) In a hydraulic press the smaller piston has a diameter of 14 cm while the larger has a diameter of 280 cm. If a force of 77 N is exerted on the smaller piston, calculate the force exerted the larger piston.

(b) With a help of diagram, describe how hydraulic brake works.

28. **(a)** Explain why large water reservoirs are much wider at the base than at the top.

(b) Figure 9.24 shows the structure of a force pump.

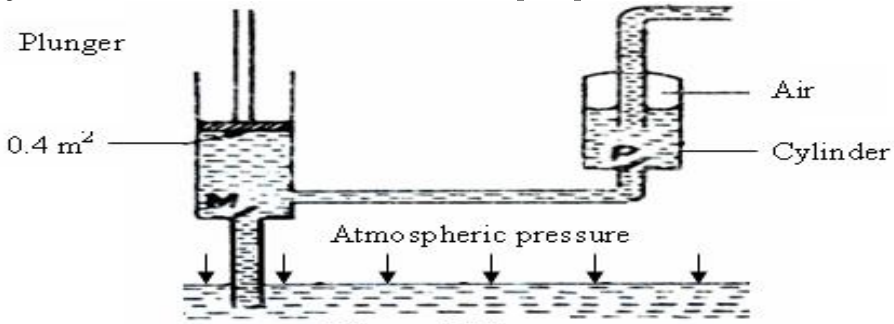


Figure 9.24

(i) Describe the action of the pump.

(ii) If a downward of 500 N is exerted of the plunger whose surface area is 0.4 m^2, calculate the pressure which forces water into cylinder C.

29. **(a)** Define term pressure and state its unit.

(b) (i) Describe how a simple mercury barometer can be set up to measure the atmospheric pressure.

(ii) The difference between the atmospheric pressure at the top and bottom of a mountain is 1 x 10^4 N m^{-2}. If the density of air is 1.25 kgm^{-3}, calculate the height of the mountain.

CHAPTER TEN
ARCHIMEDES'S PRINCIPLE

LEARNING OBJECTIVES

By the end of this chapter you should be able to:

1. *State: - Archimedes's Principle.*
2. *Describe an experiment to verify the principle.*
3. *State: - Applications of Archimedes's Principle.*
4. *Solve problems involving Archimedes's Principle.*
5. *State: - The law of Flotation.*
6. *Describe an experiment to verify the law of Flotation.*
7. *State: - Applications of the law of Flotation.*
8. *Solve problems involving the law of Flotation.*
9. *Define: - Terminal Velocity*
10. *(a) Describe: - What happens to a body released to fall in a liquid in a tall measuring cylinder until it hits the bottom.*
    (b) Sketch a velocity- time graph for a body falling in a fluid.

10.1 Archimedes' Principle

Archimedes' Principle states that:

When a body is wholly or partially immersed in a fluid, it experiences an upthrust equal to the weight of fluid displaced.

Terms Used.

Upthrust:	- a force in a fluid that acts upwards.
Apparent weight:	- weight on an object in liquid.

Experiment 10.1 To verify Archimedes's Principle

Apparatus/Requirements
Displacement can, water, Light and thin string, spring balance, A Suitable solid.

Procedure:

- Fill a displacement can with water till water flows through the spout and wait until the water ceases (stops) dripping.
- Suspend a solid object using a light string from a spring balance and weigh it in air.
- Place a beaker of known weight under the spout of the displacement can.
- Immerse the solid object in to the water in the displacement can and wait until water ceases dripping into the beaker.
- Read and record the apparent weight.
- Reweigh the beaker and the displaced water.

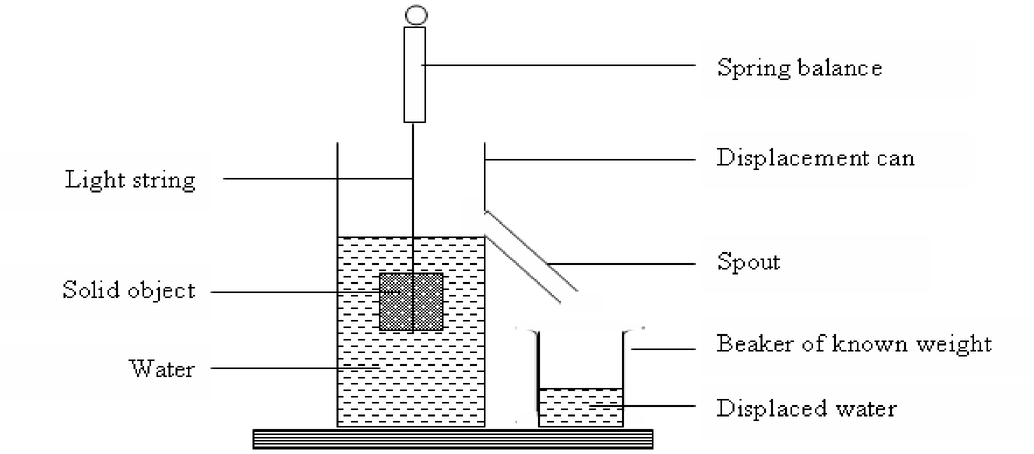


Figure 10.1

Results:

Let: Weight of solid object in air $= W_a$ N
Weight of solid object in water (Apparent weight) $= W_w$ N
Weight of empty beaker $= W_b$ N
Weight of empty beaker + Displaced water $= W_{(b+w)}$ N

Calculation:

Uptrust (Loss in weight of object) $= W_a - W_w$ N
Weight of water displaced $= W_{(b+w)} - W_b$ N

Conclusion: If $W_a - W_w = W_{(b+w)} - W_b$,
i.e Upthrust = Weight of fluid displaced, then Archimedes's principle is verified.

10.2 Application of Archimedes's Principle

Archimedes's Principle is applied in the measurement of relative density of solid and liquid substances.

Experiment 10.2 To measure the Relative Density of Solid

Apparatus/Requirements

A beaker, water, Light and thin string, spring balance and suitable solid.

Procedure:

- Suspend the solid whose relative is to be determined from a spring balance by means of light string in air and record its weight.
- Immerse the solid wholly in water and record its apparent weight.

Results:

Let: Weight of solid object in air $= W_a$ N
Weight of solid object in water (Apparent weight) $= W_w$ N

Calculation:

Uptrust (Loss in weight of object) = Weight of water displaced
$= W_a - W_w$ N

But Volume of water displaced = Volume of the solid immersed

From Relative Density $= \dfrac{\textit{Weight of substance}}{\textit{Weight of an equal volume of water}}$

$$= \frac{W_a}{W_a - W_w}$$

I.e. ***Relative Density*** $= \dfrac{Weight\ of\ substance\ in\ air}{Upthrust\ in\ water}$

Examples

1. A glass stopper weighs 44 N in air and 24N when completely immersed in water. Calculate the relative density of glass.

Solution

Weight of solid object in air = 44 N
Weight of solid object in water (Apparent weight) = 24 N
Weight of water displaced = Uptrust in water = 44 – 24 N
= 20 N

$$\text{Relative Density} = \frac{Weight\ of\ substance\ in\ air}{Upthrust\ in\ water} = \frac{44}{20} = \mathbf{2.2}$$

Experiment 10.3 To measure the Relative Density of Liquid

Apparatus/Requirements

A beaker, water, Light and thin string, spring balance and the liquid.

Procedure

- Suspend a solid from a spring balance by means of light string in air and record its weight.
- Immerse the solid wholly in water and record its apparent weight.
- Wipe the surface of the solid with a piece of dry cloth and immerse it wholly in the liquid whose relative density is to be found.
- Read and record its weight.

Results: Let: Weight of solid object in air = W_a N
Weight of solid object in water (Apparent weight) = W_w N
Weight of solid object in liquid (Apparent weight) = W_l N

Calculation: Weight of water displaced = Uptrust in water
= $W_a – W_w$ N
Weight of liquid displaced = Uptrust in liquid
= $W_l – W_w$ N

But in each case, the solid displaces its own volume of liquid.
But Volume of water displaced = Volume of the solid immersed
= Volume of the liquid displaced.

From Relative Density $= \dfrac{Weight\ of\ substance}{Weight\ of\ an\ equal\ volume\ of\ water}$

$$= \frac{Weight\ of\ liquid}{Weight\ of\ an\ equal\ volume\ of\ water}$$

$$= \frac{W_a - W_l}{W_a - W_w}$$

I.e. Relative Density $= \dfrac{Upthrust\ in\ liquid}{Upthrust\ in\ water}$

Worked Example

1. A piece of iron weighs 145 N in air. When completely immersed in water, it weighs 120 N and weighs 125N when completely immersed in alcohol. Calculate the relative density of alcohol.

Solution: Weight of solid object in air = 145 N
Weight of solid object in water (Apparent weight) = 120 N
Weight of solid object in liquid (Apparent weight) = 125 N

Calculation: Weight of water displaced = Upthrust in water
= 145 – 120 N
= 25 N

Weight of alcohol displaced = Upthrust in liquid
= 145 – 125 N
= 20 N

Relative Density $= \dfrac{Upthrust\ in\ liquid}{Upthrust\ in\ water} = \dfrac{20}{25}$ **= 0.8**

10.3 Flotation

A body floats in a liquid if its density is less than the density of the liquid. A floating body sinks deeper in liquids of less density than in liquids whose densities are high. This is seen when ships move from fresh water to sea or ocean water. The depth to which the ship sinks is shown by lines called *plimsoll line* marked on the side of ship.

The law of Flotation

The law of flotation states that:
A floating body displaces its own weight of fluid in which it floats.

(a) Facts about a floating body

- ❖ The weight of the floating body = the weight of the fluid displaced.
- ❖ The weight of the fluid displaced = Upthrust (Archimedes's Principles)
- ❖ Therefore, the Upthrust = the weight of the floating body
- ❖ The mass of the floating body = the mass of the fluid displaced.

Experiment 10.4 To verify the Law of Flotation

Apparatus: Beaker, water, light string, spring balance, a suitable solid.

Procedure:
- Fill a beaker with water.
- Suspend a solid object in air using a light string from a spring balance and record its weight.
- Immerse the solid object completely in to the water in the beaker.
- Read and record the apparent weight (weight when the solid object is in liquid).

Observation: The string becomes slack (loose).

Results: Let: Weight of solid object in air = W_a N
Weight of solid object in water = 0.0 N

Calculation: Weight of water displaced = Uptrust in water = W_a – 0.0 N

Conclusion: If $W_a - 0.0 = W_a$,

I.e. Upthrust = Weight of fluid displaced, then law of flotation is verified.

(b) Relationship between the Density of floating body, Density of liquid and the Fraction submerged

The density of a floating body is related to the density of the liquid and the fraction submerged is by the formula:

Density of floating body* = *Density of liquid* x *Fraction submerged

Derivation of the formula

Points to note:

- A floating body is supported by the upthrust of the liquid.
- Therefore, in equilibrium,
  - the upthrust = the weight of the floating body.
- The mass of the floating body = the mass of the fluid displaced.
- The fraction submerged = the volume of the liquid displaced

Now let;

The mass of the body floating $= m_b$ g

The mass of the liquid displaced = The mass of the floating body (*Law of flotation*)

$= m_b$ g

The density of the floating body $= \rho_b$ g/cm^3

The density of the liquid $= \rho_l$ g/cm^3

The volume of the body above the liquid surface $= v_x$ cm^3

The volume of the body below the liquid surface $= v_y$ cm^3

The volume of the body floating, v $= (v_x + v_y)$ cm^3

The fraction submerged $= \frac{v_b}{(v_x + v_y)}$

The density of the floating body, $\rho_b = \frac{m_b}{(v_x + v_y)}$ g/cm^3

$m_b = \rho_b(v_x + v_y)$ ……………………. 1

The density of the liquid, $\rho_l = \frac{m_l}{v_y}$ g/cm^3 *But* $m_l = m_b$

$\rho_l = \frac{m_b}{v_y}$ g/cm^3

$m_b = \rho_l \, v_y$ ……………………. 2

Equating equation (1) to (2) we have:

$$\rho_b(v_x + v_y) = \rho_l\, v_y$$

$$\rho_b = \rho_l \times \left(\frac{v_y}{(v_x + v_y)}\right)$$

∴ **_Density of floating body = Density of liquid_ x _Fraction submerged_**

Note that $\frac{v_y}{(v_x + v_y)}$ = Fraction of the floating body submerged

Rearranging the equation, we have:

Fraction submerged $= \dfrac{\textit{Density of floating body}}{\textit{Density of liquid}}$

Worked Examples

1. A piece of wood of volume floats with $\frac{4}{5}$ of its volume under a liquid of density 800 kgm^3. Find the density of the wood in kgm^{-3}.

Solution: Fraction submerged $= \frac{4}{5}$, Density of the liquid = 800 kgm^{-3}

Using the formula

Density of floating body = *Fraction submerged* x *Density of liquid*

We have: $= \frac{4}{5} \times 800 = \frac{3200}{5}$ **= 640 kgm^3**

2. A piece of wood of volume 240 cm^3 floats with three quarters of its volume under water. Calculate the density of the wood if the density of water is 1000 kgm^{-3}.

Solution: Fraction submerged = ¾, Density of the water = 1000 kgm^{-3}

Density of floating body =*Fraction submerged* x *Density of liquid*

$= ¾ \times 1000 = \frac{3000}{4}$ ***= 750*** kgm^{-3}

10.4 Application of the law of flotation

The practical application of the law of flotation is seen in the following:

(i) Ships

Ships are able to float on water although they are made of metal. This is because their average densities are less the density of water.

Ships float deeper in fresh water than in salty (sea water). This is because the density of fresh water is less than the density of salty water.

(ii) Submarines

These are ships that float submerged in water. They are equipped with periscope for viewing and tanks which can be filled with air or water to alter the density of the submarine. Thus making the submarine to float afloat, awash or submerged.

(iii) Balloons

Balloons are airships used for recording meteorological measurements.

The balloon is filled with a light gas e.g hydrogen gas until it displaces a weight of air greater than its own weight. The greater upthrust then pushes the balloon upwards.

It continuous to rise until the upthrust on it is equal to the weight of the balloon plus its contents and then floats sideways.

$\left(The\ lifting\ power\ of\ balloon\right)$ = Upthrust -

$\left(Weight\ of\ balloon + Weight\ of\ its\ contents\right)$

Example

1. A balloon has a capacity of 10 m^3 and is filled with hydrogen. The balloon's fabric and the container have a mass of 1.25 kg. Calculate the maximum mass load the balloon can lift. (Density of hydrogen = 0.089 kgm^{-3} and of air = 1.29 kgm^{-3}, gravity, g = 10 ms^{-2}).

Solution: Volume of balloon, v = 10 m^3, Density of hydrogen, ρ_h = 0.089 kgm^{-3},

Density of air, ρ_a = 1.29 kgm^{-3}, g = 10 ms^{-2},

Volume of air displaced, v_a = volume of balloon = 10 m^3

Volume of hydrogen, v_h = Volume of balloon = 10 m^3.

Mass of (balloon + container) = 1.25 kg

Let the mass of the load = x kg

Total mass = Mass of balloon + Mass of Hydrogen + Mass of the load

= 1.25 + $v_h\rho_h$ + x

= 1.25 + 10 x 0.089 + x

= (1.25 + 0.89 + x) kg

Weight of balloon = Total mass x gravity

= (2.14 + x)g N

Weight of displaced air = mg

= $v\rho g$

= 10 x 1.29 x g

= 12.9g N

For the balloon just to rise, upward force must be equal to the downward force i.e.

Upthrust = Weight of the load

12.9$\cancel{g}$ = (2.14 + x)$\cancel{g}$

x = 12.9 - 2.14

$\therefore$ x **= 10.76 kg**

.5 Hydrometer

A hydrometer is an instrument used to measure relative density of liquids. It consists of three main parts, as shown in the diagram below.

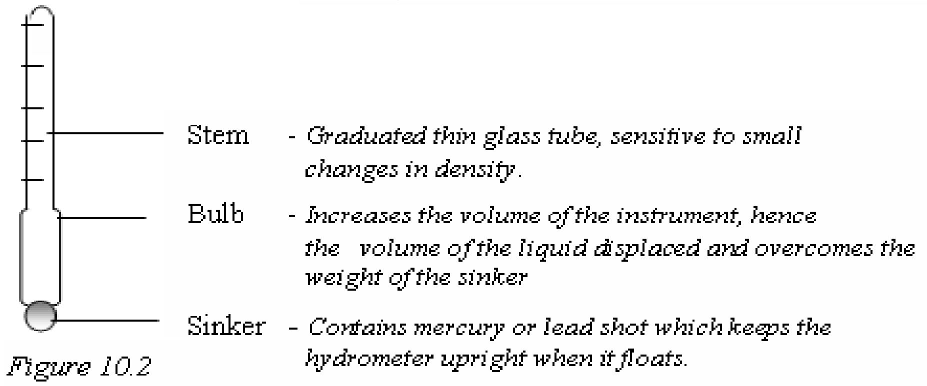


Figure 10.2

The hydrometer is placed in the liquid whose R.D is to be measured and the scale read at the level of the liquid surface.

NB:

- ❖ Sensitivity of the hydrometer increased by making the stem very thin.
- ❖ The buoyancy is increased by making the bulb large.
- ❖ The smaller scales are at the top and the bigger scales are at the bottom. This is because the hydrometer floats deeper in lighter liquids than in denser liquids.

Common examples of hydrometers are:

- *Car hydrometer (battery tester)* - Used to test the state of the charge of a car battery
- *Lactometer* - Used to test the purity of milk.

10.6 Terminal Velocity

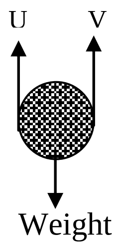
Terminal velocity – *is the constant velocity attained by a body falling through a fluid.*

Facts about a body falling in a fluid

(i) The body first accelerates i.e. its velocity increases with time.

(ii) After some time, it attains a constant velocity called *terminal velocity* and falls with this velocity until it hits the bottom of the cylinder.

A diagram showing a ball bearing falling through a liquid



Where: U = Uthrust
V = Viscosity (fluid friction)

Explanation

As the ball bearing falls through the liquid, the liquid resistance (viscosity) exerts an upward force opposing the gravity. Since viscosity in fluids increases with velocity (i.e. the higher the velocity the greater the viscosity), the viscosity or viscous drag increases with velocity eventually the combined upward force of upthrust and the viscosity is equal to the weight (force of gravity) on the ball bearing. At this point the resultant force on the ball bearing is zero and its. As no force acts on the ball bearing, it has no *acceleration*, but it acquires a constant velocity called *terminal velocity*.

When the values of velocity is plotted against time the shape of the graph below is obtained

The graph of velocity against time for a body falling through a fluid

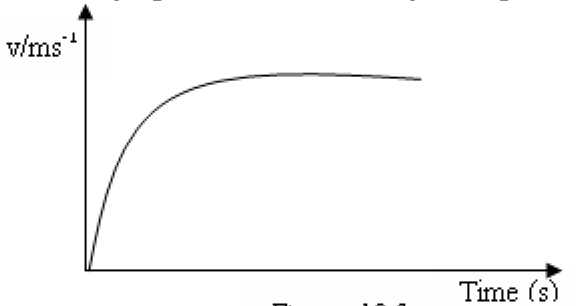


Figure 10.3

Self-Check 10.0

1. A block of metal (density 2700 kg/m^3) has volume 0.09 m^3. Calculate the up thrust force when it is completely immersed in brine (density 1 200 kg/m^3)
A. 600 N B. 1 080 N C. 180 N D. 1200 N

2. An iron box of mass 90 g and density 0.9g/cm^3 floats on brine (density 1200kg/m^3). What is the volume immersed in the brine?
A. 40 cm^3 B. 60 cm^3 C. 90 cm^3 D. 75 cm^3

3. The envelope of a hot air balloon contains 1500 m^3 of hot air of density 0.8 kg/m^3. The mass of the balloon (not including the hot air) is 420 kg. the density of surrounding air is 1.3 kg/m^3. What is the lifting force?
A. 3 300 N B. 4 400 N C. 5 500 N D. 6 600 N

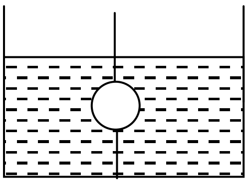
4. What fraction of a wooden cube of one side 10cm will be below the water level if the cube is floating in water? (d_{water} = 1g/cm^3, d_{wood} = 0.6 g/cm^3)
A. $^3/_5$ B. $^2/_5$ C. $^1/_5$ D. $^1/_{10}$

5. An ice cube of volume 600 cm^3 floats in water. What is the volume of the part above the water level? (d_{ice} = 0.9 g/cm^3, d_w = 1 g/cm^3)
A. 60 cm^3 B. 80 cm^3 C. 520 cm^3 D. 540 cm^3

6. A cubical brass block floats in mercury. What fraction of the block lies above the mercury surface? (the densities of brass and mercury are 8 600 kg/m^3 and 13 600 kg/m^3 respectively)
A. $^{86}/_{136}$ B. $^{50}/_{86}$ C. $^{136}/_{86}$ D. $^{50}/_{136}$

7. A large, hollow plastic sphere is held below the surface of a fresh water pool by a cable attached to the bottom of the pool. The sphere has a volume of 0.400 m^3.

Plastic sphere

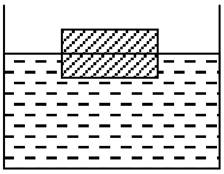


Calculate the buoyancy force exerted by the water on the sphere.
A. 4 000 N B. 800 N
C. 3 200 N D. 4 800 N

8. A lead sphere has a total mass of 45.2 kg. If it is put into water, what will be the upthrust force on it? (Density of lead is 11 300 kg/m^3)
A. 10 N B. 20 N C. 30 N D. 40 N

9. A block of wood of volume 80cm^3 and density 0.5 g/cm^3 floats in water. What is the volume immersed in water (density of water = 1g/cm^3)
A. 50 cm^3 B. 40 cm^3 C. 30 cm^3 D. 20 cm^3

10. Figure 2 shows a block of volume 40 cm^{-3} floating in water with only half of its volume submerged. If the density of water is 1000 kg m^{-3}, determine the mass of the wood.



A. 40 x 1000 kg. B. 20 x 1000 kg.
C. 40 x 10^{-6} x 500 kg. D. 20 x 10^{-6} x 500 kg.

SECTION B

11. (a) State Archimedes's Principle.
(b) Describe an experiment to verify Archimedes's Principle.
(c) A piece of iron weighs 355 N in air. When completely immersed in water, it weighs 305 N and weighs 315 when completely immersed in methylated spirit. Calculate the relative density of methylated spirit.
(d) State the application of Archimedes's principle.

12. (a) State the law of flotation.
(b) Describe an experiment to verify the law of flotation.
(c) A piece of wood of volume 40 cm^3 floats with only half of volume submerged. If the density of water is 1000 kg m^{-3}, calculate the density of wood.

13. A balloon has a capacity of 20 m^3 and is filled with hydrogen. The balloon's fabric and the container have a mass of 2.5 kg. Calculate the maximum mass load the balloon can lift. (Density of hydrogen = 0.089 kgm^{-3} and of air = 1.29 kgm^{-3}, gravity, g = 10 ms^{-2}).

14. (a) Define terminal velocity.
(b) Explain with a help of velocity-time graph, what happens to a USA soldier parachuted from a war plane flying at a high altitude in an area free of enemies in Iraq from the time he leaves the plane till he lands to the ground.

CHAPTER ELEVEN
MOTION

LEARNING OBJECTIVES

By the end of this chapter, you should be able to:

1. ***(a)*** *Define: - Speed, Velocity, Uniform velocity, Acceleration and uniform acceleration.*

(b) *State: - The S.1 units of: Speed, Velocity and Acceleration.*
- The difference between speed and velocity.

(c) State: - The equations of linear motion.

(d) Solve problems using the equations of linear motion.

2. (a) Represent linear motion using: - Displacement-time graph and
- Velocity-time graph.

(b) Describe from: - Displacement-time graph and
- Velocity-time graph.

3. (a) Calculate: - Average speed/velocity and
- Acceleration from a sample of ticker-tape.

(b) Describe: - Motion from a sample of a ticker tape.

4. (a) Solve problems involving: - Motion under gravity and
- Projectile motion.

(b) State: - The forces acting on a body moving in a circle.

11.1 Speed velocity and Acceleration

(a) Speed

Speed is defined as the rate of change of distance moved with time.

OR *Speed is the distance traveled in a unit time.*

Mathematically, it is expressed as: $\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$

S.I unite of speed is **m/s or ms^{-1}.**

(b) Displacement

Displacement is defined as distance moved in a specified direction.

For example if a body moves a long a straight line in a given direction such as 100m due east or 50m due north.

(c) Velocity

Velocity is defined as the rate of change of displacement with time.

Or Velocity is the rate of change of distance moved with time in a specified direction.

Or Velocity is defined as speed in a specified direction.

The S.I unite of velocity is **ms^{-1} or m/s.**

Difference between speed and velocity

Velocity is a vector quantity whereas speed is a scalar quantity.

NB: *Vector quantity is a quantity that has both magnitude and direction.*
Scalar quantity is a quantity that magnitude only.

Uniform Velocity

Uniform velocity is the velocity when the rate of change of displacement is constant.

Non-uniform velocity

Non-uniform velocity is the velocity when the rate of change of displacement is not constant.

Acceleration

Definition *Acceleration is defined as the rate of change of velocity with time.*

$$\text{Acceleration} = \frac{\textit{Change of velocity}}{\textit{Time Taken}}$$

$$= \frac{\textit{Final velocity} - \textit{Initial velocity}}{\textit{Time Taken}}$$

The S.I unit of acceleration

The SI unit of acceleration = m/s^2 or ms^{-2} . Read as metres per sec squared.

When the velocity of a body is changing, the body is said to be accelerating.

Acceleration is: *Positive* *- if the velocity is increasing and*
Negative *- if the velocity is decreasing.*

A negative acceleration is called deceleration or retardation.

Uniform Acceleration

Uniform acceleration is the acceleration when the rate of change of velocity is constant.

11.2 The Equations of Linear Motion

The following are the equations for uniformly accelerated body.

$\mathbf{v = u + at}$	*Where:*	***u*** *= Initial velocity i.e velocity when t = 0*
$\mathbf{s = \frac{1}{2}(u + v)t}$		***v*** *= Final velocity i.e velocity at time t*
$\mathbf{s = ut + \frac{1}{2}at^2}$		***a*** *= Constant acceleration*
$\mathbf{v^2 = u^2 + 2as}$		***s*** *= Distance traveled from the starting*

Notes**:** 1. When using the equations, it is necessary to bear in mind that ***u***, ***v***, ***a***, and ***s*** are vectors. If, say, the positive direction is taken to be *up*, then:

(i) The velocity of a body which is moving *down* (i.e in opposite direction) is negative.

(ii) Points below the starting point have negative values of **s.**

(iii) *Downward* directed accelerations are negative.

2. This facts will help a learner to understand a velocity-time graph with negative values of velocity.

Derivation of the Equations of Motion

Suppose that a body is moving with constant acceleration ***a*** and that in a time interval ***t*** its velocity increases from ***u*** to ***v*** and its displacement increases from 0 to ***s***. Then, from the definition of acceleration we have:

$$\text{Acceleration} = \text{Rate of change of velocity}$$

$$\text{Acceleration} = \frac{Change\ of\ velocity}{Time\ Taken}$$

$$= \frac{Final\,Vlocity - Initial\,Velocity}{Time\ Taken}$$

$$a = \frac{v-u}{t}$$

$$\boldsymbol{v = u + at} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad 1$$

Since the acceleration is uniform,

$$\text{Average velocity} = \frac{Final Vlocity + Initial Velocity}{2}$$

$$= \frac{v-u}{2} \text{ and therefore}$$

$$\text{Displacement} = \text{Average velocity x Total time}$$

$$s = \frac{(v+u)t}{2}$$

$$\boldsymbol{s = \tfrac{1}{2}(v + u)t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad 2$$

Eliminating ***v*** from equation gives equation 3. This is done by substituting equation 1 **(v = u + at)** in equation 2.

$$s = \frac{(u + at + u)t}{2}$$

$$= \frac{(2u + at)t}{2}$$

$$= \frac{(2ut + at^2}{2}$$

$$\mathbf{s = ut + \frac{1}{2}at^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad 3$$

Eliminating ***t*** from equation (**2**) gives equation (**4**).

This is done by substituting $t = \left(\frac{v-u}{a}\right)$ from equation (**1**) in equation (2).

I.e. $$s = \left(\frac{v+u}{2}\right) \text{ x } \left(\frac{v-u}{a}\right)$$

$$s = \frac{v^2 - u^2}{2a}$$

$$2as = v^2 - u^2$$

$$\boldsymbol{v^2 = u^2 + 2as} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad \mathbf{4}$$

Worked Examples

1. An object starts from rest and moves with an acceleration of 10 m/s^2 for four seconds.
   Calculate: (a) The velocity
   (b) The distance traveled

   Solution: **(a)** $u = 0$, $a = 10\ ms^{-2}$, $t = 4$ s, $v = ?$

   Using the formula
   $$\begin{aligned} v &= u + at \\ &= 0 + 10 \times 4 \\ \mathbf{v} &\mathbf{= 40 m/s} \end{aligned}$$

   (b) Using the formula
   $$\begin{aligned} s &= ut + \tfrac{1}{2}at^2 \\ &= 0 \times 4 + \tfrac{1}{2} \times 10 \times 4 \times 4 \\ &= 0 + 80\text{ m} \\ \therefore\ \mathbf{s} &\mathbf{= 80\ m} \end{aligned}$$

2. A car moving with uniform acceleration of 4 m/s^2 increases its velocity from 20 m/s to 60 m/s, Calculate: (a) the total time taken during this change
   (b) the total distance moved

   Solution: $a = 4\ m/s^2$, $u = 20$ m/s, $v = 60$ m/s, $t = ?$, $s = ?$

   (a) Using the formula
   $$\begin{aligned} v &= u + at \\ 60 &= 20 + 4 \times t \\ 60 &= 20 + 4t \\ 4t^2 &= 60 - 20 \\ 4t^2 &= 40 \\ \frac{4t}{4} &= \frac{40}{4} \\ \therefore\ \mathbf{t} &\mathbf{= 10\ s} \end{aligned}$$

   (b) Using the formula
   $$\begin{aligned} s &= ut + \tfrac{1}{2}at^2 = 20 \times 10 + \tfrac{1}{2} \times 4 \times 10 \times 10 \\ &= 200 + 200 \\ \therefore\ \mathbf{s} &\mathbf{= 400\ m} \end{aligned}$$

3. A body traveling with a velocity of 10m/s is uniformly retarded to rest in 5 seconds
   Find (a) its acceleration.
   (b) the distance traveled during the retardation.

   Solution: $u = 10$ m/s, $v = 0$, $t = 5$ s, $a = ?$, $s = ?$

   (a) Using the formula
   $$\begin{aligned} v &= u + at \\ 0 &= 10 + 5 \\ \frac{5a}{5} &= \frac{-10}{5} \\ \therefore\ \mathbf{a} &\mathbf{= -2\ m/s^2} \end{aligned}$$

   The minus sign means that the body is accelerating in the opposite direction to its initial velocity.

   (b) Using the formula:
   $$\begin{aligned} v^2 &= u^2 + 2as \\ 0 &= 10^2 + 2 \times 2 \times s \\ 0 &= 100 + 4s \\ 4s &= 100 \\ \frac{4s}{4} &= \frac{100}{4} \\ \therefore\ \mathbf{s} &\mathbf{= 25\ m} \end{aligned}$$

11.21 Changing velocity from km/h to m/s

Velocity given in km/h can be changed to m/s. This is simply done by applying the formula for speed.

I.e Speed $= \dfrac{Distance\,(km)}{time\ (h)}$

Steps followed

- ❖ Interpret the value of the speed given.
  E.g. *x km/h* means that the body travels *x km* in *1 hour*.
- ❖ Change the numerical value of the velocity or speed to metres and also change 1 hour to seconds.
- ❖ Divide the distance in metres by the time in seconds.

Example

Change the speed 36 km/h to m/s.

The speed *36 km/h,* means that the body travels *36 km* in *1 hour*.
To change the speed from km/h to m/s, we change the *36 km* to *m* and *1 hour* to *seconds*.

Changing 36 km to m: 1 km = 1000 m
36 km = x

$$\mathbf{x} = \frac{36\,km\ x\ 1000\ m}{1\ km} = \mathbf{36\ 000\ m}$$

Changing 1 hour to seconds: 1 h = 60 min
= (60 x 60) s
= 3600 s

Now we substitute the distance in m and time in seconds in the formula:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{36000}{3600} = \mathbf{10\ m/s}$$

Self-Check 11.1

1. An object starts from rest and moves with an acceleration of 10 m/s^2 for four seconds.
   Calculate: (a) The velocity
   (b) The distance traveled .
2. A car moving with uniform acceleration of 4 m/s^2 increases its velocity from 20 m/s to 60 m/s, Calculate: (a) the total time taken during this change
   (b) the total distance moved
3. A body traveling with a velocity of 10m/s is uniformly retarded to rest in 5 seconds
   Find: (a) its acceleration.
   (b) the distance traveled during the retardation.
4. Change the following speeds from km to m/s.
   (a) 72 km/h (b) 108 km/h (c) 180 km/h (d) 288 km/h

11.3 Graphical Representation of Linear Motion

Graphs can be used to represent the motion of a body which is moving in a straight line.

(a) Types of Graphs of Linear Motion

There are two types of graphs namely:

- Distance-time graph and
- Velocity-time graph.

(b) Distance-Time graph

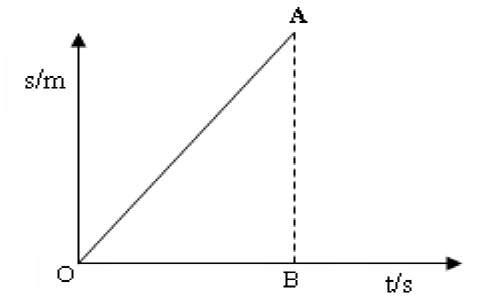
A distance-time graph is the type of graph obtained by plotting distance/displacement travelled against time taken.

The common graphs are for a body:

- Moving with a uniform velocity
- Non-uniform velocity and
- At rest.

By definition, velocity is rate of change of displacement and therefore the slope of a graph of displacement against time represents velocity.

(i) Uniform velocity

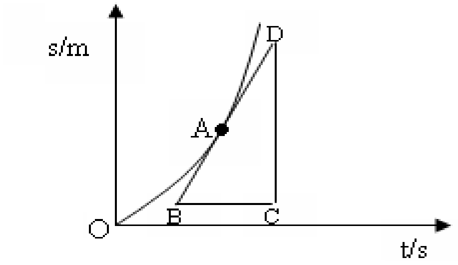


$$\text{Velocity} = \frac{Distance}{Time\ taken}$$

$$= \frac{AB}{OB}$$

I.e Velocity = Gradient of the line OA

(ii) Non-uniform velocity

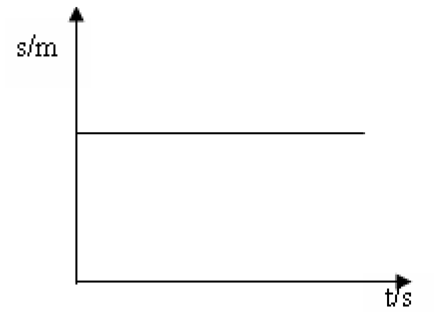


$$\text{Velocity} = \frac{Distance\ represented\ by\ DC}{Time\ interval\ represented\ by\ BC}$$

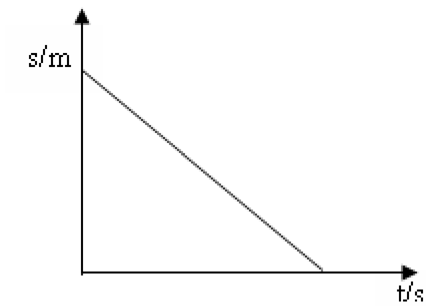
$$\text{Velocity at A} = \frac{DC}{BC}$$

= Gradient at A

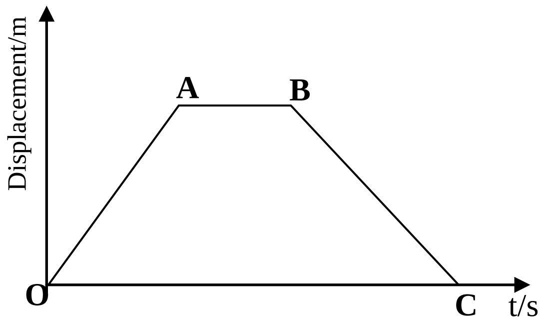
(iii) A body at rest



(iv) A body on return journey



However, these graphs can be combined to form a single graph with the different stages of the motion as shown below.



OA - The slope is positive
- The body is moving constant velocity during this period

AB - The slope is zero i.e the velocity is zero
- The body is stationary

BC - The slope is negative
- The body is moving constant velocity in the opposite direction during this period

Velocity-Time graph

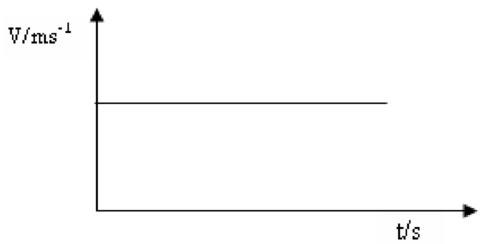
A velocity-time graph is the type of graph obtained by plotting velocity against time.

The common graphs are for a body:

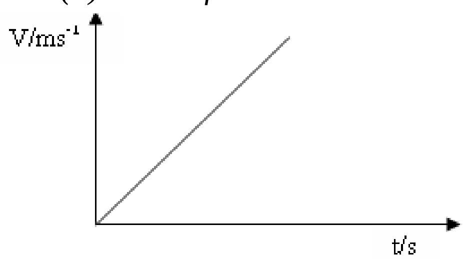
- ❖ moving with uniform velocity (constant velocity)
- ❖ Non-uniform velocity,
- ❖ Uniform acceleration,
- ❖ Non-uniform acceleration and
- ❖ Uniform deceleration

By definition, acceleration is rate of change of velocity and therefore, the slope of a graph of velocity against time represents acceleration.

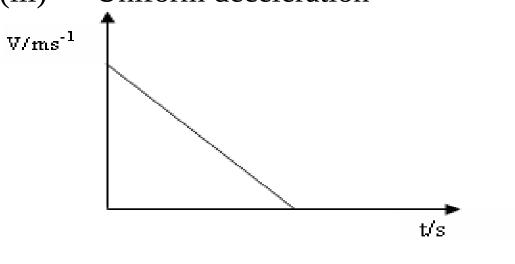
(i) *Uniform velocity,*



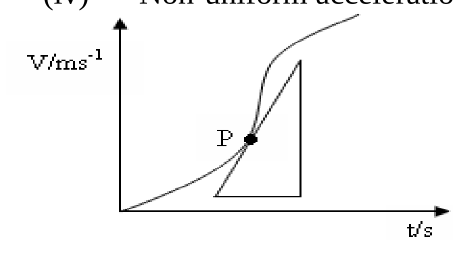
(ii) *Uniform acceleration*



(iii) Uniform deceleration



(iv) Non-uniform acceleration



Acceleration $= \dfrac{Change\,in\,velocity}{Change\,in\,time}$

$= \dfrac{AB}{OB}$

Acceleration = *Gradient of the line OA*

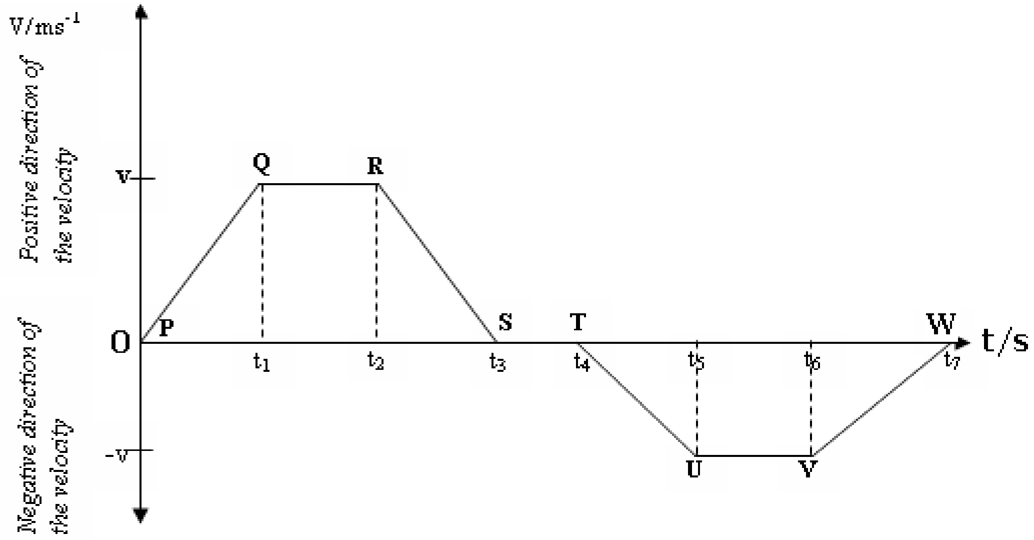
Acceleration $= \dfrac{Change\,in\,velocity}{Change\,in\,time}$

Acceleration at A $= \dfrac{DC}{BC}$

= *Gradient at A*

However, the graphs for uniform acceleration, velocity and deceleration can be combined to represent a linear motion for a body starting from rest or at a certain initial velocity. Then the total distance travelled is equal to the area under the graph and can be calculated using equations of motion or using appropriate formulae.

A velocity-time graph for a body that travelled a given distance and returned and stopped on the way.



Notes: 1. When interpreting a graph, it is necessary to bear in mind that ***u***, ***v***, ***a***, and ***s*** are vectors.

If, say, one direction is taken to be positive, then:

(i) The velocity of a body which is moving in the opposite direction is negative.

(ii) Points below the starting point have negative values of **s.**

(iii) *Downward* directed accelerations are negative.

(iv) The magnitude and the direction of the slope of the graph give the magnitude and direction of the acceleration.

2. These facts will you to understand the interpretation of the above velocity-time graph where there are negative values of velocity.

Stage	State of the body	Interpretation
P - Q	The body starts from rest and moves with a uniform acceleration during this period.	- The velocity is positive and increasing. - The slope is positive and constant. - The acceleration is positive and constant.
Q - R	The body is moving with a constant velocity during this period.	- The velocity is positive and constant. - The slope is zero. - The acceleration is zero.
R - S	At t_2, the body brakes and is uniformly decelerated to rest at t_3.	- The velocity is positive and decreasing. - The slope is negative and constant. - The acceleration is negative and constant. - The negative acceleration means that it acts in the opposite direction the direction of velocity.
S - T	The body is stationary at this period	- The velocity is zero (the body is stationary) - The slope is zero - The acceleration is zero
T - U	At time t_4, the body returns and moves with a constant acceleration in the opposite direction.	- The velocity is negative and increasing. - The slope is negative and constant. - The acceleration is negative and constant.

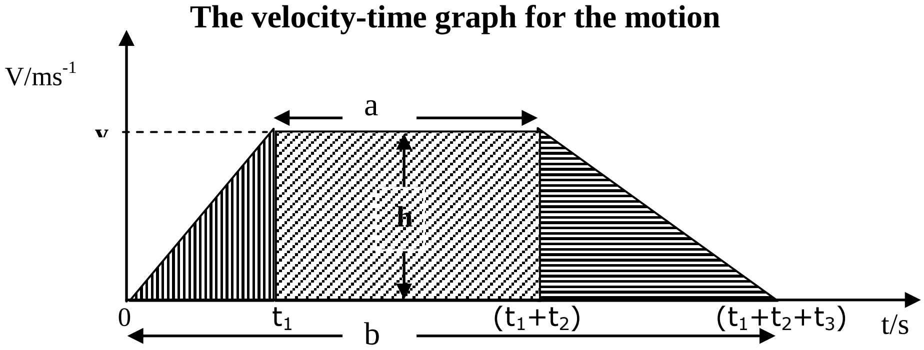
U - V	The body is moving with a constant velocity in the opposite direction during this period.	- The velocity is negative and constant. - The slope is zero. - The acceleration is zero.
V - W	At time t_6, the body brakes and decelerated uniformly until it comes to rest at time t_7.	- The velocity is negative and decreasing. - The slope is positive and constant. - The acceleration is positive and constant.

Worked Examples

1. A car starts from rest and accelerates uniformly for time t_1 seconds to attain a maximum velocity v ms^{-1}. It maintained this velocity for t_2 seconds, then the brakes were applied and the car uniformly retarded to a rest for t_3 seconds.
    - (i) Represent the above motion in a velocity-time graph.
    - (ii) On your graph show by shading the total distance travelled.

Solution: Initial velocity = 0 (from rest), Final velocity = v ms^{-1}

The velocity-time graph for the motion



Total distance travelled = Area under the graph
= Area of the trapezium
= $\frac{1}{2}(a + b)h$

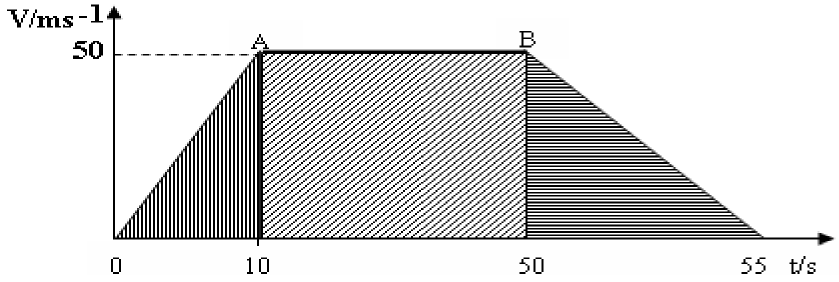
Or Total distance travelled = Area A + Area B + Area C
= $\frac{1}{2}bh + lw + \frac{1}{2}bh$

2. A car starts from rest and is accelerated uniformly at the rate of 5 ms^{-2} for time 10 seconds to attain a maximum velocity. It then maintained this velocity for 40 seconds. The brakes were then applied and the car uniformly retarded to rest in 5 seconds.
    Calculate: (i) the maximum velocity attained.
    (ii) Represent the motion on velocity-time graph.
    (iii) Find the total distance travelled in metres.

Solution **(i)** u = 0, v = ?, a = 5 ms^{-2}, t = 10 s

$v = u + at$
$= 0 + 5 \times 10$
$\therefore$ **v = 50 ms^{-1}**

(ii) The velocity-time graph for the motion



(iii) Method I $a = 50 - 10 = 40$, $b = 55$, $h = 50$

Total area = Area trapezium OABC
= ½(a + b)h
= ½(AB + OC) x AD
= ½(50 – 10 + 55) x 50
= ½(40 + 55) x 50
= ½ x 95 x 50
= 2375 m

∴ **Total distance travelled = 2375 m**

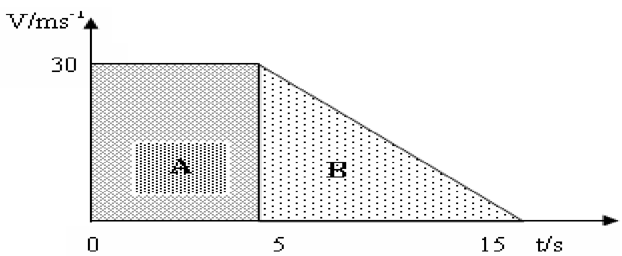
Method II Total distance travelled = Area A + Area B + Area C
= ½bh + lw + ½bh
= ½ x 10 x 50 + (50 – 10) x 50 + ½ x (55 – 50) x 50
= 250 + 40 x 50 + ½ x 5 x 50
= 250 + 2000 + 125
= 2375 m

∴**Total distance travelled = 2375 m**

3. A car travels with a uniform velocity of 30 ms^{-1} for 5 seconds. The brakes are then applied and the car comes to a rest with a uniform retardation in a further 10 seconds.
   (i) Draw a sketch graph of velocity-time graph.
   (ii) Show by shading the total distance travelled by the car.
   (iii) Find the distance travelled by the car after the brakes are applied.

Solution: $u = 30\ ms^{-1}$, $t_1 = 5$ s, $t_2 = 10$ s

(i) Velocity-time graph for the motion



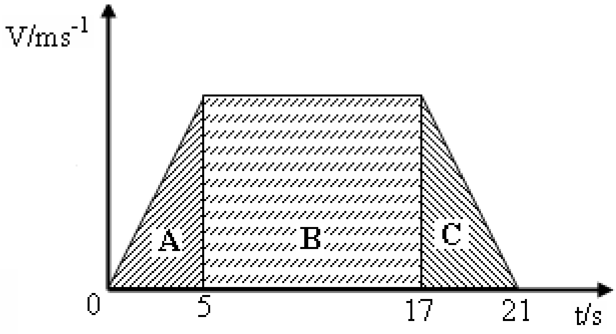
(ii) The distance travelled after the brakes were applied
= Area B
= ½bh
= ½(15 – 5) x 20
= ½ x10 x 30
= 150 m

4. A body starts from rest and accelerates at 5 ms^{-2}, for 5 seconds. Its velocity remained constant at the maximum value so reached for 12 seconds and it finally came to a rest with uniform retardation after another 4 seconds.
   Find by graphical method:
   (i) The distance travelled during each stage of the motion.
   (ii) The average velocity for the whole journey.

Solution $u = 0$, $v = ?$, $a = 5\ ms^{-2}$ $t_1 = 5\ s$, $t_2 = 12\ s$, $t_3 = 4\ s$

$v = u + at = 0 + 5 \times 5$ **$= 25\ ms^{-1}$**

Velocity-time graph for the motion



Dist. travelled at the first stage = Area A

$= \frac{1}{2}bh$

$= \frac{1}{2} \times 5 \times 25$

$= 62.5\ m$

$\therefore$ **Distance travelled** **= 62.5 m**

Distance travelled at the second stage = Area B

$= lw$

$= (17 - 5) \times 25$

$= 12 \times 25$

= 300 m

Distance travelled at the third stage = Area C

$= \frac{1}{2}bh$

$= \frac{1}{2} \times (21 - 17) \times 25$

$= \frac{1}{2} \times 4 \times 25$

$\therefore$ **s** **= 50 m**

(ii) Average velocity

$$= \frac{Total\ distance\ travelled}{Total\ time\ taken}$$

$$= \frac{Area\ A + Area\ B + Area\ C}{t_1 + t_2 + t_3}$$

$$= \frac{62.5 + 300 + 50}{5 + 12 + 4}$$

$$= \frac{412.5}{21}$$

$\therefore$ **Average velocity** **$= 19.64\ ms^{-1}$**

Self-Check 11.2

1. A body moves with uniform acceleration if
A. its momentum remains constant
B. it covers equal distances in equal times
C. the velocity changes by equal amount in equal times
D. the net force on the body is zero

2. A cyclist traveling at a constant acceleration of $2ms^{-2}$ passes through two points A and B in a straight line. If the speed at A is $10ms^{-1}$ and the points are 75m apart, find the speed at B.
A. $15.8ms^{-1}$ B. $17.3ms^{-1}$ C. $20.0ms^{-1}$ D. $400.0ms^{-1}$

3. A body moves with a uniform acceleration of Pms^{-2}. If its initial velocity is xms^{-1} and it travels for t s to attain a final velocity of $yms^{-1,}$ find the value of p in terms of x, y and t.
A. $x + yt$ B. $\frac{y-x}{t}$ C. $\frac{y+x}{t}$ D. $y + xt$

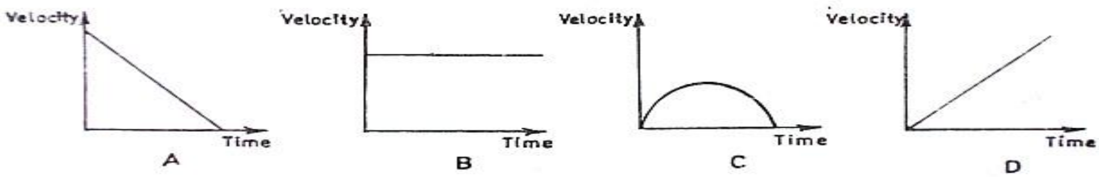
4. A body is said to be moving with a constant velocity if;
(i) Its momentum remains constant
(ii) It covers equal distances in equal times
(iii) The velocity changes by equal amount in equal times
(iv) The net force on the zero
A. All B. (i), (ii) and (iv) C. (i) and (ii) only D. (ii) only

5. A car of mass of mass 1200 kg moving with a constant velocity of 60 ms^{-1} is retarded uniformly to rest in 12 sec .Calculate the retarding force.
A. (1200 x 12) N B (1200 x 5) N C. (1200 x 10) N D. (1200 x 60) N

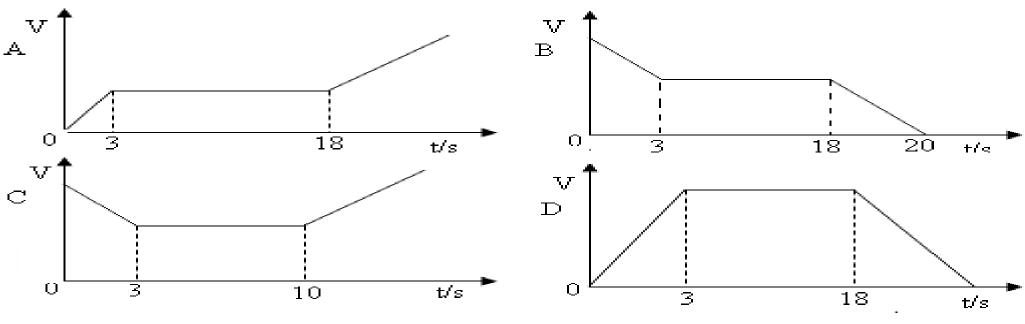
6. The gradient of a velocity-time graph represents the
A. Speed of the body B. Velocity of the body
C. Acceleration of the body D. The distance covered by the body

7. A body is said to be moving with uniform velocity when the rate of change of
A. acceleration with time is constant B. velocity with time is constant
C. distance with time is constant D. displacement with time is constant.

8. Which one of the above sketches represents uniformly accelerated motion?



9. A lift accelerates uniformly from rest for 3s. It then moves at uniform velocity for 15s then
decelerates uniformly for 2s before coming t rest. Which of the following velocity-time graph represents the motion of the lift.



10. Figure 4 shows a velocity-time graph for a moving body. Which one of the following statements is true about the motion of the body?

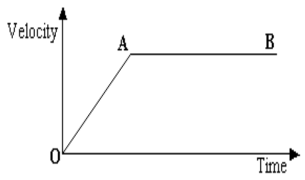


Figure 4

A. Velocity of the body is constant between O and A.
B. Velocity of the body is constant between A and B.
C. The body is accelerating between A and B.
D. The body is not accelerating between O and A.

11. The graph in figure 1 describes the motion of particle. Between which points is the particle at rest?

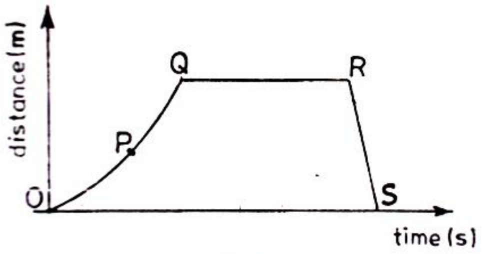


Fig.1

A. O and P.
B. P and Q.
C. Q and R.
D. R and S.

12. A car starts from rest and accelerates uniformly at the rate of $2ms^{-2}$ from 1/4 of a minute. Find the velocity of the car after this time.
A. $0.5ms^{-1}$ B. $12ms^{-1}$ C. $15ms^{-1}$ D. $30ms^{-1}$

13. The in the figure shows a speed-time graph for a body.

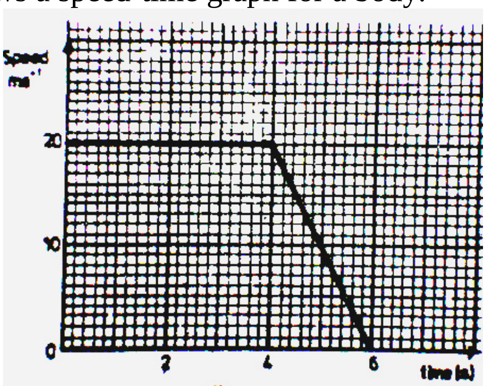
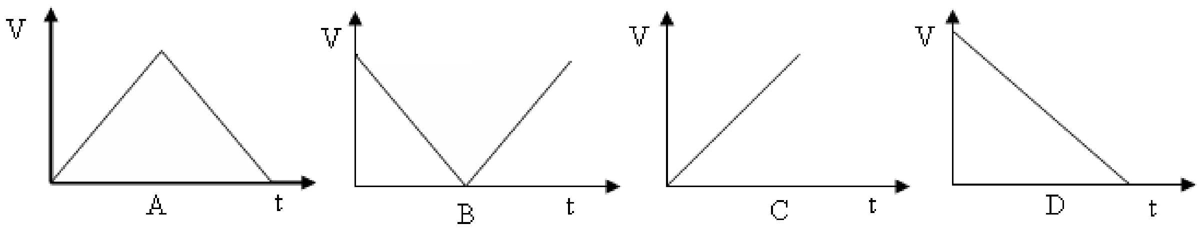


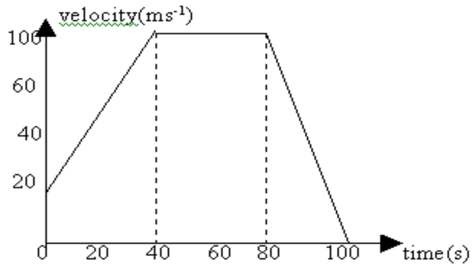
Fig. 1

Calculate the distance traveled during retardation
A. 20m B. 40m C. 80m D. 100m

14. A boy throws a ball in the air and it goes up and falls back to his hand. Which one of the following sketches of velocity-time graph represents the motion of the ball up to the time it is received back?

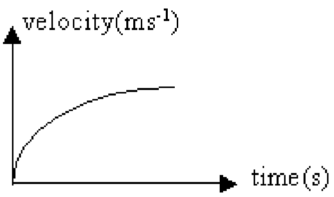


15. The velocity-time graph for a car is as shown in fig. Find the total distance the car travels.



A. 2.08×10^3m
B. 3.0×10^3m
C. 4.0×10^3m
D. 7.0×10^3m

16. Which of the following best describes the motion represented by the velocity-time graph shown in the diagram?



A. Decelerated
B. Uniformly accelerated motion
C. Non-uniformly accelerated motion
D. Uniform velocity motion

17. A car is uniformly accelerated from rest and after 10s acquires a speed of $20ms^{-1}$. How far does it move during the eleventh second?

A. 20m B. 21m C. 100m D. 121m

18. Which one of the following is not true about a body moving with a constant velocity?

A. its acceleration is zero
B. Its momentum is constant
C. Its kinetic energy is constant
D. There is a resultant force on it

19. A car travelling at $20ms^{-1}$ is brought to rest in 10s. Find the distance it travels

A. 100m B. 200m C. 300m D. 400m

20. Use the velocity-time graph in the figure to find the distance over which there is deceleration.

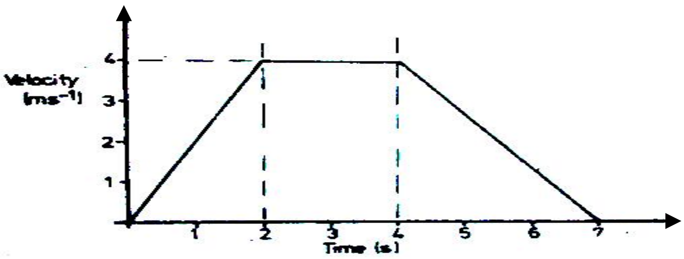


Fig. 1

A. 4m
B. 6m
C. 8m
D. 10m

SECTION B

21. **(a)** What is meant by the following:

(i) uniform velocity (ii) uniformly accelerated motion ?

(b) A body starts from rest and reaches a speed of 5m/s after travelling with uniform acceleration in a straight line for 2 seconds. Calculate the acceleration of the body.

22. A body starts from rest and moves with uniform acceleration of 2m/s in a straight line.

(a) What is its velocity after 5 seconds?
(b) How far has it travelled in this time?
(c) How long will he body be after the starting point?

23. (b) The table below shows the variation of velocity with time for a body, which has been thrown vertically up wards from the surface of a planet.

Velocity(ms^{-1})	8	6	4	2	0	-2
Time	0	1	2	3	4	5

(i) What does the negative velocity mean?
(ii) Plot a graph of velocity against time.
(iii) Use the graph in b (ii) to find the acceleration due to gravity on the planet.
(iv) Use the graph in b (ii) to find the total distance traveled.

24. (a) What is meant by acceleration?
(b)

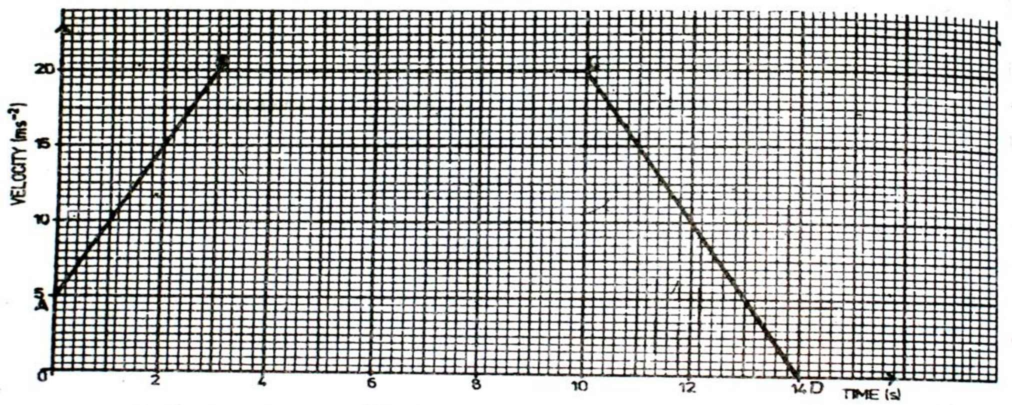


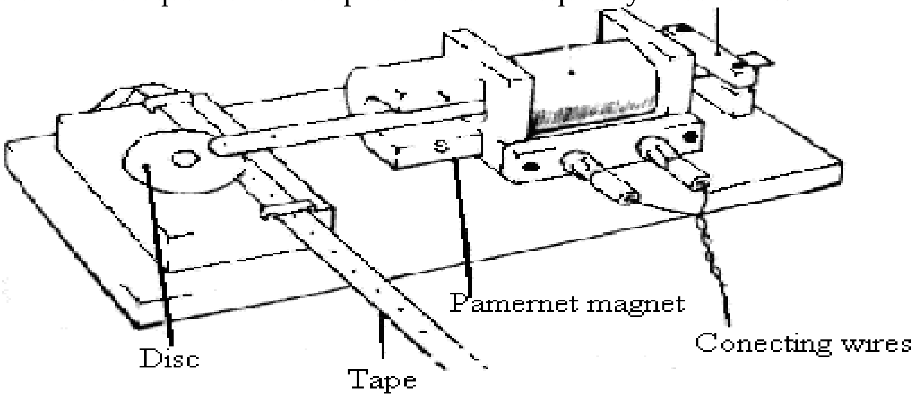
Fig.1

The above figure shows a speed-time graph of a cyclist.
(i) Find the acceleration of the cyclist between A and B.
(ii) Describe the motion of the cyclist between B and C
(iii) Explain what is happening along CD.
(iv) Calculate the distance travelled by the cyclist during the first 10 seconds

26. (a) What is the difference between speed and velocity?
(b) The graph in the figure shows the variation of distance with time for a body. Describe the motion of the body.

11.4 Ticker-Tape Timer (ticker-tape vibrator)

This is a device used to investigate speed, velocity and acceleration of a moving body. It consists of a steel strip which vibrates at mains frequency, *f*, making dots on paper tape which is pulled by the body. The time interval between any two successive dots is called a *tick* and is equal to the reciprocal of the frequency in seconds.



Calculations

To calculate speed, velocity or acceleration, the following steps are followed.

First determine: (i) The distance between the reference points (dots) on the ticker tape

(ii) The tick using the formula

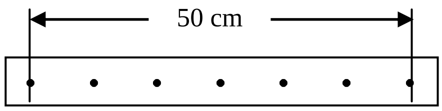
$$One\ tick = \frac{1}{Frequency}\ \text{second.}$$

(iii) The time taken to make the dots from the formula

$$Total\ time\ taken = One\ tick\ \text{x} \begin{pmatrix} No.\ of\ spaces\ between \\ reference\ points \end{pmatrix}$$

(a) To calculate velocity

1. The diagram below shows equally spaced dots on a ticker tape.



If the vibrator of the ticker time has a frequency of 50 Hz, find the average speed of the trolley in m/s.

Solution: Frequency = 50Hz, One Tick = ?

One tick $= \frac{1}{f} = \frac{1}{50} = 0.02$ s

Total distance covered = 50 cm $= \frac{50}{100}$ m = ***0.5 m***

$$Total\ time\ taken = One\ tick\ \text{x} \begin{pmatrix} No.\ of\ spaces\ between \\ reference\ points \end{pmatrix}$$

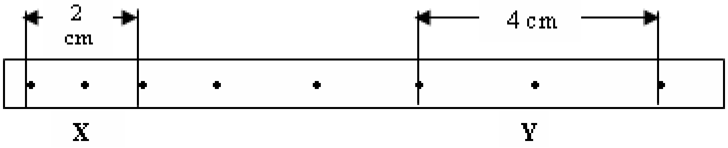
$= 0.02 \text{ x } 6$

t = ***0.12 s***

Using Speed $= \frac{\text{Distance}}{\text{Time}} = \frac{0.5}{0.12} = 4.166$ $\mathbf{= 4.2\ ms^{-1}}$

(b) To calculate the acceleration

1. The tape shown in the diagram below was made by a trolley moving with a constant acceleration. If the frequency of the ticker-timer is 100 Hz, find the acceleration in ms^{-2}.



Solution: F = 100 Hz One tick $= \frac{1}{f} = \frac{1}{100} = 0.01$ s

Initial Velocity, u, at X: Distance, s = 2 cm = $\frac{2}{100}$ = 0.02 m

Time taken = *One tick* x *No. of spaces between the dots*
= 0.01 x 2
= 0.02 s

Initial velocity, $u = \frac{s}{t} = \frac{0.02}{0.02} = 1\ ms^{-1}$

Final Velocity, v, at Y: Distance, s = 4 cm = $\frac{4}{100}$ = 0.04 m

Time taken = *One tick* x *No. of spaces between the dots*
= 0.01 x 2
= 0.02 s

Initial velocity, $u = \frac{s}{t} = \frac{0.04}{0.02} = 2\ ms^{-1}$

Time taken from X to Y is calculated from the formula

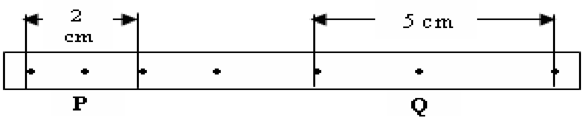
Time taken = *One tick* x *No. of spaces between* XY.
= 0.01 x 5
t = 0.05 s

The acceleration is calculated from the formula:

$$Acceleration = \frac{Velocity\ at\ Y - Velocity\ at\ X}{Time\ taken}$$

$$\boldsymbol{a} = \frac{v-u}{t} = \frac{4-2}{0.05} = \frac{2}{0.05} = \boldsymbol{40\ ms^{-2}}$$

2. Calculate the acceleration, in ms^{-2}, for the motion shown in the diagram below. (Take frequency = 50Hz)



Solution: *F = 50 Hz* *One tick* $= \frac{1}{f} = \frac{1}{50}$ *= 0.02 s*

Initial Velocity, u, at P: *Distance, s = 2 cm =* $\frac{2}{100}$ *= 0.02 m*

Time taken = *One tick* x *No. of spaces between the dots*
= 0.02 x *2*
t = 0.04 s

Initial velocity, $u = \frac{s}{t} = \frac{0.02}{0.04} = 0.5\ ms^{-1}$

Final Velocity, v, at Q: *Distance,* $s = 5\ cm = \frac{5}{100} = 0.05$ m

Time taken, t = *One tick* x *No. of spaces between the dots*
= 0.02 x 2
= 0.04 s

Time taken from P to Q is calculated from the formula

Time taken, ***t*** = *One tick x No. of spaces between P and Q*
= 0.02 s x 4
t = 0.08 s

Initial velocity, v $= \frac{s}{t} = \frac{0.05}{0.04} = 1.25\ ms^{-1}$

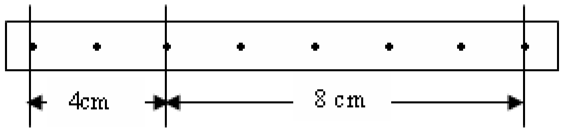
The acceleration is calculated from the formula

$$Acceleration = \frac{Velocity\,at\,P - Velocity\,at\,Q}{Time\,taken}$$

$$\boldsymbol{a} = \frac{v - u}{t} = \frac{1.25 - 0.5}{0.08} = \frac{0.75}{0.08} = \mathbf{9.375\ ms^{-2}}$$

(c) To Describe Motion

The figure below shows dots made on a ticker tape pulled by a trolley through a ticker timer.



Describe the motion of the trolley if the frequency is 50Hz?

To describe motion from a sample of ticker tape, you find the speeds at the two stages of the motion and their respective time taken. Then relate the speeds to the time taken for the two stages as shown below.

Speed at the first stage

Frequency, f = 50 Hz, Tick $= \frac{1}{f} = \frac{1}{50} = 0.02$ s, s = 4 cm,

Time taken = One tick x No. of spaces
= 0.02 x 2
= 0.04 s

Speed $= \frac{s}{t} = \frac{4}{0.04} = 100\ cms^{-1}$

Speed at the second stage

s = 8 cm, time taken = One tick x No. of spaces
= 0.02 x 5
= 0.1 s

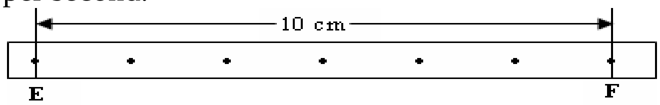
Speed $= \frac{s}{t} = \frac{8}{0.1} = 80\ cms^{-1}$

Description of the motion

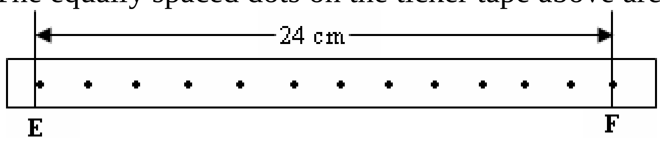
In the first 0.04 s, the body was traveling with a speed of 100 cms^{-1}. The body then decelerated and the speed reduced to 80 cms^{-1} in the next 0.1 s.

Self-Check 11.3

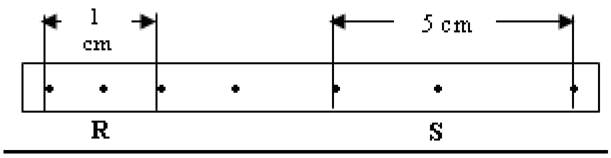
1. A ticker timer is connected to the mains supply of frequency 40Hz. Find the time it takes to print three consecutive dots.
   A. 0.08 s   B. 0.25 s   **C**. 0.050 s   D. 0.75 s
2. The ticker tape shown in the figure was pulled through a ticker timer, which makes 50 dots per second.



   The speed at which the tape was pulled is
   A. 10cm s^{-1} B. 25cm s^{-1} C. 50cm s^{-1} D. 100cm s^{-1}
3. A tape is pulled through a ticker-timer, which has a frequency of 50Hz. If the distance
   between successive dots is 2cm, calculate the speed of the body
   A. 0.01cms^{-1} B. 50cms^{-1} C. 100cms^{-1} D. 250cms^{-1}
4. The equally spaced dots on the ticker tape above are made by a vibrator of

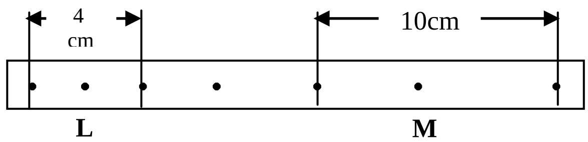


   frequency 50 Hz. The speed of the tape in m/s is
   A. 0.1   B. 0.01   C. 0.001   D. 1
5. The tape shown in the diagram below was made by a trolley moving with a constant
   acceleration. If the frequency of the ticker-timer is 50 Hz, find the acceleration in m/s^2.



   A. 75 ms^{-2} B. 2 ms^{-2} C. 5 ms^{-2} D. 12.5 ms^{-2}

6. Calculate the acceleration, in ms^{-2}, for the motion shown in the diagram below.
   (Take frequency = 100Hz)



   A. 75 ms^{-2} B. 2 ms^{-2} C. 5 ms^{-2} D. 0.04 ms^{-2}

11.5 Motion under Gravity

Types of Motion

There are different forms of motion under gravity, depending how the motion is started.

(i) Free-falling objects - If an object is released from rest (i.e. when the initial velocity is zero) and allowed to fall.

(ii) Projectile motion - If an object is thrown (projected) at an angle with an initial velocity greater than zero.

(a) Free-falling objects

Objects released from rest at a height above the surface of the earth fall downwards. This is because of force of gravity pulling them downwards.

An object thrown vertically upwards with an initial velocity greater than zero moves with a negative acceleration since the motion is away from the earth surface. The velocity reduces until it becomes zero at the maximum height reached. At this point the object is momentarily at rest and then begins to accelerate downwards with a positive acceleration. The velocity increases until it hits the ground. This acceleration arises from the *force of gravity* acting on the body, and is called *acceleration due to gravity* (symbol *g).*

If air resistance or friction is neglected, all objects near to the surface of the earth, (regardless of their mass) accelerate at the same rate called *acceleration due to gravity*. The acceleration due to gravity (symbol *g)* is about 9.81 ms^{-2}. But the value varies slightly from place to place. See chapter five. For simple calculations g is usually rounded up to 10 ms^{-2}. This means that objects falling freely under gravity increase their velocity by about ms^{-1} every second.

Calculations for free-falling objects

The equations of motion, apply to objects moving vertically downwards or upwards under the force of gravity, except that the acceleration a is replaced by the acceleration due to gravity, g.

Equations for Linear Motion	**Equations for Motion under gravity**
$v = u + at$	$v = u + gt$
$s = ut + \frac{1}{2}at^2$	$s = ut + \frac{1}{2}gt^2$
$v^2 = u^2 + 2as$	$v^2 = u^2 + 2gs$

Distance moved by a free-falling body

Distance moved by a body falling freely from rest is related to time of fall by the formula: $s = \frac{1}{2}g\,t^2$

The formula is derived from the third equation of motion

$$s = ut + \frac{1}{2}at^2$$

Where u = 0 (i.e. from rest)
a = g acceleration due to gravity

NB: *An object moving vertically upwards has a negative acceleration or retardation. Therefore the acceleration due to gravity, $g = {}^{-}10\ ms^{-2}$ while for an body moving downwards, the acceleration due to gravity is positive, $g = 10\ ms^{-2}$*

Worked Examples

1. A coconut fruit falls from the coconut tree and takes 5 seconds to hit the ground. Calculate: (a) its velocity.
(b) the total distance traveled. (Take $g = 10 m/s^2$)

Solution (a) $g = 10 m/s^2$, $u = 0\ ms^{-1}$, $t = 5$ s

$$v = u + at$$
$$= 0 + 10 \times 5$$
$$\therefore \mathbf{v = 50\ ms^{-1}}$$

(b) Using the formula $s = ut + ½at^2$

$$= (0 \times 5) + ½ \times 10 \times 5^2$$
$$= 0 + 5 \times 25$$
$$\therefore \mathbf{s = 125\ m}$$

2. A stone is dropped from a top of a cliff. If it takes 8 seconds to hit the ground, calculate: (a) the height of the cliff.
(b) the velocity at impact. (Take $g = 10\ ms^{-2}$)

Solution (a) Using the formula $g = 10m/s^2$, $u = 0\ ms^{-1}$, $t = 4$ s

$$s = ut + ½gt^2$$
$$= (0 \times 4) + \frac{1}{2} \times 10 \times 4^2$$
$$= 0 + 5 \times 16$$
$$\therefore \mathbf{s = 80\ m}$$

(b)
$$v = u + at$$
$$= 0 + 10 \times 4$$
$$\therefore \mathbf{v = 40\ ms^{-1}}$$

3. A stone is thrown vertically upwards with an initial velocity of $40\ ms^{-1}$. Calculate: (a) the maximum height reached.
(b) the time taken to reach the maximum height.
(Take $g = 10\ ms^{-2}$ and neglect air resistance)

Solution

$u = 40\ ms^{-1}$, $v = 0\ ms^{-1}$(*At maximum height the stone is momentarily at rest),*
$a = g = -10\ ms^{-2}$, $s = ?$

(a)
$$v^2 = u^2 + 2as$$
$$v^2 = u^2 + 2gs$$
$$0^2 = 40^2 + 2 \times -10 \times s$$
$$0 = 1600 – 20s$$
$$20s = 1600$$
$$\mathbf{s = 80\ m}$$

(b) $u = 40\ ms^{-1}$, $v = 0\ ms^{-1}$, $a = g = -10\ ms^{-2}$, $s = ?$

$$v = u + at$$
$$v = u + gt$$
$$0 = 40 + -10 \times t$$
$$0 = 40 – 10t$$
$$10t = 40$$
$$\therefore \mathbf{t = 4\ s}$$

4. A ball is thrown vertically upwards with an initial velocity of 20 ms^{-1}. Neglecting air resistance, calculate:

(a) the maximum height reached,

(b) the time taken before it reaches the ground. (Take g = 10 ms^{-2})

Solution u = 20 ms^{-1}, v = 0 ms^{-1}*(At maximum height the stone is momentarily at rest),* a = g = -10 ms^{-2}, s = ?

(a)

$$V^2 = u^2 + 2as$$
$$V^2 = u^2 + 2gs$$
$$0^2 = 20^2 + 2 \text{ x } -10 \text{ x } s$$
$$0 = 400 - 20s$$
$$20s = 400$$
$$\therefore \mathbf{s} = \mathbf{20\ m}$$

(b) u = 20 ms^{-1}, v = 0 ms^{-1}, a = g = -10 ms^{-2}, t_1 = ?

$$v = u + at$$
$$v = u + gt$$
$$0 = 20 + -10 \text{ x } t_1$$
$$0 = 20 - 10t_1$$
$$10t_1 = 20$$
$$\mathbf{t_1} = \mathbf{2\ s}$$

Time taken in the return journey, t_2 = ? u = 0 ms^{-1}, v = 0 ms^{-1}, a = g = 10 ms^{-2}, s = 20 m

Using the formula

$$s = ut + ½at^2$$
$$20 = (0 \text{ x } t_2) + ½ \text{ x } 10 \text{ x } t_2^2$$
$$20 = 5t_2^2$$
$$\sqrt{4} = \sqrt{t_2^2}$$
$$\mathbf{t_2} = \mathbf{2\ s}$$

Time taken before it reaches the ground = $t_1 + t_2$ = 2 + 2 **= 4 s**

NB*:* *Note that the time taken to reach the maximum height is equal to the time taken to return to the ground.*

Further Examples

1. A stone is thrown vertically upwards with an initial velocity of 30 ms^{-1} from a tower 20 m high. Neglecting air resistance, calculate:

(a) The time taken to reach the maximum height.

(b) The maximum height reached.

(c) The total time taken which elapses before it just hits the ground (Take g = 10 ms^{-2})

Solution

(a) u = 30 ms^{-1}, v = 0 ms^{-1} *(At the maximum height the stone is momentarily at rest),* a = g = -10 ms^{-2}, t = ? s = ?

$$v = u + gt$$
$$0 = 30 + -10 \text{ x } t$$
$$0 = 30 - 10t$$
$$10t = 30$$
$$\mathbf{t_1} = \underline{\underline{\mathbf{3\ s}}}$$

(b) $v^2 = u^2 + 2as$

$$v^2 = u^2 + 2gs$$

$$0^2 = 30^2 + 2 \times -10 \times s$$

$$0 = 900 - 20s$$

$$20s = 900$$

$$\therefore \mathbf{s = 45\ m}$$

(c) Total height = Height of cliff + Maximum height reached

$= 20 + 45$

$\therefore$ Total height $\mathbf{= 65\ m}$

Time to fall from the maximum height to the ground =?, g = 10 ms^{-2}

Using the formula: $s = ut + ½at^2$

$$65 = (0 \times t) + ½ \times 10 \times t^2$$

$$65 = 5t_2^{\ 2}$$

$$\sqrt{13} = \sqrt{t^2}$$

$$\mathbf{t = 3.60\ s}$$

$$\text{Total time taken} = \begin{pmatrix}\text{Time taken from the top}\\ \text{of the cliff to reach the}\\ \text{the maximum height}\end{pmatrix} + \begin{pmatrix}\text{Time taken from the top}\\ \text{of the maximum height}\\ \text{to reach the ground}\end{pmatrix}$$

$= 3 + 3.60$

$\therefore$ Total time taken $=$ **6.60 s**

2. A small marble chip of mass 5 g is dropped from the top of a cliff and takes 2.5 s to reach the sandy beach below it. Calculate:

(a) (i) the velocity with which it strikes the sand.

(ii) the kinetic energy with which it hits the sand.

(b) The height of the cliff.

(c) If the marble chip penetrates the sand to a depth of 12.5 cm, calculate:

(i) its average retardation.

(ii) the retarding force. (Take g = 10 ms^{-2})

Solution: **(a)** $u = 0\ ms^{-1}$, $v = ?$, $a = g = 10\ ms^{-2}$, $t = 2.5$ s,

(i) $v = u + at$

$$v = u + gt$$

$$= 0 + 10 \times 2.5$$

$$= 0 + 25$$

$$\mathbf{= 25\ ms^{-1}}$$

(ii) $v = 25\ ms^{-1}$, $m = 5\ g = \frac{5}{1000}$ kg, K.E = ?

$$K.E = \tfrac{1}{2}mv^2$$
$$= \frac{1}{2} \times \frac{5}{1000} \times 25^2$$
$$= \frac{5 \times 625}{2000}$$
$$= \frac{3125}{2000}$$
$$\mathbf{K.E = 1.56\ J}$$

(b) $g = 10 m/s^2$, $u = 0\ ms^{-1}$, $t = 2.5$ s, s = ?, $v = 25\ ms^{-1}$

$$s = ut + \tfrac{1}{2}gt^2$$
$$= (0 \times 2.5) + \tfrac{1}{2} \times 10 \times 2.5^2$$
$$= 0 + 5 \times 2.5 \times 2.5$$
$$\mathbf{= 31.25\ m}$$

(c) **(i)** Depth of penetration, s = 12.5 cm $\frac{12.5}{100}$ m, a = ?

u = initial velocity with which the marble starts to penetrate the sand
= velocity before it just hits the sand
= $25\ ms^{-1}$

v = 0 (after penetration the marble was brought to rest)

Using the formula $v^2 = u^2 + 2as$

$$v^2 = u^2 + 2gs$$
$$a = \frac{v^2 - u^2}{2s}$$
$$= \frac{0^2 - 25^2}{2 \times 0.125}$$
$$= \frac{625}{0.25} \mathbf{= 2500\ ms^{-2}}$$

$m = \frac{5}{1000}$ kg, $a = 2500\ ms^{-2}$, F = ?

$$F = ma$$
$$= \frac{5}{1000} \times 2500$$
$$= \frac{5000}{1000}$$
$$\mathbf{= 5\ N}$$

Experiment 5.31

To Determine Acceleration due to Gravity, *g*, in a place

Apparatus/Requirements

2 G-clamps, a retort stand, 2 m ticker tape, a ticker timer, a standard mass of 100 g, a soft material.

Procedure

- Clamp a ticker timer about 2 m or more at the edge of a table so that there is a clear drop below it as shown in figure 5.1 below.

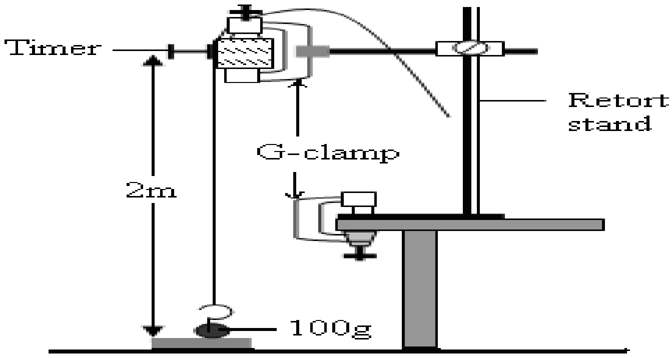


Figure 5.1

- Pass a 2 m ticker tape through the paper guides and under the carbon disc.
- Place a soft material on the floor for a 100 g mass to fall on.
- Attach one end of the tape to 100 g.
- Switch on the ticker timer and allow the 100 g mass to drop to the ground.
- Construct a tap chart using two-tick strips of the tape.

The chart may appear similar to the one shown in figure 5.2 below.

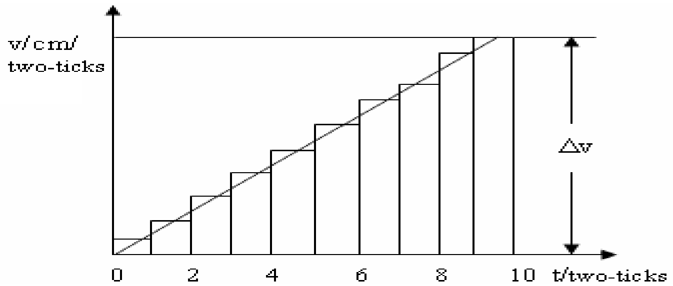


Figure 5.2

Note that: *The steps showing increasing velocity may vary slightly and become rather shorter at higher velocities. This is due to the effect of friction, particularly the drag of the tape through the ticker timer, which increases with speed.*

To overcome these irregularities, draw a straight-line graph which passes through the centre of the tops of the tapes taking more notice of those which best fit a straight line through the origin.

Measure Δv and Δt from your tape or tape chart.

Since initial velocity, $u = 0$, Δv = Final velocity, v and Δt = time taken.

Calculate the acceleration due to gravity from the formula:

$$\text{Acceleration, g} = \frac{\textit{Final Velocity}}{\textit{Time taken}} = \frac{\Delta v}{\Delta t}\ \text{ms}^{-2}$$

How to calculate the value of g from given data.
(*The steps may be understood full after covering ticker timer in Chapter 11).*

Let: Frequency of vibrator of the ticker timer = 50 Hz
Number of two-tick tapes used to construct the chart = 10 pieces

The main steps in the calculations
Step I*: *Calculate the time interval for the two-tick tape

Time interval of a two-tick = Number of spaces x Tick

$$= 2 \times \frac{1}{f}$$

$$= 2 \times \frac{1}{50}$$

$$= \frac{2}{50}$$

= 0.04 s

***Step II*:** *Using the time interval for a two-tick tape and the number of pieces of tapes used calculate the time taken.*

$$\text{Time taken, } t = \begin{pmatrix} \textit{Number of spaces} \\ \textit{of tapes used} \end{pmatrix} \times \begin{pmatrix} \textit{Time interval for} \\ \textit{two-tick tape} \end{pmatrix}$$

$$= 10 \times 0.04$$

$\therefore$ **t = 0.4 s**

Step III: *Calculate the velocity, from the ticker tape.*

Data: Final velocity, v = Δv = 14 cm in each 0.04s
s = 14 cm = 0.14 m, t = 0.04 s, v = ?

Using the formula $v = \frac{s}{t}$

$$= \frac{0.14}{0.04}$$

= 3.5 ms^{-1}

Step IV: *Calculate the acceleration due to gravity, g.*

Acceleration, g $= \dfrac{\textit{Final velocity}}{\textit{Time taken}} = \dfrac{3.5}{0.4}$ ms^{-1}/s **= 8.75 ms^{-2}**

Note: *In the absence of all resisting forces such as the drag of the tape and air resistance, the acceleration of freely falling object at the surface of the earth is about 9.8 ms^{-2}.*

Self-Check 11.4

1. A stone is dropped a deep hole. If it takes 10 seconds to hit the bottom of the hole.
Calculate: (a) its velocity.
(b) the depth of the hole. (Take $g = 10m/s^2$)

2. A stone is dropped from a top of a cliff. If it takes 6 seconds to hit the ground, calculate: (a) the height of the cliff.
(b) the velocity at impact. (Take $g = 10\ ms^{-2}$)

3. A tennis ball is thrown vertically upwards with an initial velocity of $40\ ms^{-1}$.
Calculate: (a) the maximum height reached.
(b) the time taken to reach the maximum height.
(Take $g = 10\ ms^{-2}$ and neglect air resistance)

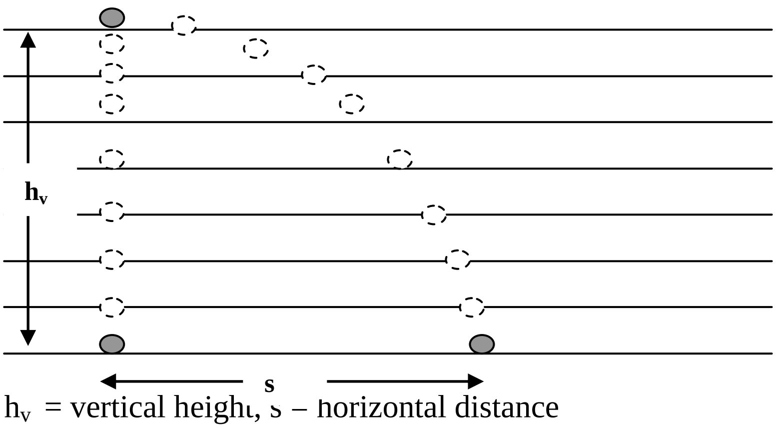
4. A bullet is fired vertically upwards with an initial velocity of $300\ ms^{-1}$. Neglecting air resistance, calculate:
(a) the maximum height reached,
(b) the time taken before it reaches the ground. (Take $g = 10\ ms^{-2}$)

5. A stone is thrown vertically upwards with an initial velocity of $20\ ms^{-1}$ from a tower 80 m high. Neglecting air resistance, calculate:
(a) the maximum height reached,
(b) the time taken to reach the maximum height,
(c) the total time which elapses before it just hits the ground
(Take $g = 10\ ms^{-2}$)

11.6 Projectile Motion

A projectile is any body projected through space. When two tennis balls placed at the same height are let to fall under gravity, one dropped vertically downwards and the other projected horizontally, it is noticed that all the balls hit the ground at the same time.

A diagram showing the motion of two tennis balls moving downwards from a table top (one dropped vertically downwards and the other projected horizontally)



h_v = vertical height, s = horizontal distance

Facts about the motion

(i) The initial velocity of the ball falling vertically is zero.
(ii) The vertical accelerations (due to gravity) of the two balls are equal.
(iii) A projectile falls like a body is dropped from rest.
(iv) The two balls take the same time to reach the ground.
(v) The horizontal velocity of the projectile is independent of the vertical motion (i.e. does not affect the vertical motion).

The path of a projectile is called *trajectory*. It is (almost) the arc of a parabola from the following reasoning.

Consider a projectile with an initial velocity, u vertical velocity zero, initial horizontal acceleration zero and vertical acceleration downwards, g. After time, *t*, the horizontal distance, *s,* and vertical distance, *h*, traveled by the projectile are obtained by using the equation: $s = ut + ½gt^2$

For horizontal motion, g = 0 $\therefore$ $s = ut$ ………………………….. 1

For vertical motion, = 0 $y = ½gt^2$ 2

Since $x \propto t$ and $y \propto t^2$, *then:*

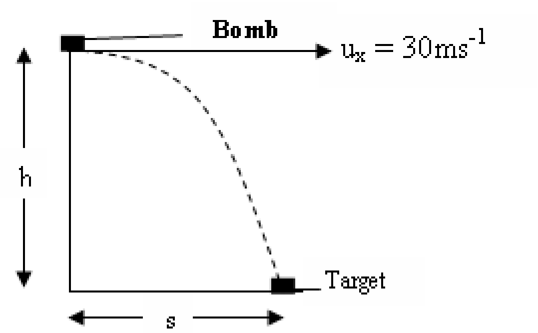
$y \propto x^2$

When this is plotted on a graph it is a parabola. That is the same as that of the graph in diagram for projectile motion but is inverted because the projectile falls and does not rise.

Worked Examples

1. A ball is hit horizontally off a cliff with a velocity of 30 ms^{-1}. If it takes 5 seconds to reach the ground, calculate:
   (a) The height of the cliff.
   (b) The horizontal distance from the foot of the cliff to the point where the ball lands.
   (c) The velocity with which the ball hits the ground.

Solution: $u_y = 0$ $u_x = 30\ ms^{-1}$



(a) To calculate the height of the cliff, we consider the vertical motion.
$u_y = 0$, $g = 10\ ms^{-2}$, $t = 5$ s, $s = h = ?$
Using the formula $s = u_yt + ½gt^2$
$= (0x5) + ½ \times 10 \times 5^2$
$= 0 + 5 \times 25$
s = h = 125 m

(b) The horizontal distance, s, is got by considering the horizontal motion.
$u_x = 30$, $g = 0\ ms^{-2}$, $t = 5$ s, $s = ?$
Using the formula $s = u_xt + ½gt^2 = 30 \times 5 + ½ \times 0 \times 5^2 = 150 + 0$ **= 150 m**
Therefore, the ball lands 150 m from the foot of the cliff.

(c) The velocity with which the ball hits the ground is got by considering the vertical motion.
This is because the horizontal velocity of the ball remains unchanged throughout the motion.

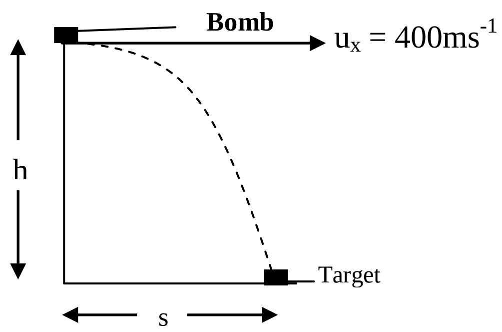
Vertical initial velocity, $u_y = 0$, Vertical final velocity, $v_y = ?$,
$g = 10\ ms^{-2}$, $t = 5\ s$,

Using the formula: $v = u + at$

$$\begin{aligned} v_y &= u_y + at \\ &= 0 + 10 \times 5 \\ &= \mathbf{50\ ms^{-1}} \end{aligned}$$

2. A bomb is released from a jet fighter plane moving with velocity of 400 ms^{-1} to hit a rebel camp in northern Uganda. If the bomb took 10 seconds to hit the target, calculate:
   (a) The altitude at which the bomb was released.
   (b) The horizontal distance from the vertical point of the plane to the target.
   (c) The velocity with which the bomb hits the target. (Take $g = 10\ ms^{-2}$)

Solution $u_y = 0\ ms^{-1}$, $g = 10\ ms^{-2}$, $t = 10\ s$, $s = h = ?$



(a) To calculate the altitude of the plane, we consider the vertical motion.

Using the formula: $s = u_y t + \frac{1}{2} g t^2$

$$\begin{aligned} &= (0 \times 5) + \frac{1}{2} \times 10 \times 10^2 \\ &= 0 + 5 \times 100 \\ s = h &= \mathbf{500\ m} \end{aligned}$$

(b) The horizontal distance, s, is got by considering the horizontal motion.
$u_x = 400$, $g = 0\ ms^{-2}$, $t = 10\ s$, $s = ?$

$$\begin{aligned} s &= u_x t + \tfrac{1}{2} g t^2 \\ &= 400 \times 10 + \tfrac{1}{2} \times 0 \times 10^2 \\ &= 4000 + 0 \\ \mathbf{s} &= \mathbf{4000\ m} \end{aligned}$$

(c) The horizontal velocity of the bomb remains unchanged throughout the motion (400 ms^{-1}). The vertical velocity increases as the bomb falls.
The velocity with which the bomb hits the ground is got by considering the vertical motion.

$u_y = 0\ ms^{-1}$, $g = 10\ ms^{-2}$, $t = 10\ s$,

$$\begin{aligned} v &= u + at \\ &= u_y + gt \\ &= 0 + 10 \times 10 \\ &= \mathbf{100\ ms^{-1}} \end{aligned}$$

Self-Check 11.5

1. Which one of the following statements is true when a stone of mass 2 kg and that of 1 kg are released from the same point at the same time?
A. both masses will hit the ground at the same time.
B. the 2kg mass will hit the ground first.
C. the 1 kg mass will hit the ground first.
D. they fall with different speeds.

2. A mass is projected upwards with a velocity of 10 m/s. If the acceleration due to gravity is 10 m/s ^{2}, what is the maximum height reached in meters?
A. 1   B. 5   C. 10   D. 20   E. 100

3. A stone is hit horizontally off a cliff with a velocity of 50 ms^{-1}. If it takes 10 seconds to reach the ground, calculate:
(a) The height of the cliff.
(b) The horizontal distance from the foot of the cliff to the point where the ball lands.
(c) The velocity with which the ball hits the ground.

4. An Israel jet fighter plane moving with velocity of 300 ms^{-1} released a bomb to hit a Hezbollah base in Southern Lebanon. If the bomb took 20 seconds to hit the target, calculate:
(a) The altitude at which the bomb was released.
(b) The horizontal distance from the vertical point of the plane.
(c) The velocity with which the bomb hits the target. (Take g = 10 ms^{-2})

5. A stone falls from rest from the top of a high tower. Ignoring air resistance and taking g =10m/s^2, what is the velocity and distance travelled after:
(i) 1 s,
(ii) 2s,
(iii) 3s
(iv) 5s?

11.7 Circular Motion

One of the effects of force is that it keeps bodies moving in a circular path about a fixed point. This kind of motion is described as *Circular Motion*.

Examples of circular motion include:
- a stone/pendulum bob whirled at the end of a string in a vertical or horizontal plane,
- a vehicle negotiating a corner,
- an electron orbiting a nucleus,
- a planet or satellite orbiting the earth.

Facts about a body moving in a circle:

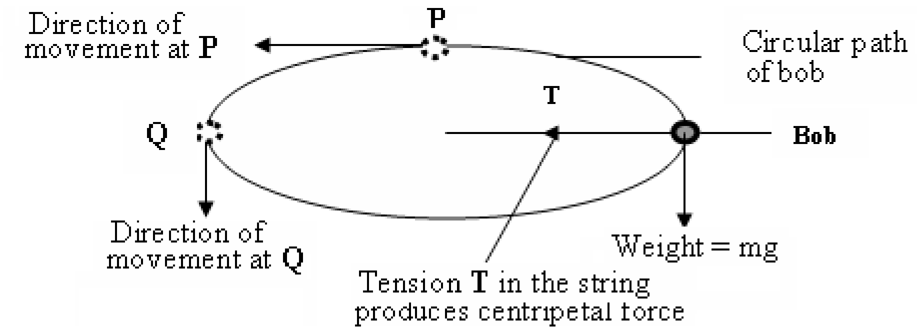
(i) Its direction changes every second.
(ii) Its velocity is constantly changing due to the change in direction of the motion.
(iii) It has an acceleration called *centripetal acceleration* due to the constant change in velocity.
(iv) It has a force called *Centripetal force*, acting on it producing the acceleration.
(v) The centripetal force is perpendicular to the direction of the motion.
(vi) Both the *centripetal force* and the *centripetal acceleration* are acting towards the centre of the circle.

Experimental results show that the centripetal force required to keep a body in a circular path increases with:

(i) an increase in the mass of the body.
(ii) an increase in the speed of the body and
(iii) a decrease in the radius of the circular path.

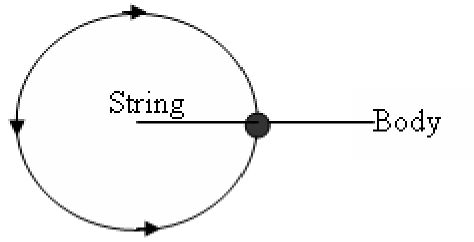
When the object is released, it moves in a straight line. The direction of the motion is along the tangent to the circle as shown in the diagram below.

A diagram showing a bob tied at the end of a string being whirled in a horizontal circle



Self-Check 11.6

1. **(a)** Define the term acceleration.



(b) A body attached to a string is swung in a vertical circular path in air as shown in the figure below.

Copy the above diagram and on it indicate and name all the forces acting on the body if the body is moving in an anti-clockwise direction.

(c) Explain why the weight of an object on the earth's surface may vary from one place to another.

CHAPTER TWELVE
FORCE AND MOTION

LEARNING OBJECTIVES

By the end of this chapter, you should be able to:

1. *(a) State: - Newton's laws of motion and give examples for laws 1 and 3.*
   (b) Define: - Inertia and state the effects of inertia.
2. *Define and the state the SI unit of momentum.*
3. *(a) State: (i) - The Principle of Conservation of momentum.*
   (ii) - The types of collision
   (b) Describe: -
   Experiments to verify the principle (law) of conservation of momentum.
   (c) State: - Applications of the principle of the principle of conservation.
   (d) Explain: - The mechanism of:
   (i) The rocket propulsion
   (ii) The jet engine
4. *Solve problems involving the principle of conservation of momentum.*

12.1 Force and Motion

As we have seen in chapter five that one of the effects of force when it acts on a body is that:
(i) Make a stationary object to move.
(ii) Increase the speed of a moving object.
(iii) Decrease or slow down the speed of a moving object or bring a moving object to a rest.
(iv) Change the direction of a moving object.
(v) Deform (change the shape of) an object.

The relationship between force and motion was stated by *Galileo Galilli*, an Italian scientist and died before he finished. Then *Sir Isaac Newton*, an English Scientist, continued the study of moving bodies which was started by *Galileo Galilli*.

Newton carried out series of experiments and through the experimental results; he summed up the basic principles underlining motion in three laws. These laws are known as *Newton's Laws of Motion*.

12.11 Newton's First Law of Motion

Newton's First Law of Motion States that:
Every body continuous in its present state of rest or uniform (un-accelerated) motion in a straight line unless acted on by some external force.

Common experiences have shown that objects at rest do **not** begin to move on their own accord or objects on motion do **not** come to rest instantly on their own. As a result of this, the following are sited as examples of Newton's first law of motion.

(a) A body at rest
If a pile of coins is placed on a table, the one at the bottom can be removed without disturbing the ones on the top.

Explanation: *The force applied only acts on that particular coin at the bottom. Since the rest are not acted upon by the force, they remain undisturbed.*

(b) A body on motion

(i) A person riding a bicycle along a level road does not come to a rest immediately when he stops pedaling. The bicycle continues to move forward for some time, but eventually comes to a rest.

Explanation

The bicycle continues to move because of inertia and comes to a rest after time as a result of the retarding action of the external forces such as air resistance and frictional force between the tyre and the road surface. These external forces oppose the motion and eventually come to a rest.

(ii) Collision of two vehicles or when a sharp brake is applied to a car moving at a high velocity.

In the above incidences, passengers who do not fasten their safety belts are often injured when they jack forward and hit the wind screen.

Explanation

An external force acts on the vehicle but not on the passenger who simply continue their motion in a straight line in accordance with Newton's first law of motion.

(iii) A bullet fired at an angle from a gun.
When a bullet is fired from a gun held at angle to the ground, the bullet travels and eventually falls to the ground.

Explanation

The motion of the bullet is opposed by air resistance and the gravitational force,hence, sooner or later it returns to the ground.

Note: *As per Newton's first law of motion, it is supposed that, if the external forces such as friction between solid surfaces in contact, air resistance and gravitational force in the above examples were not there, the bodies would continue to move for ever.*

12.12 Inertia

Definition: *Inertia is the tendency of a body to remain at rest or, if moving to continue its motion in a straight line.*

Or *Inertia is the reluctance of a body to start moving or to stop moving once it has started.*

For this reason, Newton's first law is sometimes called "*the law of inertia*".
A body of large mass requires a large force to change its speed or its direction i.e. the body has a large inertia. Thus, the mass of a body is a measure of its inertia.

12.13 Momentum

Momentum is defined as the product of mass and velocity of a body.

Mathematically, it is expressed as:

Momentum = Mass x *Velocity*

The SI unit of Momentum

The SI unit of momentum is **kg ms**$^{-1}$

Since the mass of a body is constant, the momentum of a body is directly proportional to its velocity. I.e. *Momentum* α *Velocity*

This fact explains the following observations

A heavy goods vehicle moving at high speed needs more powerful brakes to stop it than a light car moving at the same speed or velocity. This is because the heavier vehicle has greater momentum than the lighter one.

Like wise when the same force acts upon the same vehicles for the same time, the lighter one builds up a higher velocity than the heavy one. But their momenta are the same.

This important connection between force and momentum was recognized by Newton and he expressed it in his second law of motion.

12.12 Newton's Second Law of Motion

Newton's First Law of Motion States that:

The rate of change of momentum of a body is proportional to the applied force and takes place in the direction in which the force acts.

Newton's second law enables us to define the unit of force and establishes the fundamental equation of dynamics, ***F = ma***.

Derivation of the formula

Suppose a force, **F,** acts on a body of mass, ***m,*** for time ***t*** and causes its initial velocity, ***u,*** to change to final velocity, ***v***. The momentum changes uniformly from ***mu*** to ***mv*** in the time interval, ***t***.

The rate of change of momentum $= \dfrac{\text{Final momentum - Initial momentum}}{\text{Time taken}}$

$$= \frac{mv - mu}{t}$$

By Newton's second law, the rate of change of momentum is proportional to the applied force and hence, $F \alpha \dfrac{mv - mu}{t} \Rightarrow F \alpha \dfrac{m(v - u)}{t}$ But $\dfrac{v - u}{t} = a$

$$\therefore F \quad \alpha \; ma$$

Introducing a constant to change the sign of proportionality to equal sign, we have:

$$F = \textit{Constant} \times ma$$

If ***m*** = 1 kg and ***a*** = 1 ms^{-2}, the value of the unit of force is chosen so as to make ***F*** = 1. This implies that the value of the *constant* = 1.

The SI unit of force is called the *Newton* (*symbol*, N) and is defined as:

The force which produces an acceleration of 1 ms^{-2} when it acts on a mass of 1 kg.

Or *A force which give 1 kg mass an acceleration of 1m ms^{-2}.*

Thus, when ***F*** is in Newton, ***m*** in kilograms and ***a*** in metres per second squared, we have

$$\boldsymbol{F = ma}$$

Worked Examples

1. A force 3 N acts on a body of mass 5 kg. Find the acceleration produced.

Solution: Mass = 5 kg, F = 3 N, a =? F = ma $\Rightarrow$ a = $\frac{F}{m}$ = $\frac{3}{5}$ **= 0.6 ms^{-2}**

2. Find the force acting on a body of mass 12 kg and making it to produce an acceleration of 6 ms^{-2}.

Solution m = 12 kg, F = ?, a = 6 ms^{-2} F = ma = 12 x 6 **= 72 N**

12.13 Newton's Third Law of Motion

Newton's third law of motion states that:

To every action there is an equal and opposite reaction.

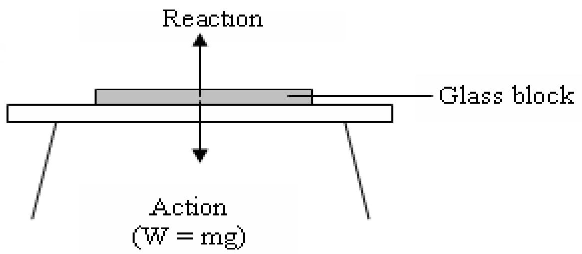


Figure 12.1

For example, a glass block placed on a table, exerts a force equal to its weight on to the table top. This force is called *action*. At the same time the table top exerts an equal force on to the glass block in the opposite direction. This force acting in the opposite direction is called *reaction*.

***Notes**:*
- *Action = Reaction*
- *Action and reaction react on different bodies.*
- *The two forces are in opposite directions.*
- *The net resultant force on the glass block is zero*

Examples of Newton's third law include:

(i) ***A man jumping from a boat***

A man jumping from a boat exerts action force on the boat and the boat exerts a reaction force. As he jumps to the river bank, the boat moves backwards.

(ii) ***Propulsion of a bullet from a gun***

When a bullet is fired from a gun, the energy of the explosion of the charge in the cartridge acts on both the bullet and the gun, thus producing equal and opposite forces acting on them. These equal forces act for the same time i.e the time taken by the bullet to travel up the barrel of the gum. The time effect of a force is called ***impulse***; thus the bullet and the gun are given equal and opposite impulses. In each case, the impulse is equal to the change in momentum.

To show that Impulse is equal to momentum change of a body

Applying $F = ma$ and substituting a = $\frac{(v - u)}{t}$ in it we have,

$$= \frac{mv - mu}{t}$$

$$= \frac{m(v - u)}{t}$$

$$\boldsymbol{Ft = mv - mu}$$

The product ***Ft*** is called *impulse*, *p*, and is equal to the change in momentum.
I.e. ***Impulse of a force on a body = Change in momentum of a body***

As the gun and the bullet were initially at rest, their initial momenta were both zero, hence the final momentum in each case is the change in momentum. Since their impulses were equal and opposite, their momenta will be equal and opposite.
The bullet leaves the gun barrel with a *muzzle velocity* and the gun kicks or reacts in the opposite with a velocity called *recoil velocity*.

Thus; ***Mass of bullet* x *Muzzle velocity = Mass of gun* x *recoil velocity***

Since the two velocities (muzzle and recoil velocities) and the two momenta are vector quantities and are acting in opposite directions, their sum is zero.
The above observation illustrates an important principle called the *principle of conservation of momentum.*

Worked Example

1. A car of mass 5000 kg initially moving at a velocity of 50 m/s accelerates to 100m/s in 2 seconds. Calculate the engine force on the car that caused the velocity change.

Solution: Mass of the car = 50 000 kg,

Initial velocity, u = 50 m/s, Final velocity, v = 100 m/s, Time, t = 2 s, F = ?

Hint: *- Think of a formula of force in this topic.*
- See if your data has all the quantities in it.
- If one quantity is missing in the formula, check if the information in the data can
help you to calculate it.
- If yes, solve for the quantity first and then substitute in the formula to get F.

We use the formula F = ma

From the data the values of u, v and t can used to solve for '*a'*. Now let us solve for '***a***'.

Using the formula $v = u + at$

$$a = \frac{(v - u)}{t} = \frac{(100 - 50)}{2} = 25 \text{ ms}^{-2}$$

Now we can now apply the formula the formula

$$F = ma = 50{,}000 \times 25 = 1\,250\,000 = 1.25 \times 10^6 \text{ N}$$

12.2 The Principle of Conservation of Momentum

The principle of conservation of momentum states that:

When two or more bodies act on one another, their total momentum remains constant, provided no external forces act on the colliding bodies.

Or *The total linear momentum of a system of interacting bodies, on which no external forces are acting, remains constant.*

For example, if two bodies of masses m_1 and m_2, initially moving with velocities u_1 and u_2 respectively collide, and after the collision their velocities change to v_1 and v_2, then by the principle of conservation of momentum,

The Sum of the Initial Momenta = The Sum of the Final momenta

$$\mathbf{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$$

(a) Collisions

Whenever two or more bodies collide, their total momentum is conserved unless there are external forces acting on them. However, the total kinetic energy usually decreases. This is because some of the kinetic energy is converted by the impact to other forms of energy such as: *heat*, *sound*, and *light* or permanently distorts the bodies leaving them with an increased amount of potential energy. The total kinetic energy of the colliding system before and after impact determines the type of collision.

(b) Types of collision

There are three types of collision, namely:

- Elastic collision,
- Inelastic collision and
- Completely inelastic collision.

(i) Elastic collision

Elastic collision is the type of collision in which there is no loss of kinetic energy. I.e. the total kinetic energy remains the same.

(ii) Inelastic collision

Inelastic collision is one in which there is loss of kinetic energy. I.e. the total kinetic energy after collision becomes less than the kinetic energy before collision.

(iii) *Completely inelastic collision*

Completely inelastic collision is one in which the colliding bodies stick together on impact and move as a single body.

Notes: *Using these types of collisions, the principle of conservation of momentum can be investigated.*

12.1 Experiments

Experiment 12.1 To investigate the Conservation of Momentum for colliding bodies moving in the same direction.

Inelastic collision of a moving body with one at rest

Apparatus/requirements

2 trolleys, a stout pin, a cork., a ticker timer with a ticker tape, standard masses and a run way.

Procedure:

- Fit trolley 1 with a stout pin and the other with a cork.
- Load trolley 1 with standard masses and weigh them.
- Attach a ticker tape passing through a ticker timer to trolley 1.
- Place the trolleys on a run way as shown in figure below.
- Give a push to trolley 1 so that it moves forward with a uniform velocity u1 and collides with trolley 2 which is at rest.

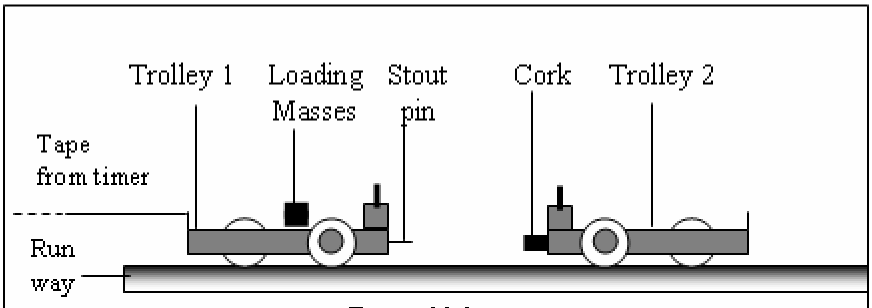


Figure 12.2

Observation:
The stout pin on trolley 1 penetrates the cork on trolley 2 and the two trolleys stick together and move on as a single body, with a common velocity *v*.

The ticker tape has two successive sets of equally spaced dots from which the velocities *u*
and *v* are calculated.

Both trolleys after collision Trolley 1 before collision
Gives common velocity, v Gives initial velocity, u_1 of
trolley 1

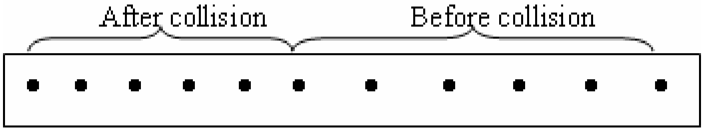


Figure 12 3

Calculation:

Mass of trolley 1 = m_1 kg
Mass of trolley 2 = m_2 kg
Initial velocity of trolley 1 = u_1 ms-1
Initial velocity of trolley 2 = u_2 ms-1
= 0 (at rest)
Final velocity of both trolleys = v (common velocity since they stick together)

Total momentum before collision $= m_1u_1 + m_2u_2$
$= m_1u_1 + m_20$
$= m_1u_1$ kg ms^{-1}
Total momentum after collision = Total mass x common velocity
$= (m_1 + m_2)$ v kg ms^{-1}

If, Momentum before collision = Momentum after collision
I.e. $m_1u_1 = (m_1 + m_2)v$,
Then law of conservation of momentum is verified.

Experiment 12.2 ***Partially elastic collision of a moving body with one at rest***

Apparatus/requirements
2 trolleys, a spring buffer, 2 ticker timers with a ticker tapes, standard masses and a run way.

Procedure
- Fit trolley 1 with a spring buffer.
- Load trolley 1 with standard masses and then weigh both trolleys.
- Attach each trolley to a tape passing through a ticker timer.

- Place the trolleys on a run way as shown in figure below.
- Give a push to trolley 1 so that it moves forward with a uniform velocity u_1 and collides with trolley 2 which is at rest.

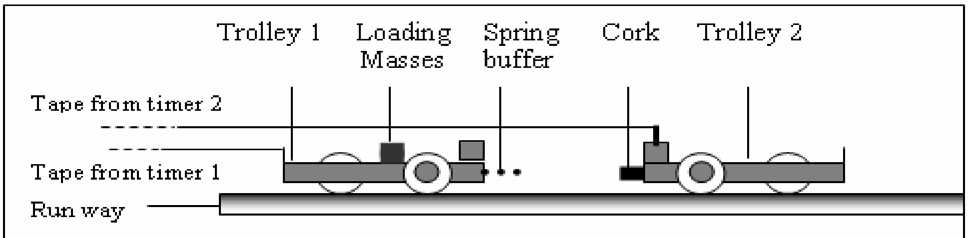
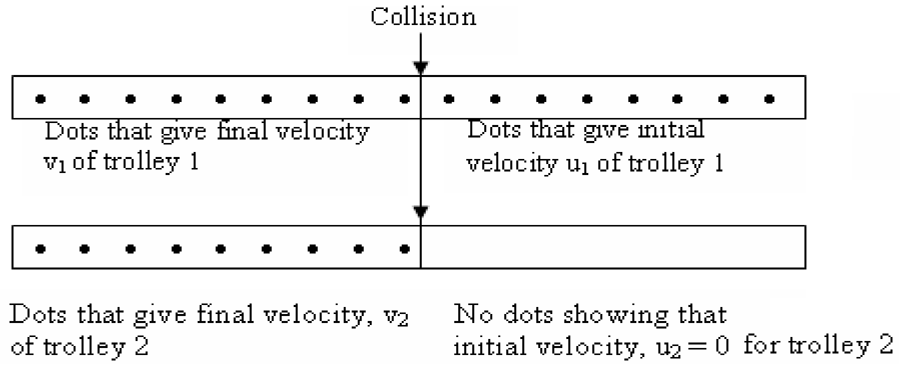


Figure 12.4

Observation:
On collision, trolley 1 is slowed up while trolley 2 moved forward with a greater velocity.



Calculation: Mass of trolley 1 $= m_1$ kg
Mass of trolley 2 $= m_2$ kg
Initial velocity of trolley 1 $= u_1\ ms^{-1}$
Initial velocity of trolley 2 $= u_2\ ms^{-1}$
$= 0$ (at rest)
Final velocity of trolley 1 $= u_1\ ms^{-1}$
Final velocity of trolley 2 $= v_2\ ms^{-1}$

Total momentum before collision $= m_1u_1 + m_2u_2$
$= m_1u_1 + m_2 0$
$= m_1u_1$ kg ms^{-1}

Total momentum after collision $= m_1v_1 + m_2v_2$

Conclusion: Momentum before collision = Momentum after collision
I.e. $m_1u_1 = m_1v_1 + m_2v_2$
Then law of conservation of momentum is verified.

Experiment 12.3 Gun and projectile experiment

Apparatus: 2 trolleys, 2 ticker timers with a ticker tapes, standard masses and a run way.

Procedure
- Load trolley 1 with standard masses and then weigh both trolleys.
- Push in the horizontal metal rod in trolley 1 (the "gun") against a spring held in place by a metal plate.
- Attach each trolley to a tape passing through a ticker timer.

- Place the trolleys on a level run way so that they are in contact as shown in figure below.
- Releases the top trigger on the "gun" to fire the projectile.

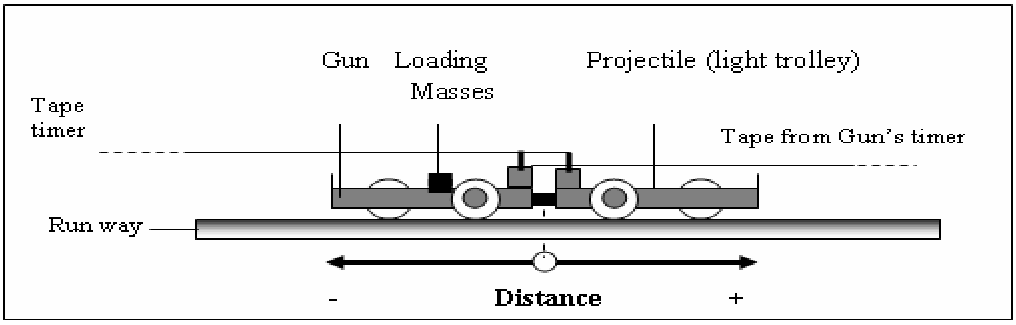


Figure 12.5

Observation:
On releasing the trigger, the projectile (lighter trolley) moved forward and the heavy trolley, the gun, moved backwards.

Note: The total momentum before and after collision is both zero.

Reason: *Momentum is a vector quantity. Since the momenta are equal and opposite, then their sum equal to zero.*

Calculation:
We represent distances and velocities to the right by a positive sign.

Let: The velocity of the gun $= -v_g$ (velocity in the opposite direction)

The velocity of the projectile $= v_p$

Momentum of the gun after firing $= m_g \times -v_g$
$= -(m_g v_g)$ kg ms^{-1}

Momentum of the projectile after firing $= m_p v_p$ kg ms^{-1}

The total momentum after firing $= m_p v_p + - m_g v_g$
$= (m_p v_p - m_g v_g)$ kg ms^{-1}

The total momentum before firing $= 0$ (Both bodies initially were at rest)

By the law of Conservation of momentum,

Momentum before collision = Momentum after collision

$0 = m_p v_p - m_g v_g$

$\boldsymbol{m_g v_g = m_p v_p}$

Worked Examples

1. Two trolleys of masses 8 kg and 5 kg are traveling on the same truck with speeds 4 ms^{-1} and 2 ms^{-1} respectively in the same direction. They collided and coupled together.
(i) State the type of the collision.
(ii) Calculate the common velocity after collision.

Solution: (i) Inelastic collision

(ii) $m_1 = 8$ kg, $u_1 = 4$ ms^{-1}, $m_2 = 5$ kg, $u_2 = 2$ ms^{-1}

Total mass $= m_1 + m_2 = 8 + 5 = 13$ kg,

Common velocity $= v$ ms^{-1}

Total momentum before collision $= m_1u_1 + m_2u_2$ kg ms^{-1}

Total momentum after collision $= (m_1 + m_2)v_2$ kg ms^{-1}

Applying the principle of conservation of momentum,

Momentum before collision = momentum after collision

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$
$$8 \times 4 + 5 \times 2 = (8 + 5) \times v$$
$$32 + 10 = 13 v$$
$$42 = 13 v$$
$$v = \frac{43}{13}$$
$$\therefore \mathbf{v = 3.2\ ms^{-1}}$$

2. A bullet of mass 6 g is fired from a gun of mass 500 g. If the muzzle velocity of the bullet is 300 ms^{-1}, calculate the recoil velocity of the gun.

Solution: Let: Mass of the gun $= m_g = 500g = \frac{500}{1000}$ kg

Mass of bullet $= m_b = 6$ g $= \frac{6}{1000}$ kg

Velocity of gun before explosion $= u_g = 0$

Velocity of the bullet before explosion $= u_b = 0$

Velocity of gun after explosion $= v_g = ?$

Velocity of the bullet after explosion $= v_b = 300\ ms^{-1}$

Applying the principle of conservation of momentum,

Momentum before firing = Momentum after firing

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$m_bu_b + m_gu_g = m_bv_b + m_gv_g$$
$$0.006 \times 0 + 0.5 \times 0 = 0.006 \times 300 + 0.5v_g$$
$$0 = 1.8 + 0.5 v_g$$
$$0.5 v_g = 1.8$$
$$\mathbf{v_g = -3.6\ ms^{-1}}$$

Since velocity is a vector quantity, the minus sign indicates that the bodies move to the left (i.e. in the original direction of body B) after collision.

Therefore, the gun kicks backwards with a velocity of 3.6 ms^{-1}

***Note**: To get the same answer but positive at once, we take the velocity of the gun to be negative in the data and then substitute in the same formula as shown below.*

Mass, m_g, of the gun $= \frac{500}{1000}$ kg, Mass, m_b, of bullet $= \frac{6}{1000}$ kg

Initial velocity, u_g, of gun before firing = 0

Initial velocity, u_b, of the bullet before firing = 0

Final velocity, v_g, of gun after firing = ?

Final velocity, v_b, of the bullet after firing = 300 ms^{-1}

Since velocity is a vector quantity, we take the velocity of the bullet to be positive and that of the gun to be negative.

Then apply the principle of Conservation of momentum, we have;

$$\begin{aligned}
\text{Momentum before firing} &= \text{Momentum after firing} \\
m_1u_1 + m_2u_2 &= m_1v_1 + m_2v_2 \\
m_bu_b + m_gu_g &= m_bv_b + m_gv_g \\
0.006 \times 0 + 0.5 \times 0 &= 0.006 \times 300 + 0.5 \times {}^{-}v_g \\
0 &= 1.8 - 0.5\, v_g \\
0.5\, v_g &= 1.8 \\
\therefore\ \mathbf{v_g} &= \mathbf{3.6\ ms^{-1}}
\end{aligned}$$

Note: - *The detailed working is to make you understand the application of the concept*
of solving the problem the first principle.
- After understanding the concept, use the shortest method.

3. A gun of mass 5 kg fires a bullet of mass 50 g at a speed of 500 ms^{-1}. Calculate the recoil velocity of the gun.

Solution: Mass of the gun, m_g = 5 kg, Mass of bullet, m_b = 50 g = $\frac{50}{1000}$ kg

Initial velocity, u_g, of the gun	= 0
Initial velocity, u_b, of the bullet	= 0
Final velocity, v_g, of gun	=?
Final velocity, v_b, of the bullet	= 500 ms^{-1}

We take the direction of the bullet to be positive and that of gun to be negative. Then we can solve the problem by using any one of the following methods.

Method I *Using the first principle*
We substitute the values in the data in the formula of the principle of conservation of momentum.

$$\begin{aligned}
\text{Momentum before firing} &= \text{Momentum after firing} \\
m_1u_1 + m_2u_2 &= m_1v_1 + m_2v_2 \\
m_bu_b + m_gu_g &= m_bv_b + m_gv_g \\
0.05 \times 0 + 5 \times 0 &= 0.05 \times 500 + 5\,{}^{-}v_g \\
0 &= 25 + 5\,{}^{-}v_g \\
5\, v_g &= 25 \\
v_g &= \frac{25}{5} \\
\therefore\ \mathbf{v_g} &= \mathbf{5\ ms^{-1}}
\end{aligned}$$

Method II Using the derived formula for gun and propectile
We substitute the values in the data in the formula:

$$\begin{aligned}
m_gv_g &= m_bv_b \\
5 \times v_g &= 0.05 \times 500 \\
v_g &= \frac{25}{5} \quad \mathbf{= 5\ ms^{-1}}
\end{aligned}$$

12.21 Applications of the principle of Conservation of momentum

The principle of conservation of momentum is applied in:

(i) Rocket propulsion
(ii) Jet engine

(a) Rocket propulsion

A space rocket carries tanks of liquid fuel and liquid oxygen and some chemicals which react to produce oxygen to enable the fuel to burn.

When the fuel burns inside the rocket engine, it crates a large force which propels a blast of hot gaseous products of the combustion out through the tail nozzle with very high velocity.

By Newton's third law of motion, the reaction to this force propels the rocket forward.

Note that although the mass of the gas emitted per second is very small, it has a very large momentum on account of its high velocity. An equal momentum is imparted on to the rocket in the opposite direction, so that in spite of its large mass, it also builds up a high velocity.

(b) Jet engine

A jet engine (used on air crafts) operates on the same principle as rocket propulsion. As the air craft does not leave the earth's atmosphere, oxygen supply is from the air.

The fuel burns to form large quantities of gaseous products. As the blast of hot gas molecules (burnt fuel and excess air or oxygen) is thrown out from the combustion chamber) through the exhaust pipe with high momentum, the jet intern, acquires an equal momentum but in the opposite direction enabling it to move forward.

Self-Check 12.0

1. When a car is suddenly brought to rest, a passenger jerks forward because of

A. inertia B. friction C. gravity D. momentum

2. A boxer while training noticed that a punch bag is difficult to set in motion and difficult to stop. What property accounts for this observation?

A. Size. B. Inertia. C. Friction. D. Weight of the bag.

3. Eggs packed in a soft, shock-absorbing box are placed in a car. When the car suddenly starts or stops moving, the eggs do not crack because

A. no force acts on them
B. the force acts on them for only a short time
C. the force is small and acts for a longer time
D. the force causes fast change of momentum.

4. A body of mass 20 kg moves with a uniform velocity of 4 m/s from rest. Find its momentum.

A. 5 kg m/s B. 80 kg m/s C. 160 kg m/s D. 320 kg m/s

5. An object of mass 2 kg moving at 5 ms^{-1}, collides with another object of mass 3 kg which is at rest. Find the velocity of the two bodies if they stick together after collision

A. 1.0 ms^{-1} B. 2.0 ms^{-1} C. 2.5 ms^{-1} D. 5.0 ms^{-1}

6. A bullet of mass 0.1 kg is fired from a rifle of mass 5 kg. The rifle recoils at a velocity of

16 ms^{-1}. Calculate the velocity with which the bullet is fired

A. 66 ms^{-1} B. 110 ms^{-1} C. 210 ms^{-1} D. 800 ms^{-1}

7. A body of mass 20 kg moves with a uniform velocity of 4 m/s from rest. Find its momentum.

A. 5kgm/s B.80 C.160 D. 320

8. When a person steps forward from rest, one foot pushes backwards on the ground. The ground will as a result push that foot

A. backwards with an equal force B. forwards with an equal force
C. backwards with a smaller force D. forwards with a smaller force

9. If the forces acting on a moving body cancel each other out (i.e. are in equilibrium) the body will

A. Move in straight line to the steady speed B. Slow down to a steady slower speed
C. Speed up a steady faster speed D. Be brought to a state of rest.

10. A body of mass 20 kg, moving with uniform acceleration, has an initial momentum of 200kg m/s and after 10s, the momentum is 300 kg m/s. What is the acceleration of the body.

A. 0.5 m/s^2 B. 5 m/s^2 C. 25 m/s^2 D. 50 m/s^2

SECTION B

1. (a) State Newton's laws of motion.

(b) Define: (i) Inertia of a body (ii) Momentum.

(d) Explain why a passenger standing on the floor of a lorry jerks backwards when the lorry starts moving forwards.

(e) A 7-tonne initially moving at a velocity of 50m/s accelerates to 80m/s in 3 seconds. Calculate the force on the truck that caused the velocity change.

2. (a) (i) What is meant by linear momentum?

(ii) State the law of conservation of linear momentum.

(b) A bullet of mass 20g is fired into a block of wood of mass 400g lying on a smooth horizontal surface. If the bullet and the wood move together with the speed of $20ms^{-1}$, calculate.

(i) The speed with which the bullet hits the wood, (ii) The kinetic energy lost.

(c) State the energy changes involved in (b) above.

3. (a) State the law of conservation of linear momentum.

(b) A water jet directed to a spot on the ground digs a hole in the ground after sometime. Explain.

(c) A moving ball P of mass 100g collides with a stationary ball Q of mass 200g. After collision, P moves backwards with a velocity of $2ms^{-1}$ while Q moves forwards with a velocity of $5ms^{-1}$. Calculate

(i) The initial velocity of P.

(ii) The force exerted by P on Q if the collision took 0.03s.

(d) Explain the principal of operation of a rocket engine

4. A sphere of mass 3 kg moving with velocity 4m/s collides head on with a stationary sphere of mass 2kg & imparts to it a velocity of 4.5 m/s. Calculate the velocity of the 3kg sphere after the collision.

5. A railway tracks of mass $4x10^4$ kg moving at a velocity of 3m/s collides with another truck of mass 2 $x10^4$ kg which is at rest. The couplings join & the trucks move off together.

(a) State the type of collision.

(b) Calculate the common velocity after collision.

CHAPTER THIRTEEN
MECHANICAL PROPERTIES OF MATTER

LEARNING OBJECTIVES

By the end of this chapter, you should be able to:

1. *State and define: - The following properties of materials*
   - Strength, Stiffness, Ductility, Brittleness and Elasticity.
2. *State: (a) - The factors affecting the strength of a material.*
   (b) - Characteristics of brittle materials.
   (c) - The factors which determine the amount of deformation of a material.
3. *(a) State: - Hooke's law.*
   (b) Describe: - Experiments to verify Hooke's law using
   - A Nuffield spring and
   - A copper wire.
4. *Define: - Stress and Strain.*
5. *(a) State: - The factors for stability and safety of structures.*
   (b) Define: - Ties and strut.
   (c) Identify: - Ties and struts in a structure.
6. *(a) Explain: - The effects of compression and tension forces on beams.*
   (b) State:- The uses of beams.
7. *(a) Describe:- How concrete is made.*
   (b) State: - The uses of concrete.
   (c) Describe:- How concrete is reinforced.

13.1 Materials

Materials are used in construction of structures like buildings, communication masts, bridges, dams, tanks, motorcars etc. Before these materials are put to use, their mechanical properties are tested. Some of the properties tested are: *strength*, *stiffness* (toughness) *ductility*, *brittleness* and *elasticity*.

1. Strength

Strength is the ability of material to withstand stress.

It is the measure of how great an applied force a material can withstand before breaking. Thus strength is a measure of the size of *breaking stress* a sample of a material has.

(a) Breaking Stress

Breaking strength is the force needed to break a piece of material of cross-sectional area of $1m^2$.

It is given by the formula: Breaking stress $= \frac{\text{Force}}{\text{Cross-sectional area}}$ N/m^2 (Pascal)

The greatest stress (breaking stress), the material can withstand before it can break is its ultimate tensile strength.

(b) Factors affecting the strength of a material

(i) Cross-sectional area (diameter)
For samples of the same substance, it requires a larger force to brake one of larger cross-sectional area.

(ii) Nature of the material
It is for example harder to break a steel rod than to break a sisal rod

(iii) Force applied
The strength of a material may depend on how a force is applied on it. For example concrete is very strong when compressed but very weak when stretched.

2. Stiffness (toughness)

Stiffness *is the ability of a material to resist bending*. Stiffness tells us about opposition a material sets up when being deformed i.e. having its size or shape changed.
Stiff materials resist forces which try to change their shapes or sizes. They can withstand large compressional forces but stretch by little amount when pulled apart.

3. Ductility

Ductility is the ability of a material to be drawn into wires or worked into any shape without breaking. Examples of ductile materials include all metals except mercury. Metals can be hammered, rolled, cut or stretched into any useful shapes.

4. Brittle materials

Brittle materials *are substances which bend very little, then suddenly crack without any warning*. Examples of brittle materials are glass, brass, bronze and other alloys.

Characteristics of brittleness

(i) Undergo very little deformation before finally breaking
(ii) Can withstand fairly large compressional forces but break when subjected to tension
(iii) Cracks form easily and are rapidly transmitted through the material once formed
(iv) When broken, the broken pieces can be fitted together accurately.

5. Elasticity

Elasticity is the ability of a substance to recover its original shape and size after deformation.
Examples of materials which have this property called elasticity are said to elastic. E.g. all metals except mercury and rubber. While materials which do not have elasticity are said to inelastic. E.g. Wood, paper, etc.

The amount of deformation depends on

(i) The nature of the material
(ii) The magnitude (Strength) of the distorting force.

There is a limit to the elasticity of the substance as it will break if the force applied is strong. The elasticity of a material depends on the force of attraction between its molecules. When a substance is stretched, its molecules are pulled apart as the distorting force overcomes the force of attraction between the molecules. On removing the distorting force, the substance will return to its original shape if the elastic limit is not exceeded. The relation between force and extension was investigated by Robert Hooke and stated it in his law called *Hooke's law*.

13.2 Hooke's Law

Hooke's Law states that:

The extension is directly proportional to the load provided the elastic limit is not exceeded.

Robert Hooke illustrated his law by carrying out four different experiments.

- Loading a spiral spring (deformation increases in length)
- Loading a wire (deformation increases in length)
- Loading a horizontal beam fixed at one end (deformation and depression of the free end).
- Tightening a watch spring (deformation = angular rotation)

Experiment 13.1 To verify Hooke's Law, Using a spring

(a) Arrange the apparatus as shown in figure 13. below.

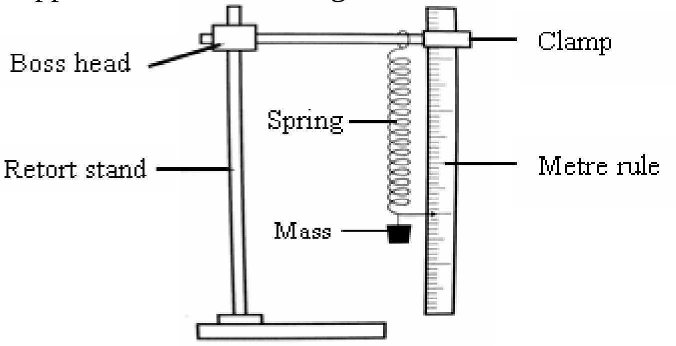


Figure 13.1

(b) Attach a pointer to a spring and suspend the spring from a support clamped up using a retort stand.

(c) Read and record the position, P_0, of the pointer on the meter rule.

(d) Suspend a mass hunger of 100 g from the spring.

(e) Read and record the new position, P_1, of the pointer.

(f) Determine the extension, e_1, of the spring.

(g) Repeat the procedures (d) to (f) for values of slotted masses of 200 g, 300 g, 400 g, 500 g and 600 g.

(h) Remove the masses in steps of 100 g and each time record the position, P_2, of the pointer until the spring returns to its original length.

(i) Determine the extension, e_2, of the spring.

(j) Enter your results in the table 13.1 below.

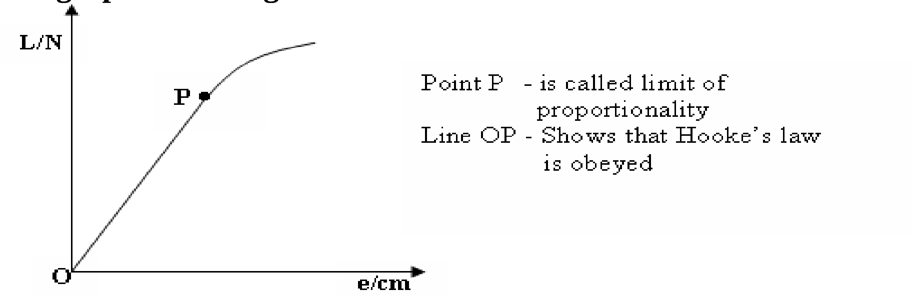
Table of results

Increasing load				Decreasing Load		Average extension
M/kg	**Load/N**	**P_1/cm**	**$e_1 = (P_1 - P_0)$cm**	**P_2/cm**	**$e_2 = (P_2 - P_0)$/cm**	**e** $= \frac{e_1 + e_2}{2}$ / cm
0.1						
0.2						
0.3						
0.4						
0.5						

Table 13.1

(k) Plot a graph of load (N) against extension, e.
If the experiment is carefully performed, and the graph is accurately drawn, the shape of the graph below is obtained.

The graph of load against extension



Observation: When the load is doubled, the extension is also doubled.

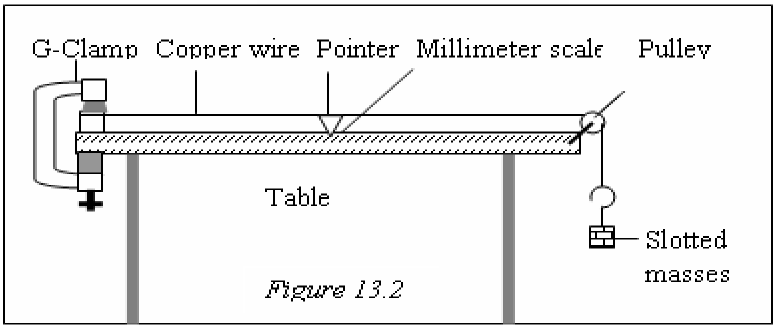
Conclusion: Hooke's law which states that:
The extension of an elastic material is directly proportional to the load provided the elastic limit is not exceeded, is verified.

Experiment 13.2

To verify Hooke's Law, Using a Copper wire

Apparatus
A smooth pulley, G-clamp, a mass hanger and slotted masses, a table, a pointer (made of paper) and a millimeter scale.

Procedure
(a) Set up the apparatus as in figure 13.3 below.



(b) Add small masses one at a time to the weight hunger and observe the scale reading.
(c) Determine the extension in millimeters.
(d) Repeat the procedure (b) to (c) by adding more masses until the wire breaks.
(e) Enter your results in table 13.2 below.

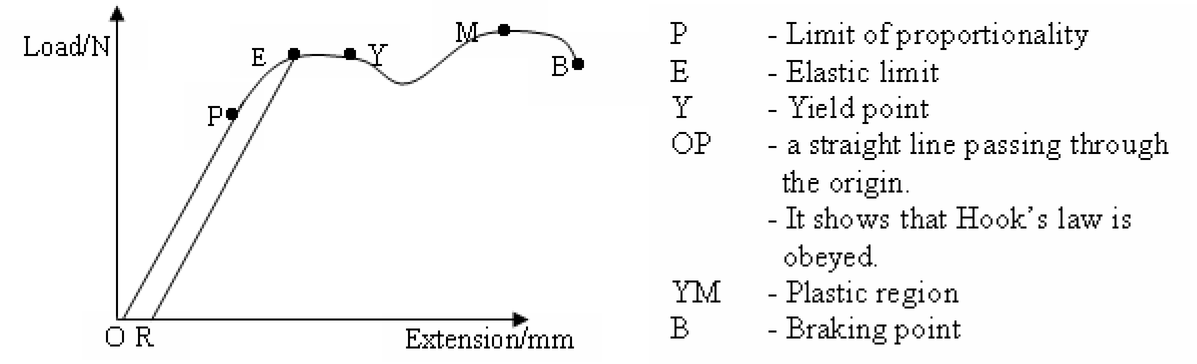
Mass/kg	Load/N	Extension/cm

Figure 13.2

(f) Plot a graph of load/force against extension for the wire.
If the experiment is carefully performed, and the graph is accurately drawn, the shape of the graph resembles that of a spring except the extension is much smaller for a given force.

Note: The extension for a given load depends on:
- The metal used,
- The length of the wire and
- The diameter or cross sectional area of the wire.

The graph of load against extension for a wire



Explanation of the shape of the graph

Line OP: Line OP is a straight line passing through the origin. It shows that the extension is directly proportional to the load. And therefore indicates that Hooke's law is obeyed. When the load is removed from the wire within this range of load, the wire returns to its original unloaded length.

Point E: Point E is called *elastic limit*.
After the elastic limit, the wire does not return to its original length when the load is removed. Some of the extension remains and the wire is deformed permanently giving a *residual extension,* OR, on the diagram.

Point Y: Just after the elastic limit is point Y. It is called *yield point*. Beyond this point, an increase on the load causes the wire to 'flow' thus producing a larger extension than the previous one. After point Y, *plastic deformation* occurs.

Point M: Point M indicates the maximum load the wire can take.
Reducing the load after reaching point M has little effect; the wire forms a waist at its weakest point and breaks on reaching point B called *breaking point*.

Worked Examples

1. A force of 20 N extends a spring by 5 cm. Calculate the force required to extend the same wire by 20 cm.

Solution: Let: $F_1 = 20$ N, $e_1 = 5$ cm, $e_2 = 20$ cm, $F_2 = ?$
20 N extends the spring by 5 cm,
1cm is extended by a force of $\frac{20}{5}$ N
The force that can extend the spring by 20cm = $\frac{20 \text{ x } 20}{5}$ **= 80 N**

2. A force of 10 N extends a wire by 2 cm. Find the extension by a force of 50 N.

Solution: Let: $F_1 = 10$ N, $e_1 = 2$ cm, $e_2 =?$ cm, $F_2 = 50$ N
10 N extends the by 2 cm,
1 N extends by 10/2
Then the extension by 50 N = 2 x $\frac{50}{10}$ = **10 cm**

13.3 Stress and strain

(a) Stress

Stress is defined as *force per unit area of cross-section*.

i.e. $\text{Stress} = \frac{\text{Force (N)}}{\text{Area (m}^2)}$

The SI unit of stress is Newton per metre squared, Nm^{-2} = Pascal (Pa).

(b) Strain

Strain is defined as *the change in length per unit length.*

$$Strain = \frac{\text{Change in length}}{\text{Original length}}$$

Strain has no units because it is a ratio of the same quantity.

13.4 Structures

Structures are generally designed and built to carry or support a load (i.e. to with stand a force). A well designed structure must be able to stay in position without collapsing for a long period of time.

13.41 Factors for stability and safety of structures

The stability and safety of structures depend on the following.

(i) The nature of the material.
(ii) Shapes of the materials.
(iii) The way of arrangement of the materials.

The behaviour of structures under stress or strain can be investigated by using models made out of straws.
The straws are arranged to give structures of different shapes and a heavy load is then attached to one end of each structure as shown in the diagrams in figure 13.3 below.

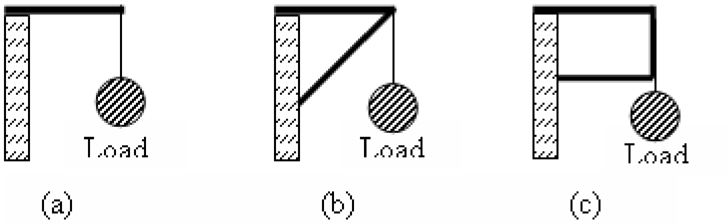


Figure 13.3

Experimental results show that the triangular structure figure 13. (b), provides better support than (a) and the rectangular structure (c) for the same load. However, the rectangular structures can be strengthened by fixing more girders diagonally. Notable examples are seen in door and window frames.

13.42 Beams and Girders

(a) Beams:

A beam refers to a large, straight piece of material with uniform cross section whose width and thickness are small compared to the length. Beams are used as major structural items in a building or structure.

(b) Girders:

These are the smaller pieces of materials used to strengthen a structure.
In any structure, some girders are under tension while others are under compression. Girders under compression bend or buckle and those under tension become thin and may break.

13.43 (a) Ties and Struts

Girders and beams are used to make the frame works of structures such as water tank supports, radio masts, roof trusses, bridges, cranes, electricity pylons etc. In these structures, materials ties and others are struts.

(i) Ties: *A tie is a girder under tension.*
(ii) Struts: *A strut is a girder under compression.*

(b) Identification of Ties and struts in a structure

To identify ties and struts in a structure can be identified by replacing them with strings. If the string is pulled tightly, the girder is in tension and therefore it is a tie. And if it becomes slacked, it is under compression and therefore it is a strut.

Worked Examples

1. The diagram in figure 13.4 (a) and (b) shows a structure supporting a heavy load hanging from a nail at A. Identify ties and struts in the diagrams below.

(a) Figure 13.4 (b)

Answer: (a) P, Q, S and R are ties
Only T is a strut.

(b) Tie: W
Struts: X, Y and Z

(c) Applications of struts and Ties

Ties and struts are applied in:

(i) Roof supports
(ii) Water tank/reservoir supports
(iii) Girder bridge
(iv) Communication masts

13.5 Effect of Compression and Tension forces in beams

Beams are used to support loads in buildings and bridges.

(a) Effect of Compression forces in a beam

If a beam is acted on by two compression forces (two opposing forces that act toward each other) as shown in figure 13. (a) below, the particles in the body are pushed closer together. The body becomes shorter compared to the original length and the body is said to be in a state of compression.

(a) Figure 13.5 (b)

(b) Effect of Tension forces in a beam

If the beam is acted on by two tension forces (two opposing forces that act away from each other) as shown in figure 13. (b) above, the particles in the body are moved apart. The body becomes slightly longer than the original length and the body is said to be in a state of tension.

(c) Bending of beams

When a beam is subjected to a bending force, as shown in figure 13. below, three things happen to it.

Diagrams showing the behaviour of a beam under a load

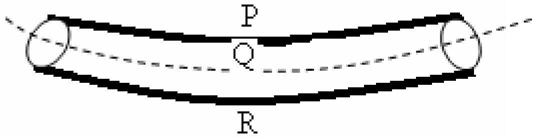


Figure 13.6

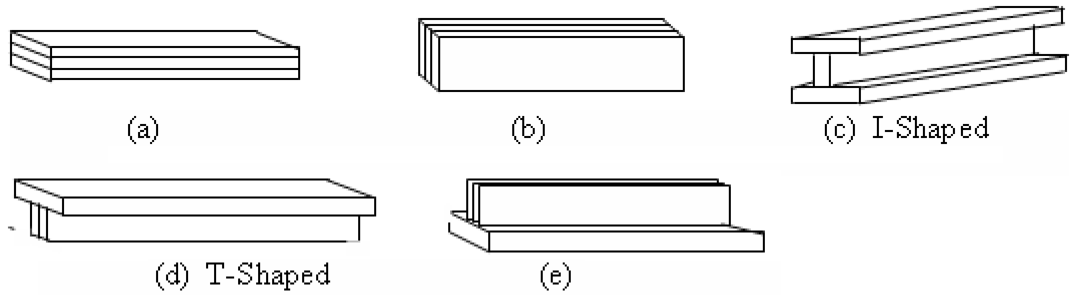
(i) The particles in the upper region marked P become closer and shorter. The part region is under compression.

(ii) In the lower region, R, the particles become further apart and the region becomes longer. It is under tension.

(iii) Near the middle point Q, the distance between the particles remains the same. That is the region is neither under compression nor under tension. The length of this surface has not changed. This region is called the *neutral plane* or *neutral surface*.

13.51 (a) Stiffness of *three* wooden beams

Three wooden beams can be arranged in five possible ways to give different shapes as shown in figure 13.7 below. One end of each beam is then subjected to a heavy load.



Experimental results show that the I-shaped beam is the most stiff i.e. the strongest. That is why many beams in structures such as bridges, buildings are I-shaped.

I-shaped beams can be made lighter by removing material along the neutral axis.

(b) Effect of shear force on a body

When a body is acted upon by *sheer forces* (two equal and opposite forces whose line of action are parallel to each other) the body becomes twisted and deformed. See the diagrams in figure 13.8 below.

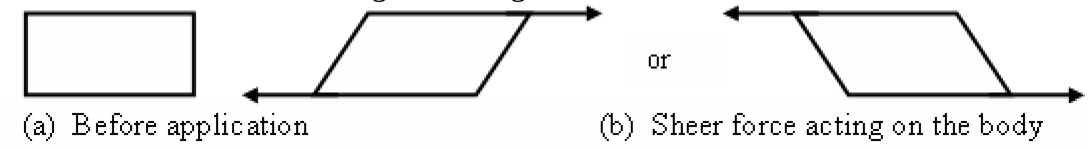


Figure 13.8

(c) Uses of beams

(i) I-Shaped beam: I-Shaped beam resists bending and sagging under load. They are used in building constructions such as buildings, bridges, masts, cranes etc.

(ii) Joist. A joist is a wooden beam which helps to support a floor or ceiling. The joists are laid on their edges since they are then stiffer and bend less.

13.52 Other materials used in construction of structures are the following:

1. **Natural stone:** Stone is an inorganic mineral or soil concretion of the earth. The commonly used types of stones in building, civil engineering, manufacturing and art are: sedimentary, igneous, or metamorphic origin. Some of the building stones are basalt, flint, granite, limestone, marble, porphyry, sandstone, slate, and flagstone.

2. **Bricks:** Brick is block of clay or other ceramic used for construction and decorative facing. They are made by mixing together clay and water. The mixture is molded into different shapes and baked in a kiln (i.e. fired at high temperature) or may be dried in the sun. Well known places for making bricks is Lweza Clays at Kajjansi in Wakiso district.

3. **Mortar:** Mortar is a mixture of sand and cement made into paste by adding water. When dried it produces a hard stony material used for *bonding* bricks. They resist dampness and heat, and can last longer than stone. Some bricks are made of special fireclays for use in fireplaces or ovens.

4. **Concrete:** Concrete is made by mixing gravel or small stones, sand, cement and water in right proportions. The gravel or stones make the concrete very strong; the sand fills up the spaces between the stones.

(a) Uses of concrete

It is used where heavy loads have to be supported, e.g. in

- the foundations of tall buildings,
- airport runways,
- dams,
- the piers of bridges
- etc.

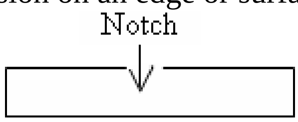
Concrete has great compressive strength but little tensile strength. Its tensile strength is increased by reinforcing it.

(b) Reinforcement of concrete

Concrete used in most construction work is reinforced with steel. Steel is embedded in the concrete in the form of a mesh. A bond forms between the steel and the concrete. When reinforced concrete is subjected to extreme tensile stresses, the steel supplies the necessary strength and stresses can be transferred between both components.

13.6 Notches

A notch is an indentation or incision on an edge or surface.



Notches and cracks spread more easily when brittle materials such as glass and concrete are under tension. This effect is applied in cutting glass where a notch is made on the surface of the glass and the glass is subjected to bending from below.

Self-Check 13.0

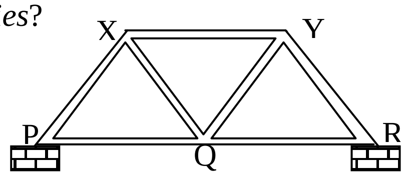
1. The strength of a material depends on the:
   (i) Nature of the material (ii) Diameter of the material (iii) Length of the material
   A. (i) only B. (i) and (ii) only C. (ii) and (iii) only D. (i), (ii) and (iii)

2. A mass of 0.2 kg produces an extension of 8 cm in a spring. The force required to produce an extension of 6cm is
   A. 0.75 N B. 1.50 N C. 2.70 N D. 24.00 N

3. A ductile material is that which
   A. Is fragile B. Is not elastic
   C. Can be molded into any shape D. Easily breaks under compression.

4. In a wire supporting a load, stress is given by
   A. $\frac{strain}{area}$ B. force x area C. $\frac{area}{force}$ D. $\frac{force}{area}$

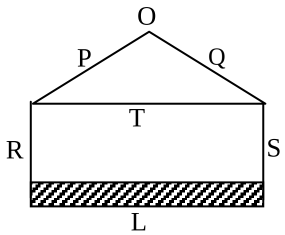
5. The diagram in figure below shows a framework of a bridge. Which of the girders are *ties*?



A. XQ, QY, PX, YR B. PQ, QR, XY C. XQ, QY D. PX, YR

6. Which of the following are all brittle materials?
A. Leather, rubber, thread.
B. Clay, glass, wood.
C. Glass, cast iron, stone.
D. Rubber, polyester, copper wire.

7. The diagram below shows a structure of wooden beams P, Q, R, S and T supporting a heavy road L. Which of the beams can be replaced by ropes if the same shape is to be maintained?



A. P, R, S and T
B. P, Q, S and T
C. Q, R, S and T
D. P, Q, R and S

8. A rod of cross-sectional area $40cm^2$ needs a tensile force of 2N to break it. What is its breaking stress?
A. 0.005 Nm^{-2} B. 0.05 Nm^{-2} C. 5 Nm^{-2} D. 500 Nm^{-2}

9. A mass of 0.5kg causes a spiral spring to extend by 4cm. The force that would cause an extension of 6cm would be
A. 2.0 N B. 3.3 N C. 4.8 N D. 7.5 N

10. If a load 1N extends a spring by 5cm, what extension will a load of 0.6 N produce?
A.1.2 cm B. 3.0 cm C. 8.3 cm D.30.0 cm

11. Which of the following are brittle substances?
A. Dry clay, steel, chalk, wood
B. Chalk, steel, plastic, glass.
C. Glass, chalk, concrete and steel
D. Dry clay, glass, chalk and concrete.

12. Reinforced concrete is stronger that ordinary concrete because concrete and the steel are
A. Both brittle materials.
B. Strong in tension and compression respectively.
C. Both ductile materials.
D. Strong in compression and tension respectively.

13. The graph above represents the extension of a wire with increasing load. Where does the yield point occur?
A. between point P and Q
B. between point Q and R
C. between point R and S
D. at point S

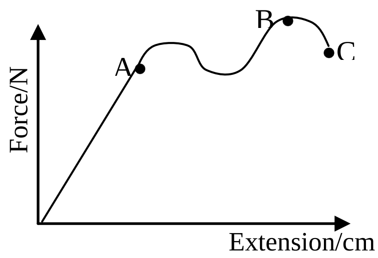
14. A load of 4 N stretches a spring by 0.5 cm. Calculate the extension when a load of 8 N is applied.
A. 0.25 cm B. 1.0 cm C. 2.0 cm D. 4.0 cm

15. A notch on a material spreads more rapidly when the metal is
A. in tension B. in compression C. pre-stressed D. reinforced

SECTION B

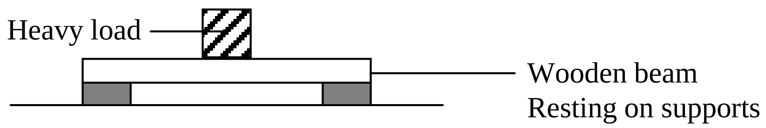
1. (a) With the aid of a diagram, describe the effect of a shear force on a body.
   (b) (i) What is meant by strength as applied on a material?
   (ii) State the factors on which strength on a material depends.
   (c) (i) Describe a simple experiment to describe Hooke's law using a spring.
   (ii) State any three characteristics of concrete which make it a desirable building material.

2. (a) Define the following terms
   (i) Strain. (ii) Stress.
   (b)



The curve in the figure shows the force versus extension graph for a copper wire.
Describe what is happening between points A and B.

3. (a) Explain with the aid of a sketch diagram, how a notch weakens a beam of a brittle
   material.
   (b) Sate two ways in which concrete may be made stronger.
   (c) A force of 100 N stretches an elastic spring by 2 cm. what force would stretch the same spring by 3.5 cm?
   (d) The diagram in the above figure shows a simple bridge on a stream.



   (i) Mark the neutral axis
   (ii) Label the part that will be in tension
   (iii) Indicate on the diagram how the bridge can be strengthened.

CHAPTER FOURTEEN
BEHAVIOUR OF LIGHT

LEARNING OBJECTIVES

By the end of this chapter, you should be able to:

1. Define: (a) A ray and a beam.
(b) State and draw the types of beams.
2. (a) State (i) The rectilinear propagation of light.
(ii) The types of shadow.
(b) Describe an experiment for the formation of the types of shadows.
(c) State: The types of eclipses.
(d) Describe with a help of diagrams the formation of the types of eclipses.
3. (a) Describe (i) The structure of the Pin-hole camera.
(ii) The formation of image in a pin-hole camera.
(b) State: (i) The properties of the image formed in a pin-hole camera.
(ii) The effects of the object distance and the size of the pin hole, on the image formed in a pin-hole camera.
4. (a) Define magnification.
(b) Solve problems using the formula of magnification.

14.1 INTRODUCTION

Optics *is a branch of physical science dealing with the propagation and behavior of light.*

Light *is a form of energy that travels in a straight line*.

(a) Rays and Beams of Light

(i) Rays: *A ray is the direction of the path taken by light.*

Diagrammatically it is represented by a straight line with an arrow.

a ray

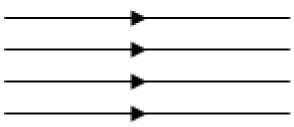
(ii) Beam: *A beam is a stream of light energy.*
It is represented by a number of rays.

(b) Types of Beams

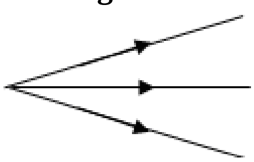
There are three types of beams namely;

- Parallel beam,
- Convergent beam and
- Divergent beam.

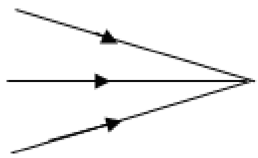
Diagrams showing the three types of beams



(a) Parallel beam
The rays are parallel to each other.



(b) Divergent beam:
The rays start from a common point and separate in different directions.



(c) Convergent beam:
The rays from different directions come together and meet at a common point.

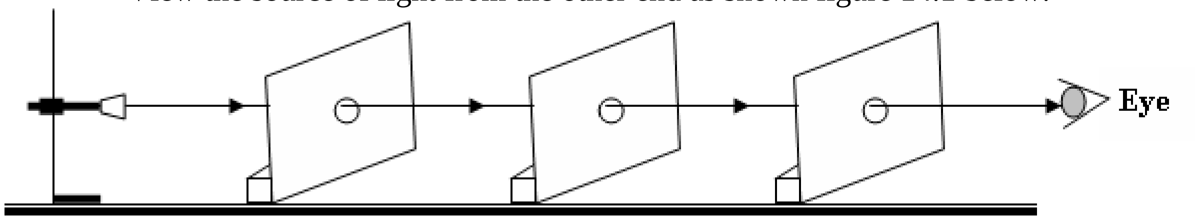
14.11 Rectilinear Propagation of Light

The rectilinear propagation of light states that: *Light travels in a straight line*.

Experiment 14.1 To verify the rectilinear propagation of light

Apparatus:
Source of light, three slits, screen, retort stand

Procedure:
- Stand the slits such that they are in a straight line.
- Place the source of light at one end such that it is at the same level with the three holes.
- View the source of light from the other end as shown figure 14.1 below.



Fifure 14.1

Result: The source of the light can be seen.

Procedure continued
- Now displace any one of the slits such that the holes are not in a straight line.
- Observe the source of the light as before.

Result: The source of the light can no longer be seen.

Explanation: Since the holes are no longer in a straight line, therefore the light ray strikes the surface of the displaced slit.

Conclusion: Light travels in a straight line.

14.12 Applications of the Rectilinear Propagation of Light
The rectilinear propagation of light is applied in:
- Shadows
- Eclipses and
- Pinhole camera.

(a) Shadows
A shadow is a dark area caused by an opaque object blocking a ray or a beam of light. A shadow may be descried as umbra and penumbra.

(i) Umbra:
The fully shaded inner region of shadow cast by an opaque object, especially the area on the earth or moon experiencing totality in an eclipse.

In the laboratory, it may be obtained when a pint source of light is enclosed in a box with very small hole on one its sides and an opaque object is placed in between the source of light and a screen as shown in figure 14.2 on page 213.

A diagram showing the formation of umbra using a point source of light

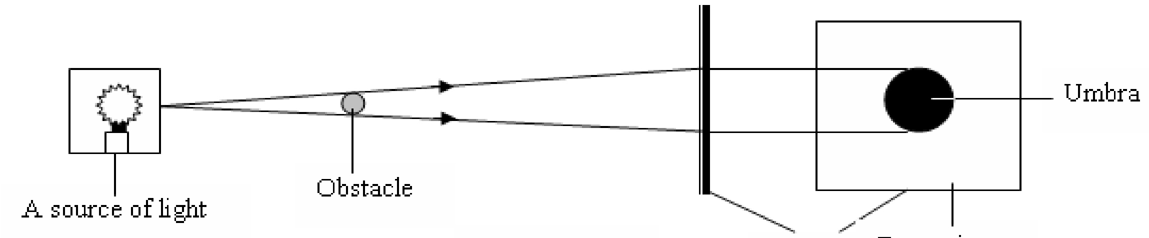


Figure 14.2

(i) Penumbra: The partially shaded outer of the shadow cast by an opaque object, especially the area on the earth or moon experiencing totality in an eclipse.

It can be obtained when of source of light is enclosed in a box with extended or larger hole on one its sides and an opaque object is placed in between the source of light and a screen as shown in figure 14.3 below.

A diagram showing the formation of penumbra using an extended source of light

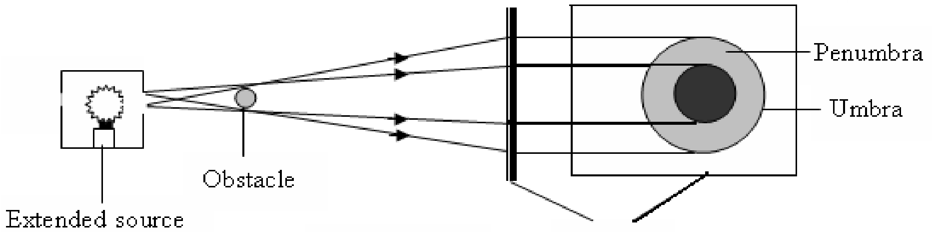


Figure 14.3

(b) Eclipses

***Eclipse**, in astronomy, is the obscuring or blocking of light from the sun by a celestial body (e.g the moon or the earth).*

14.13 Types of Eclipse

Two kinds of eclipses involve the earth. These are:

(i) Solar eclipse and

(ii) Lunar eclipse

(a) Solar eclipse

A solar eclipse is an eclipse occurs when the moon is between the sun and the earth and its shadow moves across the face of the earth.

Solar Eclipse is divided into two, namely;

(i) Total solar eclipse and

(ii) Annular eclipse

(i) Total solar eclipses

Total solar eclipse occurs when the moon's umbra reaches the earth as shown in the figure 14.4 below.

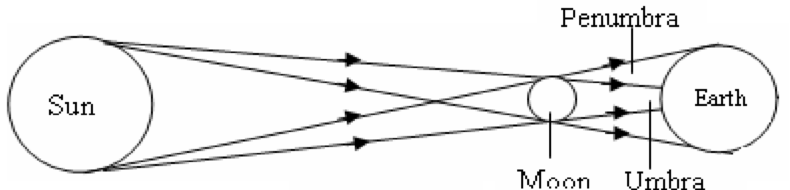


Figure 14.4

(ii) Annular eclipse (Partial Eclipse)

The Moon's orbit around Earth is slightly elliptical, or egg-shaped. Therefore the distance between Earth and the Moon varies slightly as the Moon orbits Earth. When the moon is farther from Earth than usual, it appears smaller and may not cover the entire sun during an eclipse. A cone of umbra forms in upper atmosphere and does not reach the earth surface. As a result the whole area on the earth surface is covered by penumbra. See figure 14.5 below. This type of eclipse produced is called an ***annular eclipse***.

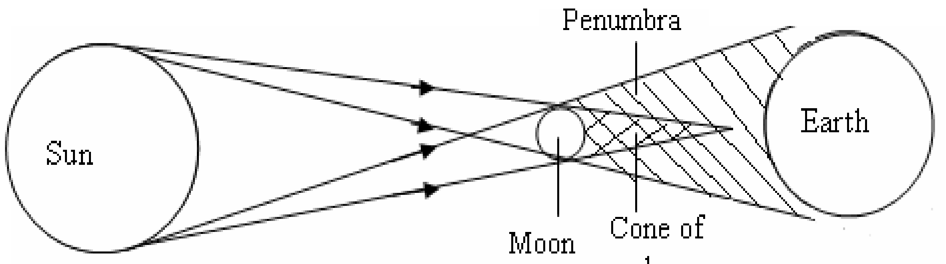
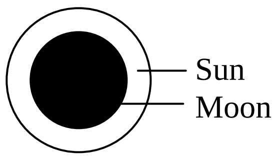


Figure 14.5

Off to the sides of the path of a total eclipse, a partial eclipse, in which the sun is only partly covered, is visible. As such a bright ring of the solar disk appears around the black disk of the moon.

At the beginning of a total eclipse, the moon begins to move across the solar disk about 1 hour before totality.



Notes: (i) *In areas outside the band swept by the moon's umbra but within the penumbra, the sun is only partly obscured, and a partial eclipse occurs.*

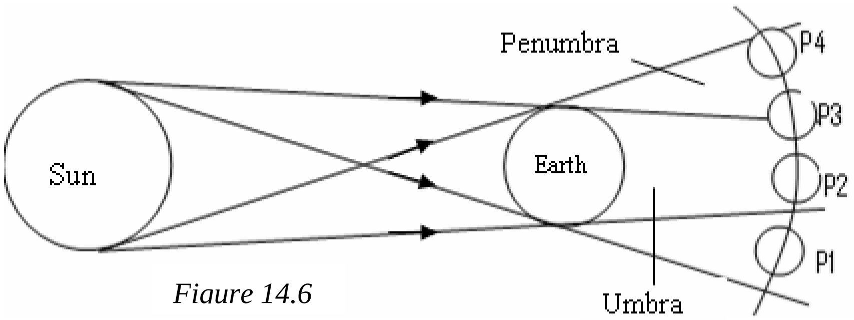
(ii) *The illumination from the sun gradually decreases and during totality (and near totality) declines to the intensity of bright moonlight.*

(iii) *A total solar eclipse occurs about every 18 months*.

(b) Lunar Eclipse (Partial or total obscuring of the moon by the earth's shadow)

A lunar eclipse occurs when the earth is between the sun and the moon and its shadow is cast on the surface of the moon.

The earth, lit by the sun, casts a long, conical shadow called *umbra* in space. At any point within that cone the light of the sun is wholly obscured. Surrounding the shadow cone, is an area of partial shadow called the *penumbra*. See figure 14.6 below.



Fiaure 14.6

Lunar Eclipse is divided into two types namely;

- Partial lunar eclipse and
- Total lunar eclipse.

(i) Partial lunar eclipse

A partial lunar eclipse occurs when only a part of the moon enters the umbra and is obscured. This is the case when the moon is in position P3. Figure 14.1 .

(ii) Total lunar eclipse

A total lunar eclipse occurs when the moon passes completely into the earth's umbra. That is when the moon is in position P2, figure14.1. In positions P1 and P4 there is no eclipse, but the moon is less bright.

Notes:

❖ *Before the moon enters the umbra in either total or partial eclipse, it is within the penumbra and the surface becomes visibly darker.*

❖ *The portion that enters the umbra seems almost black, but during a total eclipse, the lunar disk is not completely dark; it is faintly illuminated with a red light refracted by the earth's atmosphere, which filters out the blue rays.*

❖ *Occasionally a lunar eclipse occurs when the earth is covered with a heavy layer of clouds that prevent light refraction; the surface of the moon is invisible during totality.*

14.14 The Pin hole Camera

(a) Structure

A pinhole camera is made of a light proof box painted black on the inside with a small pinhole in front and a piece of frosted glass or tracing paper placed at the back as the screen.

(b) Mechanism

An illuminated object is placed in front of the camera. Light rays from the object pass through the pin hole and forms an inverted, backwards image of the subject on the screen at the back of the box.

A diagram showing the formation of image in a pinhole camera

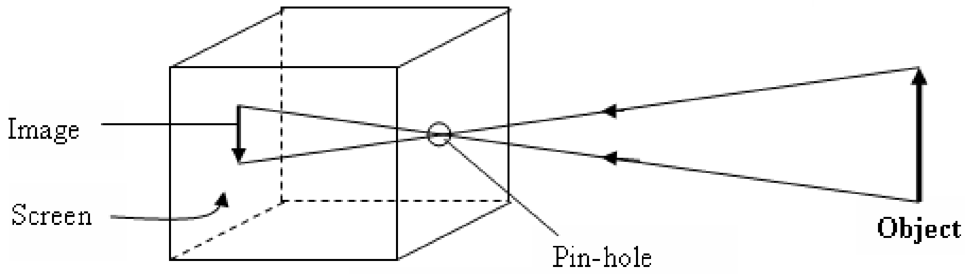
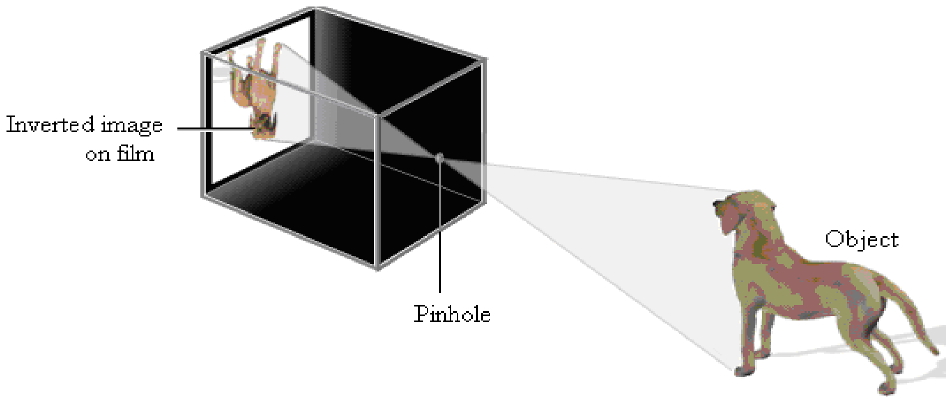


Figure 14.7

A photograph of a dog formed in a pin-hole camera



(c) The Nature of the Image formed by a Pin-hole Camera

The nature of the image formed by a pin-hole camera is described by using the following terms.

(i) Real - Formed by intersection of actual rays and can be obtained on screen.
(ii) Inverted - Upside down
(iii) Diminished - Smaller than the object and
(iv) Magnified - Bigger/larger than the object.

(d) The effect of the object distance on the size of the image

The size of the image, magnified or diminished, depends on the object distance.

(i) When the object distance is small

When the object is moved towards the pin-hole or when the camera is moved towards the object, such that the object distance is small, the image becomes magnified.

(ii) When the object distance is large

When the object is moved away from the pin-hole, or when the camera is moved away from the object, such that the object distance is large, the image becomes diminished.

(e) The effect of the size of the pin-hole on the nature of the image formed in a pin-hole camera

When the pin-hole is small, the image is sharp or clear and is said to be in focus. But when the hole is bigger, the image is blurred (brighter).

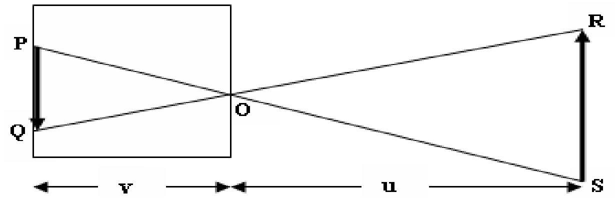
Explanation

The blurred image is explained by the fact that a large hole is believed to be made up of a collection of many small holes, each of which produces an image slightly displaced from the other images. The result is thus a brighter (blurred) image.

(f) Magnification, m, of the pin-hole camera

Magnification, m, *is the ratio of the image height to the object height*.

Consider the cross section of the pin-hole camera



$$\text{Magnification} = \frac{\text{Image Height}}{\text{Object height}} = \frac{PQ}{RS}$$

$$\therefore \ \boldsymbol{m} = \frac{h_i}{h_o}$$

Or
$$\text{Magnification} = \frac{\text{Image distance from the pin-hole}}{\text{Object distance from the pin-hole}}$$

$$\therefore \boldsymbol{m} = \frac{v}{u}$$

From the above equations, we have

$$\frac{\text{Image Height}}{\text{Object height}} = \frac{\text{Image distance from the pin-hole}}{\text{Object distance from the pin-hole}}$$

$$\frac{h_i}{h_o} = \frac{v}{u}$$

Example

1. An object of height 5 cm is placed 20 cm from a pin-hole camera which is 5 cm long. If the height of the image formed is 1.25 cm, calculate the magnification

Solution: u = 20 cm, v = 5 cm (length of the camera), Oh = 5 cm, Ih = 1.25 cm, m = ?

$$\text{Magnification} = \frac{\text{Image Height}}{\text{Object height}} = \frac{IH}{OH} = \frac{1.25}{5} = \mathbf{0.25}$$

Or
$$\text{Magnification} = \frac{\text{Image distance from the pin-hole}}{\text{Object distance from the pin-hole}} = \frac{v}{u} = \frac{5}{20} = \mathbf{0.25}$$

2. A man 1.75 m tall stands at a distance of 7.0 m from the pinhole of pinhole camera. If the film is 0.20 m behind the pinhole, find the length of the image of the man formed on then film.

A. 8.75 m B. 4.00 m C. 0.80 m D. 0.05 m

Solution: $h_o = 1.75$ m, $u = 7.0$ m, $v = 0.20$ m, $h_i = ?$

Using $\frac{h_i}{h_o} = \frac{v}{u} \Rightarrow \frac{h_i}{1.75} = \frac{0.2}{7.0}$ $h_i = \frac{0.2 \; x \; 1.75}{7.0}$ **= 0.05 m**

Therefore the answer is **D**.

Self-Check 14.1

1. When a pin-hole camera is moved nearer an object, the size of the image
   A. remains the same B. becomes smaller
   C. becomes larger D. becomes diminished

2. In a pin-hole camera, sharper and taller images are obtained by
   A. widening the hole and moving the object farther
   B. narrowing the hole and moving the object nearer
   C. using a longer camera with a wider hole
   D. using a shorter camera with a narrower hole

3. (a) Describe an experiment to show that light travels in a straight line.
   (b) With the aid of a diagram, illustrate how the shadows are formed when an opaque object is placed between an extended source of light and a screen.

4. (a) An object of height 4 cm is placed 5 cm away from a pin-hole camera. The screen is 7 cm from the pinhole.
   (i) Draw a scale a ray diagram to show the formation of an image by a pinhole camera.
   (ii) What's the nature of the image.
   (iii) Find the magnification.
   (iv) Explain what happens to the image if the pinhole is made larger.
   (b) Draw a diagrams to show the formation of a partial and total solar eclipse.

REFLECTION OF LIGHT

LEARNING OBJECTIVES

By the end of this chapter, you should be able to:

1. (a) *Define the term reflection.*
   (b) *State and describe the type of reflection.*
2. (a) *State the laws of reflection.*
   (b) *Describe an experiment to verify the laws of reflection.*
3. (a) *Draw the formation images in plane mirror.*
   (b) *State the properties of the images formed in a plane mirror.*
4. *State the effect of rotating a plane mirror to the angle of reflection.*
5. *Solve problems involving the number of images in two mirrors inclined at an angle.*

14.2 Reflection

Definition: *Reflection is the change in the direction of light ray or a beam of light after impinging on a surface.*

(a) Types of reflecting surfaces

Reflection occurs on two types of reflecting surfaces namely;

(i) Plane surface e.g. plane mirrors.
(ii) Curved surfaces e.g curved mirrors

(b) Reflection on plane surfaces

Terms used

The figure 14.21 below illustrates the terms used in the study of reflection of light.

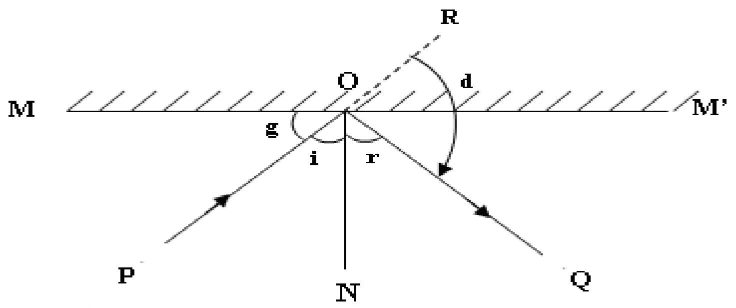


Figure 14.21

MM' - represents the plane mirror

PO - *Incident ray* is the ray of light falling to the reflecting surface**.**

OQ - *Reflected ray* is the ray of light that leaves the reflecting surface after reflection.

ON (Normal) - is a perpendicular line that meets the reflecting surface.

Point O - *Point of incidence* is the point on the reflecting surface where the incident ray, the reflected ray and the normal all meet.

$\angle$**PON =** *Angle of incidence* - is the angle made by incident ray and the normal.

$\angle$**NOQ =** *Angle of reflection* - is the angle made by the normal and the reflected ray.

$\angle$**MOP =** $\angle$**M'OP** = *Glancing angle, g:* - is the angle between the incident ray or reflected ray and the reflecting surface.

$\angle$**ROQ** Angle of deviation, ***d***.
The angle of deviation is the angle between the reflected ray and the original path of the incident ray.

14.22 Types of reflection

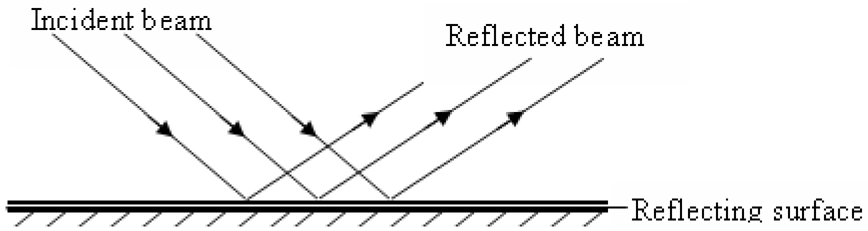
There are two types of reflection namely;

(i) Regular Reflection

(ii) Diffuse (Irregular) reflection

(a) Regular Reflection

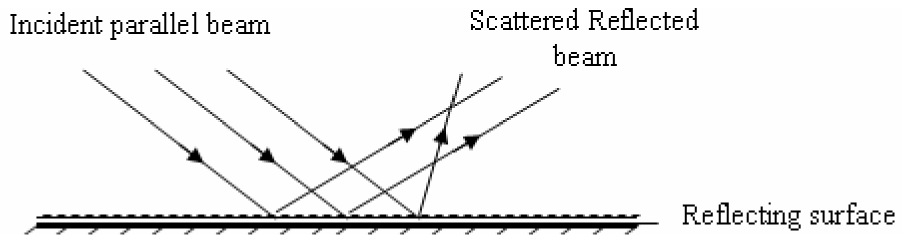
Regular reflection is the type of reflection in which a parallel beam is reflected parallel. It occurs on smooth reflecting surfaces and gives clear images of objects thus allowing them to be recognized.



(b) Diffuse (Irregular) reflection

Diffuse or Irregular Reflection is the type of reflection in which a parallel beam is scattered after reflection.

It occurs on rough reflecting surfaces. Because the reflected beam is scattered after reflection it does not form clear images of objects.



Difference between Regular and Diffuse Reflections

Regular Reflection	Diffuse Reflection
(i) Parallel incident beam is reflected parallel	- Parallel incident beam is scattered after parallel
(ii) Forms clear images of objects	- Forms blurred image of objects due to the scattering reflected rays
(iii) Occurs on smooth surfaces	- Occurs on rough surfaces
(iv) The angles of incidence and reflection are **equal** for the incident rays and reflected rays	- The angles of incidence and reflection are **not** equal for the incident rays and also **not** equal for the reflected rays
(v) The amount of light intensity after reflection is high since the reflected beam remains parallel	- The amount of light intensity after reflection is low due to the scattering of reflected rays

Note: *1. Not all of the light that strikes a mirror is reflected; some of it absorbed by the reflecting object and the object becomes warmer.*

2. The laws of reflection are obeyed for each ray.

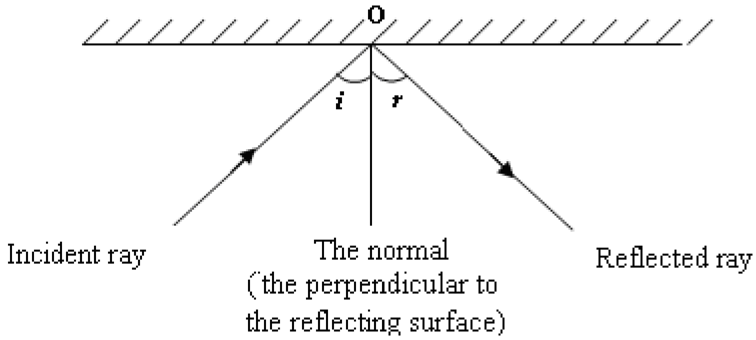
14.23 The Laws of Reflection

The reflection of light from a reflecting surface is governed by two laws. These laws are called *laws of reflection*. They are stated as follows:

LAW 1 *The incident ray, the reflected ray and the normal at the point of incidence all lie in the same plane.*

LAW 2 *The angle of incidence is equal to the angle of reflection.*

The situation is illustrated in the figure below.



Where: *i = Angle of incidence,* *r = Angle of reflection*

According to ***law 1***, point of incidence, **O**, which lies in the plane formed by the reflecting surface, is a common point for the incident ray, the reflected ray and the normal That is to say, at that, all the three lie in the same plane.
Law 2 is about the angles made by the incident ray and the reflected ray with the normal. That is $\angle \boldsymbol{i} = \angle \boldsymbol{r}$. However, the laws of reflection can be verified experimentally.

Experiment 14.1 To verify the laws of reflection

Apparatus: A strip of a plane mirror, 4 optical pins, 4 office pins, a white piece of paper and drawing board.

Procedure

- Fix the white sheet of paper on the drawing board using the office pins.
- Draw a straight thin pencil line MM' as shown in figure 14.23.

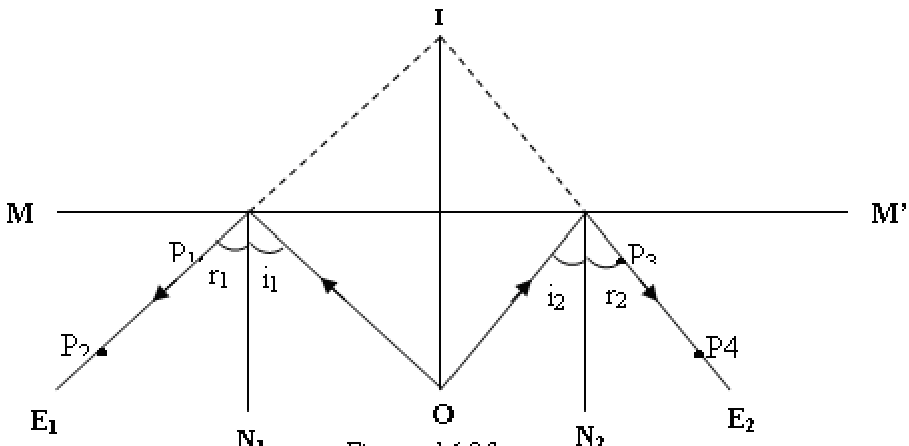


Figure 14.23

- Stand a strip of a plane mirror up vertically, with the silvered surface on the line MM' drawn on the paper.
- Fix object pin O, in front of the mirror about 10cm from line MM'.
- View the image, I, of the object, O, by placing your eye at position E_1 and fix two pins P_1 and P_2 such that they are collinear (on a straight line) with I.

- Remove the two pins P_1 and P_2.
- Repeat the above procedure with the eye in position E_2.
- Remove the mirror.
- Join the points P_1, P_2, P_3 and P_4 by to meet MM' at B_1 and B_2.
- Extrapolate the lines to intersect at I.
- Construct the normals, B_1N_1 and B_2N_2.
- Measure the angles i_1 and r_1, i_2, and r_2.

Results:
1. The incident rays (OB_1 and OB_2), the reflected rays (B_1E_1 and B_2E_2), and the normals N_1 and N_2 at the points of incidence B1 and B2 all lie in the same plane.
2. The angle of incidence, i_1 = Angle of reflection r_1 and
   The angle of incidence, i_2 = Angle of reflection r_2

Conclusion: The laws of refraction are verified.

(a) Image Formation by Plane Mirrors

The formation of an image by a plane mirror is illustrated in the figure 14.24 below.

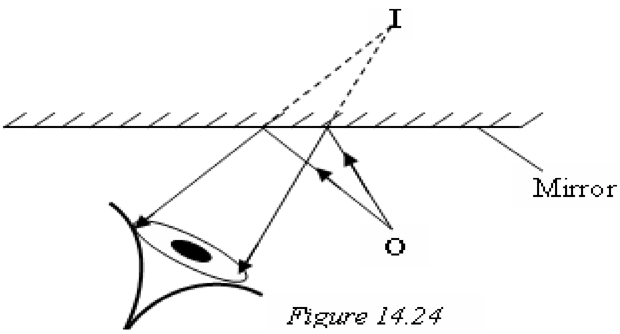


Figure 14.24

Rays of light from a real object **O** are reflected by the mirror and enter the eye in such a way that they appear to have come from **I**. It follows that the image of the object is at **I**. Since the rays appear to have come from **I**, the image is a *virtual image*.

(b) To show that the Line Joining a Point Object and its Image is Perpendicular to the Mirror

Consider the diagram in figure 14.25 below in which I is the image of point object O, and ON is perpendicular to the mirror.

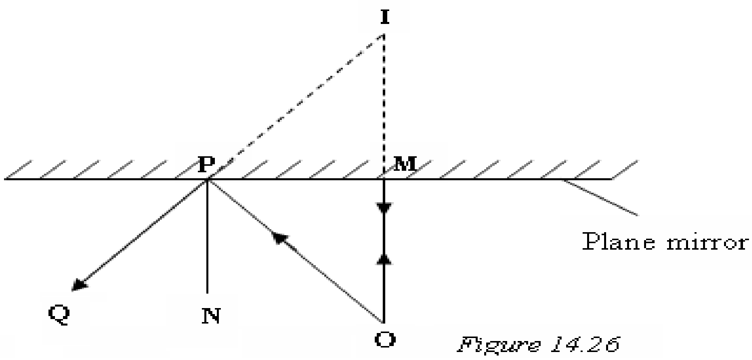


Figure 14.26

It follows from the second law of reflection that a ray incident along **OM** must be reflected back along **MO**. Since this reflected ray appears to have come from the image, **I**, it follows that **OMI** are collinear (i.e. are on a straight line) and therefore, **OI** is perpendicular to the mirror. That is to say, the line joining object and image is perpendicular to the mirror (reflecting surface). Also by measurement, **OM** = **MI.** This shows that the image is as far behind the mirror as the object is in front of it.

(c) Image Location by No-parallax

The method of no-parallax is used in light experiments to locate the images of objects formed in a mirror.

In this method the eye is moved from side to side while viewing a search pin until a position is found for which both the object pin and the image appear to coincide in the same straight line.

(d) Rotation of a Plane Mirror

Consider the diagram below in which a plane mirror M_1M_1' is rotated through an angle.

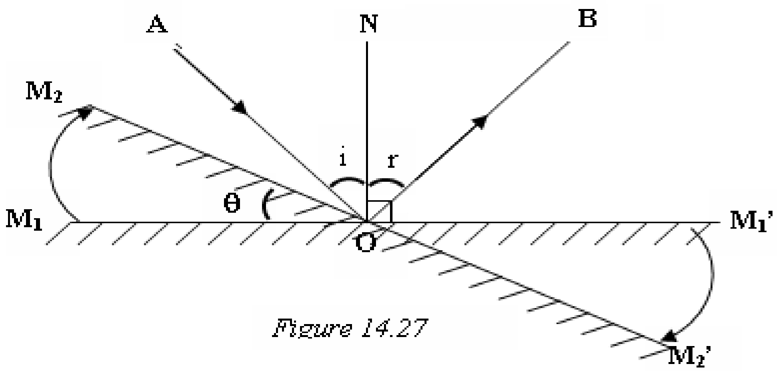


Figure 14.27

M_1M_1' = Position of mirror before rotation

M_2M_2' = Position of mirror after rotation

Angle, θ = Angle of rotation of the mirror

= $\angle \mathbf{M_1OM_2} = \angle \mathbf{M_1'OM_2'}$

When a plane mirror is rotated through an angle, θ, while keeping the direction of the incident ray along the same path, it can be proofed that:

$$\begin{pmatrix} \text{Angle of rotation} \\ \text{of the reflected ray} \end{pmatrix} = \begin{pmatrix} \text{Twice the angle of} \\ \text{rotation of the mirror} \end{pmatrix}$$

I.e. The angle of rotation of the reflected ray = 2θ

Derivation of the formula

The incident ray, AO, makes an angle, i, with the normal. The angle between the angle of incidence and the angle of reflection, AOB

= AON + NOB

= i + r   But by the second law of reflection i = r

= i + i

= 2i 1

If the mirror is rotated through an angle, θ, the normal must also turn through the same angle of rotation, θ. This gives a new angle of incidence (i + θ).

But by the second law of reflection, *the angle of incidence = angle of reflection.*

Therefore, the reflected ray has also turned through an angle (i + θ).

Hence, the new angle between the angle of incidence and the angle of reflection

= Angle of incidence + Angle of reflection

= (i + θ) + (i + θ)

= i + θ + i + θ

= 2i + 2θ 2

As the incident ray remains in the same path, the angle of rotation of the reflected ray is given by the formula:

$$\begin{pmatrix}\textit{The angle of rotation}\\ \textit{of the reflected ray}\end{pmatrix} = \begin{pmatrix}\textit{Angle between the incident}\\ \textit{ray \& the reflected ray}\\ \textit{after rotation}\end{pmatrix} - \begin{pmatrix}\textit{Angle between the}\\ \textit{incident ray \& the}\\ \textit{reflected ray}\end{pmatrix}$$

= Equation (2) - Equation (1)

$= 2i + 2\theta - 2i$

$= 2\theta$

Thus when a mirror is rotated through an angle, θ,

The angle of rotation of the reflected ray = twice the angle of rotation of the mirror

(e) Properties (Characteristics) of Images formed in Plane Mirrors

The image formed in a plane mirror has the following properties.

(i) The image is the same size as the object.
(ii) The image is as far behind the mirror as the object is in front.
(iii) The image is lateral inverted. See figure 14.28 below.
(iv) The image is virtual (can not be obtained on film) for a real object.

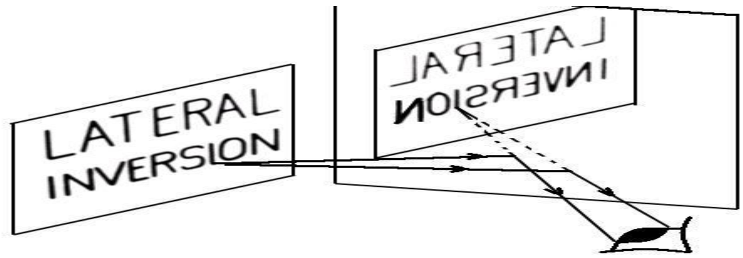


Figure 14.28

(f) Formation of a Real Image from a Virtual Object by a Plane Mirror

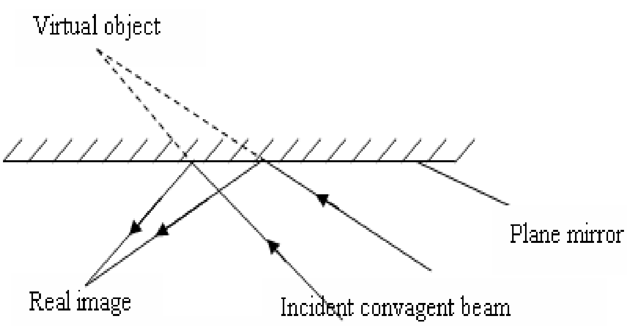


Figure 14.29

We have seen that a plane mirror produces a virtual image of a real object. It follows fom the principle of reversibility of light that a plane mirror must produce a real image of a virtual object as shown in the figure 14.29.

(g) The Number of Images formed by Two Mirrors Inclined at an Angle θ

When an object is placed between two mirrors inclined at an angle,θ, a number, n, of images is formed. The value of n is calculated from the formula:

$$n = \left(\frac{360°}{\theta}\right) - 1$$

Examples

Find the number of images of an object placed between two mirrors inclined at:

(a) 90° (b) 60° (c) 45° (d) 0°

Solution **(a)** $\theta = 90°$, n =? **(b)** $\theta = 90°$, n =?

Using $n = \left(\frac{360°}{\theta}\right) - 1$ $\qquad n = \left(\frac{360°}{\theta}\right) - 1$

$$= \frac{360°}{90°} - 1$$
$$= 4 - 1$$
$$\therefore \textbf{ n } = \textbf{3}\therefore$$

$$= \frac{360°}{60°} - 1$$
$$= 6 - 1$$
$$\therefore \textbf{ n } = \textbf{5}$$

(c) θ = 45°**,** n =?

$$n = \left(\frac{360°}{\theta}\right) - 1$$
$$= \frac{360°}{45°} - 1$$
$$= 8 - 1$$
$$\therefore \textbf{ n } = \textbf{7}$$

(d) θ = 0°, n =?

$$n = \left(\frac{360°}{\theta}\right) - 1$$
$$= \frac{360°}{0°} - 1$$
$$= \frac{360}{0}$$
$$\therefore \textbf{ n } = \infty$$

Formation of Image when the angle of Inclination is zero (i.e. θ = 0)

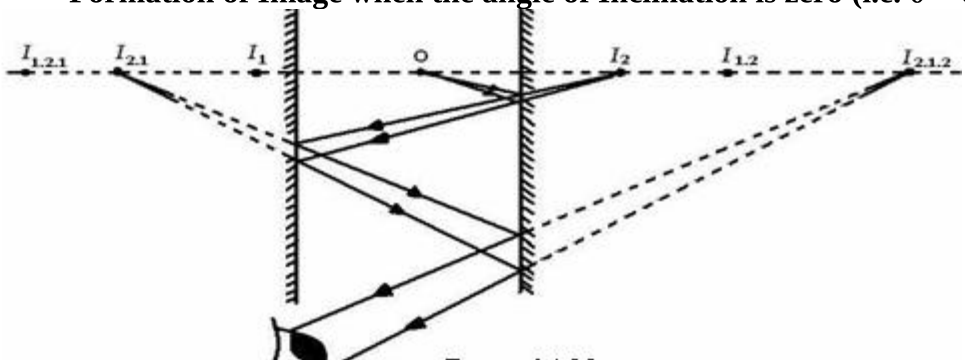


Figure 14.30

14.24 Applications of Reflection in Plane Mirrors

Reflection in plane mirrors is applied in the following:

(i) Periscope
(ii) Kaleidoscope
(iii) Saloons and Dressing rooms for admiration
(iv) Small vehicles by drivers to see the activities of passengers aboard.
(v) Bicycles for seeing traffic behind.

(a) **Periscope**, *optical instrument for conducting observations from a concealed or protected position.*

(i) Structure

A simple periscope consists essentially of reflecting mirrors at opposite ends of a tube with the reflecting surfaces parallel to each other, and at a 45° angle to the axis of the tube.

(ii) Mechanism

A ray of light from an object is successively reflected by the two mirrors inclined at 45° so that the image of an object on the other side of an obstacle is seen by an observer. See figure 14.31 below.

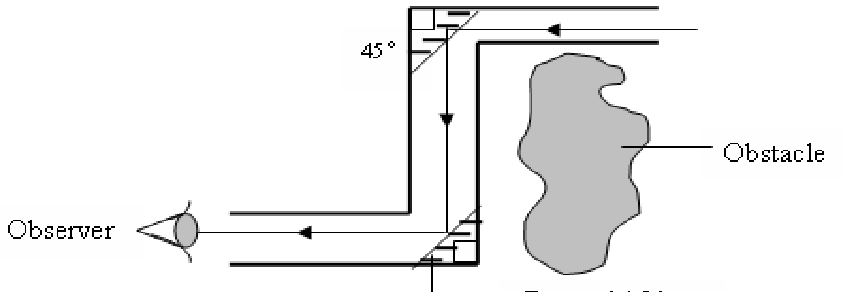


Figure 14.31

Periscopes are commonly used in:

- National theatres,
- Football pitches and
- Submarines

Notes: *- The modern submarine periscope is a larger and more complex instrument.*
- It consists of reflecting prisms at the top and bottom of the vertical periscope tube, with two telescopes and several lenses between them, and an eyepiece at the lower end.

Figure 14.32 shows a photograph of a cross section of a submarine periscope

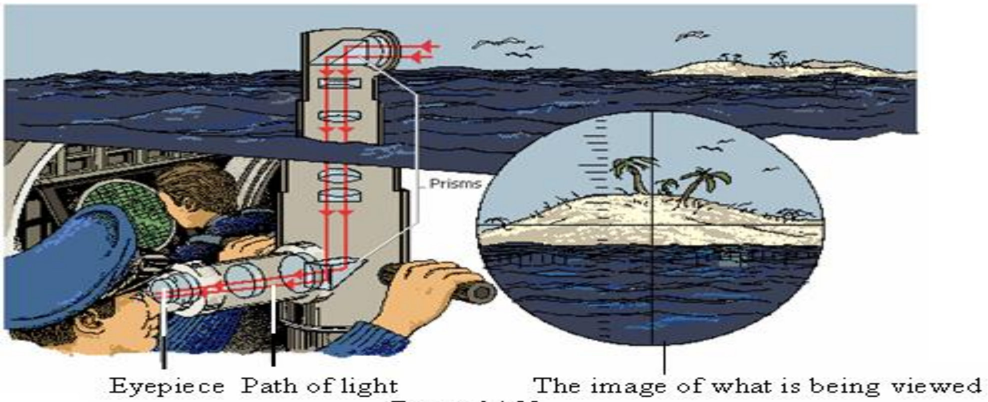


Figure 14.32

(b) **Kaleidoscope**, *optical device that uses mirrors and objects to create a variety of colorful patterns.*

(i) Structure

The Kaleidoscopes consists of three mirrors inclined at 60-degrees to each other and is fixed inside light proof tube with a small hole in one end. At the bottom of the tube is a ground glass plate to admit light into the tube.

(ii) How it works

Beads, colored glass, pieces of plastic, or mixtures of oil and water contained loosely between two glass or plastic disks is placed at the bottom of the kaleidoscope. Light enters the tube through the translucent glass or plastic, reflecting the images of the mirrors inside the tube giving multiple symmetrical images of objects inside the tube. When the tube is rotated, the tumbling motion of the items inside the compartment changes the patterns seen through the eyepiece.

Self-Check 14.2

1. Light energy is reflected when
   A. angle of incidence is greater than angle of reflection
   B. angle of incidence is equal to angle of refraction
   C. angle of incidence is equal to angle of reflection
   D. the normal at the point of incidence makes the same angle as the incident ray.

2. When reflection occurs in a plane mirror
   (i) the image is real, erect and magnified
   (ii) the angle of reflection is equal to the angle of incidence
   (iii) the incident ray and reflected ray lie in different planes
   (iv) the object and the image are at the same distance from the mirror
   A. (i), (ii) and (iii) only B. (ii) and (iv) only
   C. (i), (ii) and (iv) only D. (iv) only

3. An object is placed 30 cm in front of a plane mirror. If the mirror is moved a distance of 6 cm towards the object, find the distance between the object and its image.
   A. 24 cm B. 36 cm C. 48 cm D. 60 cm

4. Objects P and Q are placed at distances of 2 m and 3 m respectively from a plane mirror as shown in figure 14.1. Find how far the image of P is from Q.

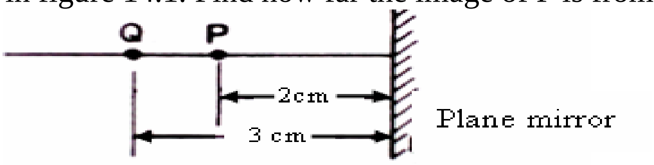


Figure 14.1

   A. 1m.
   B. 4m.
   C. 5m.
   D. 7m.

5. A person observes the image of a pin placed in front of a plane mirror as shown in the figure 14.2 below. The reflected beam from the pin reaching the observer is a

Figure 14.2

   A. virtual beam
   B. divergent beam
   C. parallel beam
   D. convergent beam

SECTION B

6. a) With the aid of diagrams, distinguish between diffuse and regular reflection.
   b) (i) State the laws of reflection.
   (ii) Describe an experiment to verify the laws of reflection of light.

7. (a) State the applications of reflection in plane mirrors.
   (b) Describe the structure and the mechanism of the periscope.

RFLECTION IN CURVED MIRRORS

LEARNING OBJECTIVES

By the end of this topic you should be able to:

1. *State the types of curved mirrors.*
2. *Define the terms used in curved mirrors.*
3. *(i) State the principal rays for constructing ray diagrams.*
   (ii) Draw and state the properties of images in curved mirrors.
4. *Solve problems using the mirror formula.*

14.31 Curved Mirrors

Curved mirrors are mirrors whose surfaces are obtained from a hollow transparent sphere or pipe and silvering one of the side.

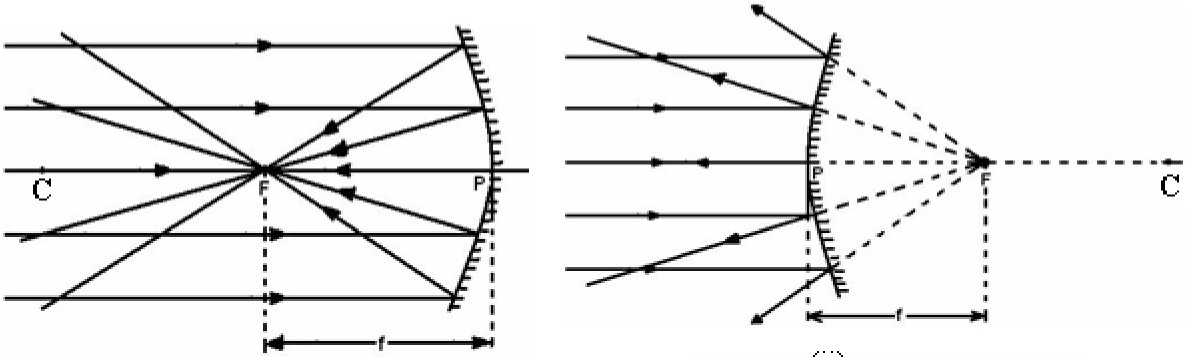
(a) Types of Curved Mirrors

There are two types of curved mirrors, namely:

(i) Concave mirror
(ii) Convex mirror

(i) Concave mirror (ii) Convex mirror

(b) Reflection of parallel beams from curved mirrors



(i) Reflection of parallel beams from Concave mirrors

Figure 14.31

(ii) Reflection of parallel beams from Convex

(c) Terms Used

(i) *The Centre of curvature*, C, is the centre of the sphere of which the mirror is a part.

(ii) *The radius of curvature*, r, is the radius of the sphere.

(iii) *The pole, P*, of a mirror is the centre of reflecting surface.

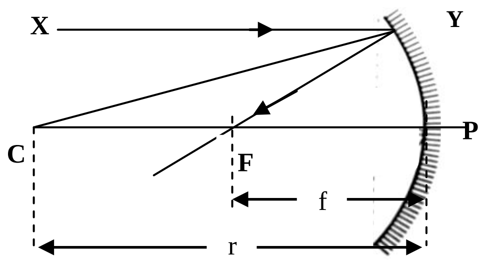
(iv) *The principal axis* is the imaginary line joining the pole and the centre of curvature produced in each case.

(v) *The principal focus, F*, is the point on the principal axis through which rays parallel and close to the principal axis pass after reflection (for the case of concave mirror) or from which the rays appear to come (for the case of convex mirror).

(vi) *The focal length, f,* is the distance between the principal focus and the pole of the mirror.

***Notes**: - **C** and **f** of a concave mirror - are real (i.e. are in front of the mirror).*
*- **C** and **f** of a convex mirror - are virtual (i.e. are behind the mirror).*

(d) The Relation between f and r



BF = PF

From the laws of reflection

$\angle ABC = \angle CBF$

But $\angle ABC = \angle BCF$ *(alternate angles)*

$\angle CBF = \angle BCF$

$\therefore BF = FC$ *(sides of an isosceles triangle)*

Now B is very close to P

PF = FC

$\therefore \boldsymbol{f} = \frac{r}{2}$ or $\boldsymbol{r = 2f}$

(e) Mirrors of Large aperture

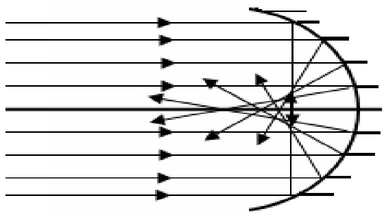
If a parallel beam parallel to the principal axis is incident on to a concave mirror, the rays which are close to the principal axis are reflected to pass through the principal focus. While those which are not close to the principal axis are refledted in such a way that they subsequently cross the axis at points which are closer to the mirror than its principal focus. The reflected rays intersect to form a surface called *caustic surface*.

Examples of bright caustic surface can be seen:

(i) on surfaces of liquids contained in vessels of circular cross section e.g. tea surface on tea surface.

(ii) at the bottom of clean containers such as cups, source pans, basins etc.

Diagram showing formation of Caustic Surface



(i) Parabolic Mirror

A parabolic mirror has the property of reflecting the light source placed at the focal point parallel to the principal axis with undiminished intensity.

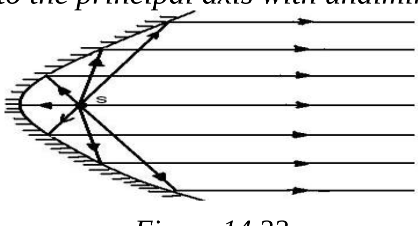


Figure 14.32

(ii) Uses of Parabolic Mirrors

Because of their property of reflecting light rays from a lamp placed at the focus parallel with a diminishing intensity, they are used as search-light reflectors.

14.32 Construction of ray diagrams

The ray diagrams for curved mirrors are drawn by using any two of the principal rays. The images formed are described by using the technical terms.

(a) Terms used to describe images formed in Curved Mirrors

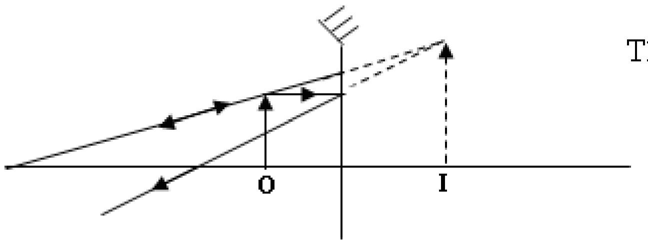
(i)	Real	- Formed by intersection of actual rays and can be obtained on screen.
(ii)	Virtual	- when not actual rays of light intersect after reflection
(iii)	Inverted	- Upside down
(iv)	Diminished	- Smaller than the object and
(v)	Magnified	- Bigger/larger than the object.
(vi)	Erect	- Upright

(b) The principal rays for a Concave Mirror

1. A ray parallel to the principal axis is reflected through the focal point.
2. A ray passing through the focal point is reflected parallel to the principal axis.
3. A ray passing through the centre of curvature is reflected back along the same path.

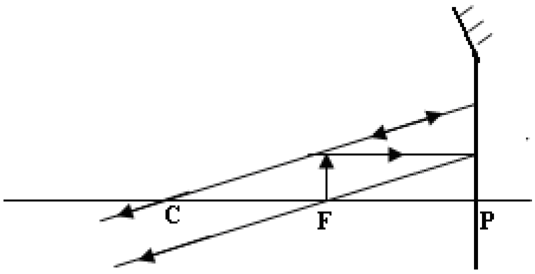
Ray diagrams for concave mirror for various positions of object

(i) Object between F and P



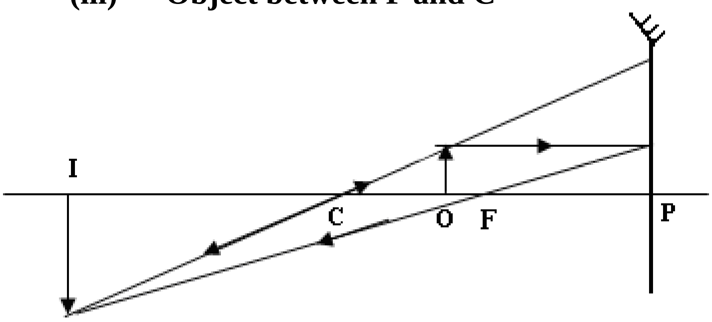
The nature of the image is
- erect
- magnified
- Virtual
- Behind the mirror

(ii) Object at F



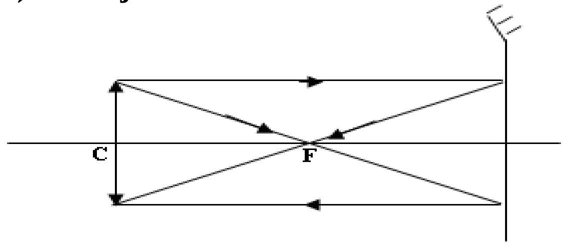
The nature of the image is
- real
- at infinite

(iii) Object between F and C



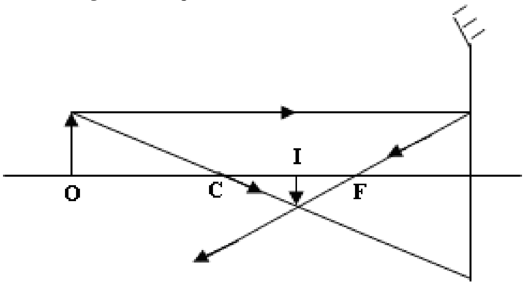
The nature of the image is
- real
- magnified
- Inverted
- Beyond C

(iv) **Object at C**



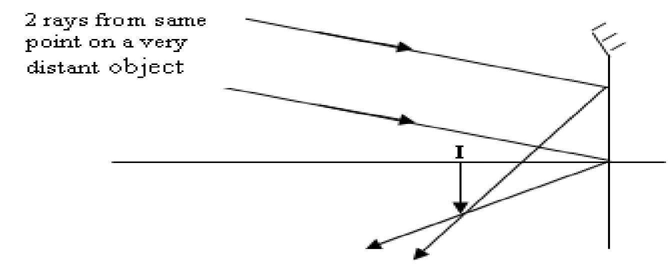
The nature of the image is
- Real
- Same size as object
- Inverted
- At C

(v) **Object beyond C**



The nature of the image is
- Real
- Inverted
- Diminished
- Between C and F

(vi) **Object at infinite**



The nature of the image is
- Real
- Inverted
- Diminished
- At F

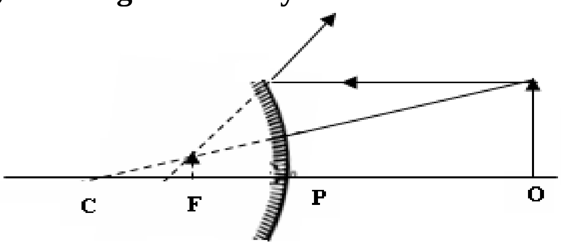
(c) **The principal rays for a Convex Mirror**

1. *A ray parallel to the principal axis is reflected to appear as though it passed through the focal point in other words the reflected ray, if produced, would pass through the focal point.*
2. *A ray which if produced would pass through the focal point, is reflected parallel to the principal axis.*
3. *A ray which if produced, would pass through the centre of curvature, is reflected back along the same path.*

Ray diagrams for convex mirror

For any position of object in front of the mirror, the nature of the image is:
- Erect
- Virtual
- Diminished
- Between P and F

(d) **Image formed by a Convex Mirror**



14.33 The Mirror Formula

The mirror formula is given by the relation:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Since r = 2f, the mirror formula can also be written as:

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r}$$

$$\text{Where:} \quad \left.\begin{array}{l} u = \text{object distance} \\ v = \text{image distance} \\ f = \text{focal length} \\ r = \text{radius of curvature} \end{array}\right\} \text{all measured from the pole of the mirror}$$

In order to distinguish between real and virtual image and the two types of curved mirrors, it is necessary to employ a *sign convention*.

Sign Convention

Real-is-Positive Convention

1. All distances are measured from the mirror as the origin.
2. Distances of real objects and images are positive.
3. Distances of virtual objects and images are negative.
4. Focal length, *f*, for concave mirror is positive and negative for convex mirror.

Examples

1. An object 1 cm tall is placed 30 cm in front of a convex mirror of focal length 20 cm. Find (i) the position (ii) the size of the image formed.

Solution: h_o = 1 cm, h_i = ?, u = 30 cm, v = ?, f = -20 cm (convex mirror)

(i) $$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-20} - \frac{1}{30} = \frac{-3-2}{60} = \frac{-5}{60} = \mathbf{-12\ cm}$$

The minus sigh means that virtual image is formed 12 cm behind the mirror

(ii) $$\frac{\text{Image Height}}{\text{Object height}} = \frac{\text{Image distance from the mirror}}{\text{Object distance from the mirror}}$$

$$\frac{h_i}{h_o} = \frac{v}{u} \Rightarrow \frac{h_i}{1} = \frac{12}{30} \qquad \therefore \mathbf{h_i = 0.4\ cm}$$

2. A small object is placed on the principal axis of a convex mirror of radius of curvature of 20 cm. Determine the position of the image when the object is 15 cm from the mirror.

Solution: r = 20 cm, u = 15 cm, v = ?, f = $\frac{r}{2} = \frac{20}{2}$ = -10 cm (convex mirror)

` $$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-10} - \frac{1}{15} = \frac{-3-2}{30} = \frac{-5}{30} \quad \therefore \mathbf{v = -6}$$

cm

The minus sigh means that virtual image is formed 6 cm behind the mirror

3. A concave mirror of radius of curvature 20 cm forms an erect image 30 cm from the mirror and 5 cm tall.

Find (i) Position and
(ii) Size of the object.
(iii) Hence or otherwise, find the magnification

Solution: $h_i = 5$ cm, $u = ?$, $h_o = ?$, $v = -30$ cm, $m = ?$, $r = 20$ cm,

$$f = \frac{r}{2} = \frac{20}{2} = 10 \text{ cm (concave mirror)}$$

(i)
$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{u} = \frac{1}{f} - \frac{1}{v} = \frac{1}{10} - \frac{1}{-30} = \frac{1}{10} + \frac{1}{30} = \frac{3+1}{30} = \frac{4}{30} \quad \therefore \mathbf{u = 7.5\ cm}$$

(ii)
$$\frac{Image\ Height}{Object\ height} = \frac{Image\ distance\ from\ the\ mirror}{Object\ distance\ from\ the\ mirror}$$

$$\frac{h_i}{h_o} = \frac{v}{u} \qquad \Rightarrow \frac{5}{h_o} = \frac{30}{7.5} = \frac{5 \times 7.5}{30} \qquad \therefore \mathbf{h_i = 1.25\ cm}$$

(iii)
$$\mathbf{m} = \frac{h_i}{h_o} = \frac{5}{1.25} = \mathbf{4}$$

or
$$\mathbf{m} = \frac{v}{u} = \frac{30}{7.5} = \mathbf{4}$$

14.34 Applications of Curved Mirrors

Curved mirrors are applied in various fields depending on the type of the mirror.

(a) Convex Mirror

Convex mirrors are used in:

(i) locomotives as driving mirrors for seeing traffic behind.
(ii) They are used to observe the activities of customers in supper markets for.

They are preferred to plane mirrors and concave mirrors for this purpose because of the following advantages.

(i) They form erect image
(ii) They have wider field of view.

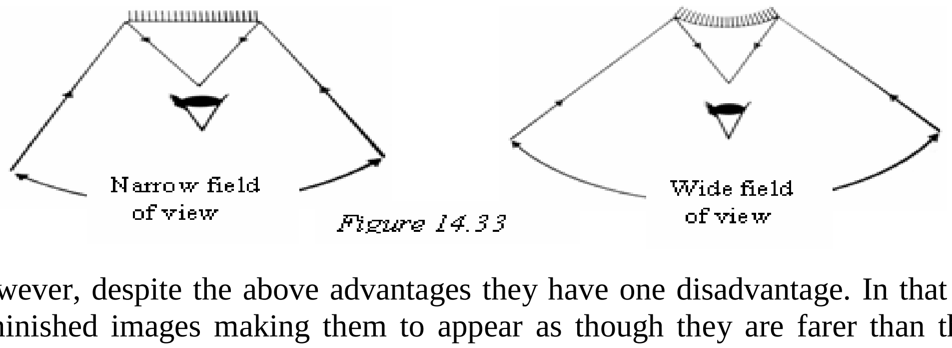


Figure 14.33

However, despite the above advantages they have one disadvantage. In that they form diminished images making them to appear as though they are farer than their actual distances from the mirror, hence, leading to accidents.

(b) Concave Mirrors

Concave mirrors are used as:

(i) dentist's mirror,
(ii) shaving mirror and

(iii) Reflectors in - telescopes for seeing far objects such as stars and planets.

- search lights, car head lights, torch light.

Advantages

For uses (i) and (ii), the image formed is:

- erect and
- magnified.

Note: *1. In uses (i) and (ii), the mirror is held so that the object distance is less than the focal length of the mirror.*

2. They are usually made of plastic or metal.

Self-Check 14.3

1. A concave mirror can be used as a shaving mirror because when an object is placed between the focus and the pole, the image formed is,
   A. magnified, virtual and erect
   B. magnified, real and inverted
   C. diminished, real and inverted
   D. diminished, virtual and erect

2. Which of the following information is true about the concave and convex mirrors?

	Concave mirror	Convex mirror
A	Converges light	Diverges light
B	Diverges light	Converges light
C	Refracts light	Reflects light
D	Has a wide field of view	Has a narrow field of view

3. The focal length of a concave mirror is the
   A. distance between the pole of the mirror and the focal point
   B. distance between the center of curvature and the mirror
   C. distance between the object and the image
   D. diameter of the mirror

4. (a) With the aid of the diagram explain why a parabolic mirror is most suitable for use in car head lights.
   (b) List three uses of a concave mirror.

5. (a) Draw a ray diagram to show the formation of an image of an object O placed in front of a convex mirror shown in the figure below. F is the principal focus of the mirror.
   (b) A convex mirror whose radius of curvature is 30 cm forms an image of a real object which has been 20 cm from the mirror.
   Calculate: (i) The position of the image
   (ii) The magnification produced.
   (c) Give reasons for use of convex mirrors in vehicles.

14.4 REFRACTION OF LIGHT

LEARNING OBJECTIVES

By the end of this topic you should be able to:

1. *Define the term refraction.*
2. *State the laws of refraction.*
3. *(i) Define refractive index,* ***n*** *of a material.*
   (ii) Describe an experiment to determine the refractive index of a glass block.
4. *(a) (i) Define the term critical angle.*
   (ii) Explain total internal reflection.
   (b) State (i) the applications of internal reflection.
   (ii) Some effects due to total internal reflection.

14.42 Refraction

Refraction is the change in the direction of light when it travels from one medium to another.

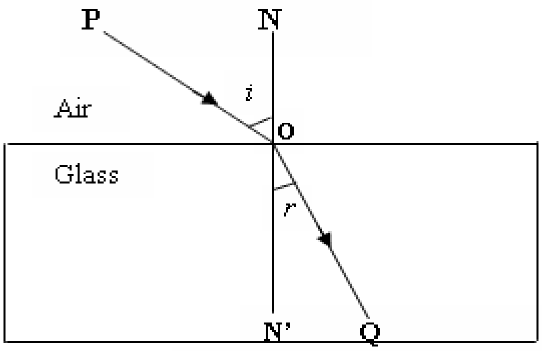
(a) Laws of Refraction

Law I States that:

The incident ray and refracted ray are on the opposite sides of the normal at the point of incidence, and all the three are in the same plane.

Law II States that:

At any boundary between any two given media, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant for rays of particular wavelength.



OP - *Incident ray*
OQ - *Refracted ray*
NN' - *Normal*
i - *Angle of incidence*
r - *Angle of refraction*

Figure 14.41

(b) (i) Refractive Index, n

It follows from the second law of refraction that:

$$\frac{Sin\,i}{Sin\,r} = \text{Constant}$$

The ratio is known as the relative refractive index, ${}_1n_2$, of the two media. Therefore, *Snell's* law can be written as: $\frac{Sin\,i}{Sin\,r} = {}_1n_2$

(ii) Absolute Refractive Index

The absolute refractive index of a medium is defined as

$$n = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$$

By analogy, the absolute refractive index, n, of medium 1 is given by

$$n_1 = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium 1}} \quad \text{.......1}$$

And the absolute refractive index, n_1, of medium 2 is given by

$$n_2 = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium 2}} \quad \text{.......2}$$

Dividing equation 1 by 2 we have: $$\frac{n_1}{n_2} = \frac{\text{Speed of light in vacuum 2}}{\text{Speed of light in medium 1}}$$

Or $$\frac{n_2}{n_1} = \frac{\text{Speed of light in vacuum 1}}{\text{Speed of light in medium 2}}$$

$$\frac{n_2}{n_1} = {}_1n_2$$

Therefore, $$\frac{Sin\,i}{Sin\,r} = \frac{n_2}{n_1}$$ Or n_1**Sin *i* = n_2Sine r**

Note: *The absolute refractive index of vacuum is equal to 1 and that of air at normal atmospheric pressure and 20°C is 1.0003. This difference is so small that it is normally ignored; hence the absolute refractive index of air is taken to be 1 as that of vacuum.*

From the equation n_1 **Sin *i* = n_2Sine r**

It follows that:

If (i) $n_2 > n_1$, the ray bends towards the normal as it enters the glass and the angle $i > r$.

(ii) $n_1 > n_2$, the ray bends away from the normal as it leaves the glass., then $i < r$.

Thus when light travels from a medium into one which has a greater absolute refractive index, it bends towards the normal as shown in figure 14.42 below.

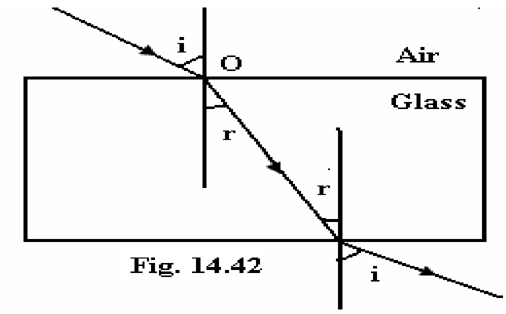


Fig. 14.42

Note:

- *If the glass block has parallel sides, the emergent ray is parallel to the incident ray, but it is laterally displaced..*
- *This means that the ray is traveling in the same direction but it has been shifted sideways when it emerges.*
- *When the incident ray meets a refracting surface at a right angle, it is not refracted at all.*

Experiment 14.41 **To determine the refractive index, *n*, of a glass block**

Apparatus

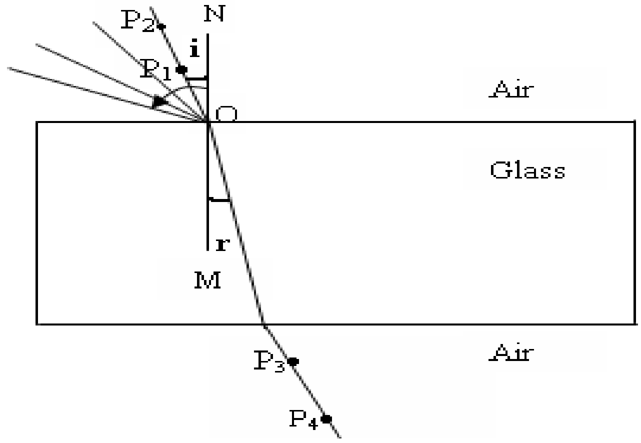
A protractor, 4 optical pins, 4 office pins, a white piece of paper and a drawing board

Procedure:

(a) Fix a white piece of paper on a drawing board using the 4 office pin.

(b) Trace accurately the outer margin of the glass block on the paper.

(c) Using a protractor, draw normal NM, in a position shown in figure 14.4, and measure from it several angles of incidence, i.



(d) Accurately aim the incident ray at O, and for each angle of incidence mark the directions
of the emergent ray by fixing 2 pins P_3 and P_4.

(e) Remove the glass block.

(f) Draw in the emergent and refracted rays.

(g) Measure the angle of refraction, r.

(h) Replace the glass block.

(i) Repeat the procedures (d) to (g) for values of $i = 30°, 45°$ and $60°$.

(j) Enter the values of the angles *i* and *r* in table 14.41 including the values of Sin *i* and Sin r.

Table 14.41

i/degree	r/degree	Sin i	Sin r	$\frac{Sin\,i}{Sin\,r}$
15		0.26		
30		0.50		
45		0.71		
60		0.87		

(k) Calculate the mean value of $\frac{Sin\,i}{Sin\,r}$.

Note: *If the experiment is accurately performed, the mean value for air to glass is about 1.5.*

Values of n for some common media

Material	Absolute refractive index n
Air	1 (1.0003)
Glass - Flint	1.65
- Crown	1.52
Water	1.33

Table 14.41

14.43 Total Internal Reflection and Critical Angle

(a) Total Internal Reflection

When light travels from an optically more dense medium to an optically less dense medium (e.g from glass to air), it undergoes both refraction and weak reflection.

Consider a ray PQ incident in glass-air boundary shown in figure 14.43 below.

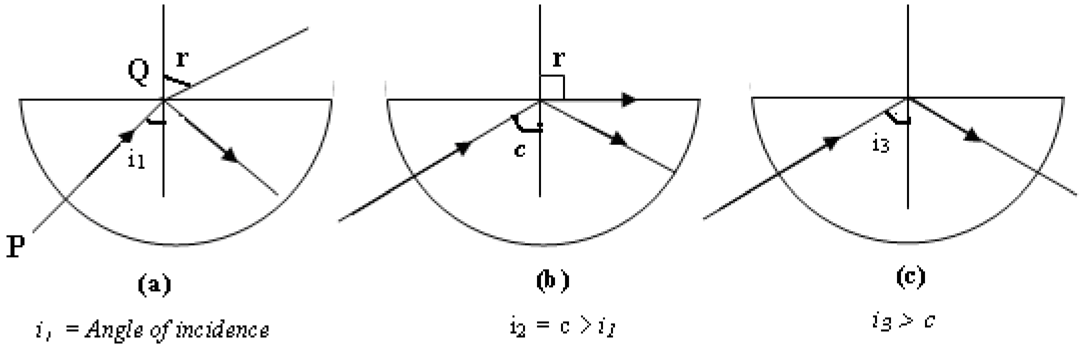


Figure 14.43

When the angle of incidence, *i*, is gradually increased, the angle of refraction, r, also increases and the weak reflected ray becomes stronger and stronger. A point is reached for which the angle of incidence is such that the angle of refraction is 90^o as in diagram (b). The angle of incidence for which the angle of refraction is 90^o is called *critical angle, c*.

If the angle of incidence, *i*, becomes greater than the critical angle (i > r), the light ray is totally reflected back into the medium as shown in diagram (c). This is known as ***Total Internal Reflection***.

(b) Critical Angle, *c*

Critical angle of a medium is the angle of incidence when a ray passes from optically denser medium into a less dense optical medium at an angle of refraction of 90°.

(c) The Relationship between c and n

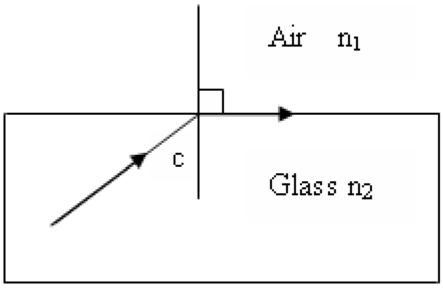


Figure 14.44

Consider the diagram below.

Applying ***nSin i = Constant***

at the glass-air boundary we have **:**

$n_1 \text{Sin } 90^o = n_2 \text{Sin } c$ But Sine $90^o = 1$

$n_1 \times 1 = n_2 \text{Sine } c$

$\text{Sine } c = \frac{n_1}{n_2}$ *But n1 = 1*

$$\therefore \textbf{Sine } c = \frac{1}{n_2}$$

Where: *c* = Critical angle of glass for a glass-air boundary ≈ 42°

n = the refractive index of the medium in which c is found.

Refractive indices and Critical Angles for Crown glass and Water

Material	Refractive index, n	Critical angle, c
Crown glass	1.5	42°
Water	1.33	49°

Table 14.42

Worked Examples

1. The refractive index of glass is 1.5. Find the critical angle.

Solution: n = 1.5, c = ?

Substuting in $Sin\ c = \frac{1}{n} = \frac{1}{1.5} = 0.6667$

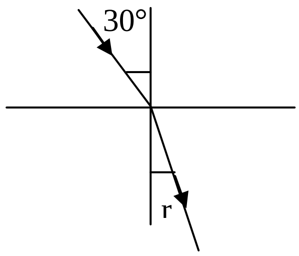
$c = \text{Sin}^{-1}\ 0.6667$

$\therefore \mathbf{c = 41.8°}$

2. A ray of light is incident in air at an angle of 30°. Find the value of angle of refraction, r, if the refractive index is 1.5.

Solution $n_g = 1.5$, $n_a = 1$, i = 30°, r = ?

Substituting in nSin i = constant



$n_a\text{Sin}\ i = n_g\text{Sin r}$

$1 \times \text{Sin } 30° = 1.5 \times \text{Sin r}$

$1 \times 0.5 = 1.5 \times \text{Sin r}$

$\text{Sin r} = \frac{0.5}{1.5}$

$= 0.3333$

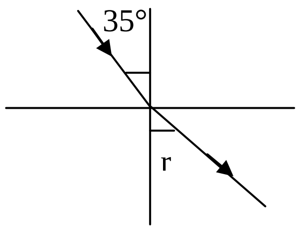
$\text{r} = \text{Sin}^{-1} 0.3333$

$\therefore \mathbf{r = 19.47°}$

3. (a) Draw a ray diagram for a yellow light traveling from glass to air at an angle of 35°.
   (b) Find: (i) The angle of refraction.
   (ii) The critical angle of the glass.
   (iii) The angle of deviation. ($n_g = 1.55$)

Solution:

(a)



(b) (i) $n_g = 1.55$, $n_a = 1$, i = 35°, r = ?

nSin i = constant

$n_a\text{Sin}\ i = n_g\text{Sin r}$

$1.55 \times \text{Sin } 35° = 1 \times \text{Sin r}$

$1.55 \times 0.5736 = \text{Sin r}$

$\text{Sin r} = 0.8891$

$\text{r} = \text{Sin}^{-1} 0.8891$

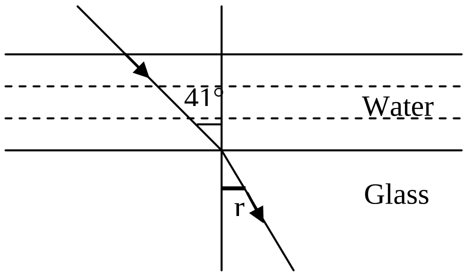
$\therefore \mathbf{r = 62.76°}$

(ii) n = 1.55, c = ? $Sin\ c = \frac{1}{n} = \frac{1}{1.55} = 0.6452 \Rightarrow c = \text{Sin}^{-1}\ 0.6452 \mathbf{= 40.18°}$

(iii) Angle of devıatıon, d = r – 1 = 62.76° - 35° **= 27.76°**

4. (a) What is meant by refractive index of a medium?

(b) A ray of light is incident on a water-glass boundary at an angle of 41° as shown in the diagram below. Calculate the angle of refraction r, if the refractive indices of water and glass are 1.33 and 1.50 respectively.



Solution: **(a)** ***See from the notes.***

(b) $n_g = 1.50$, $n_w = 1.33$, $i = 41°$, $r = ?$

$$n\text{Sin } i = \text{constant}$$

$$n_w\text{Sin } i = n_g\text{Sin r}$$

$$1.33 \text{ x Sin } 41° = 1.50 \text{ x Sin r}$$

$$1.33 \text{ x } 0.6561 = 1.5 \text{ Sin r}$$

$$\text{Sin r} = \frac{1.33 \text{ x } 0.6561}{1.5}$$

$$= 0.5817$$

$$\text{r} = \text{Sin}^{-1}0.5817$$

$$\therefore \mathbf{r = 35.57°}$$

14.44 Applications of Total Internal Reflection

(a) The fish's eye view

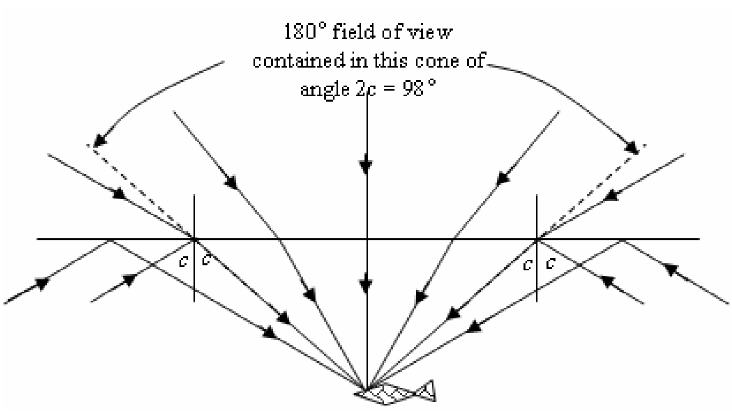


Fig. 14.45 The fish's eye view

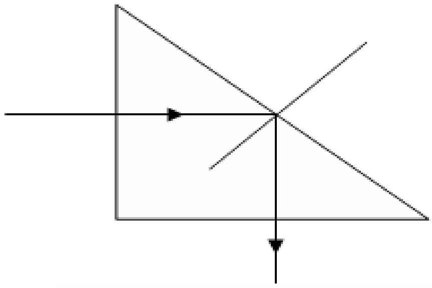
The fish has a full view of everything above or in water at whatever depth, provided that the water surface is unruffled. It enjoys a 180° field of view apparently all squeezed into a cone of angle about 98°(i.e. twice the critical angle for water).

Outside this range the fish sees objects in water and on the bottom which are mirrored in the surface of the water by total internal reflection.

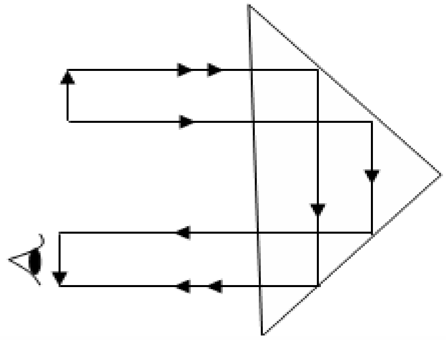
(b) Total Internal Reflection in prisms

The critical angle of glass-air boundary is about 42°. Thus, whenever light which is traveling in glass is incident on such a boundary at an angle greater than 42°, it undergoes total internal reflection leading to formation of upright or inverted image as shown in figure 14.46 below.

Note: *That in the diagrams when an incident ray or refracted ray meets the glass at right angle, it is not refracted.*

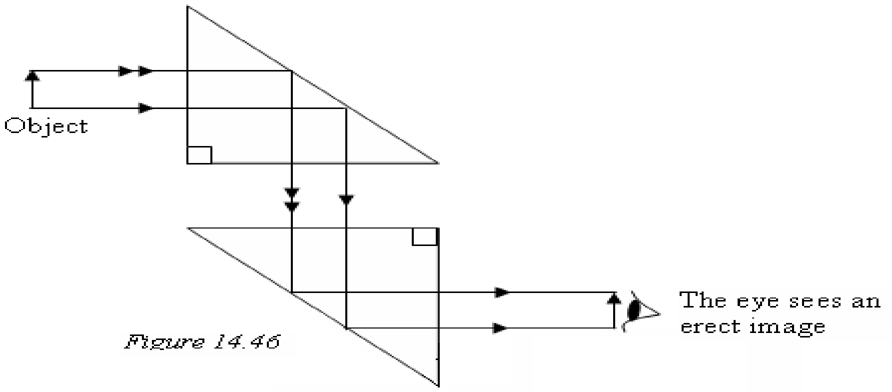


(i) Total internal reflection in a right angle prism



(ii) Formation of inverted imageThe by a single prism

(iii) Formation of upright/erect image by two prisms.



Notes: *Totally reflecting prisms find applications in optical instruments such as binoculars and periscopes used in submarines and opera theaters.*

Figure 14.47 showing arrangement of prisms in a binocular

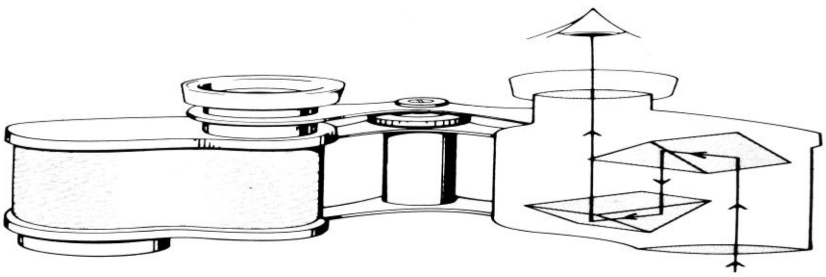


Figure 14.47

They are preferred to plane mirrors in periscopes and binoculars because they their advantages over plane mirrors.

(c) Advantages of glass prisms over plane mirrors

(i) All the incident light is reflected when total internal reflection occurs in glass prism whereas mirrors absorb some of the incident light.

(ii) The finite thickness of glass in front of the reflecting surface produces ghost, blurred, image.

14.45 The Principle of reversibility of light

States that: *The path of light ray is reversible.*

Consider the incident ray PO in figure 14.48. The ray undergoes refraction towards the normal at O giving a refracted ray OQ. The refracted ray OQ further undergoes refraction at Q giving the second refracted ray QR.

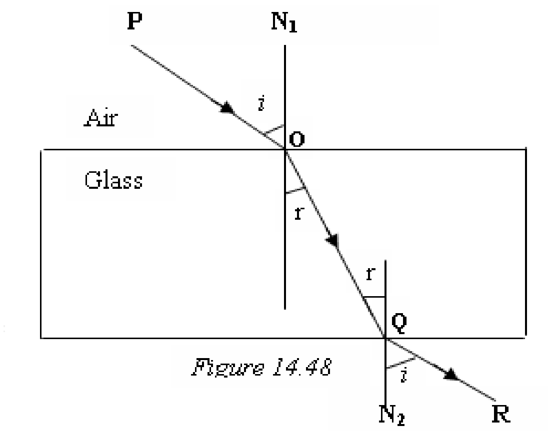


Figure 14.48

According to the principle of reversibility of light, if the direction of the light ray is reversed, the ray traces the same path in the opposite direction.

The refractive indices for a ray passing from air to glass and from glass to air may be distinguished by using symbols $_an_g$ abd $_gn_a$ respectively.

Thus for a ray passing from air to glass $\quad \dfrac{\sin i}{\sin r} = {}_an_g$ ……………………… 1

Applying the principle of reversibility of light $\quad \dfrac{\sin r}{\sin i} = {}_gn_a$

$$\frac{\sin i}{\sin r} = \frac{1}{_gn_a} \quad \text{……………………… 2}$$

Equating equation (1) to equation (2), we have: $\boldsymbol{{}_an_g} = \dfrac{1}{_gn_a}$ **or** $\boldsymbol{{}_gn_a} = \dfrac{1}{_an_g}$

(a) Real and apparent depth

Due to refraction, a thick glass block appears to be only two-thirds of its real thickness when viewed vertically above. See figure 14.49.

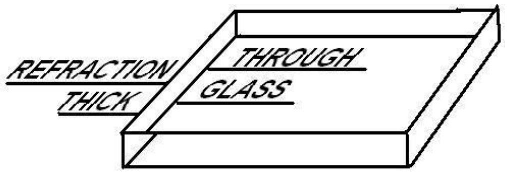


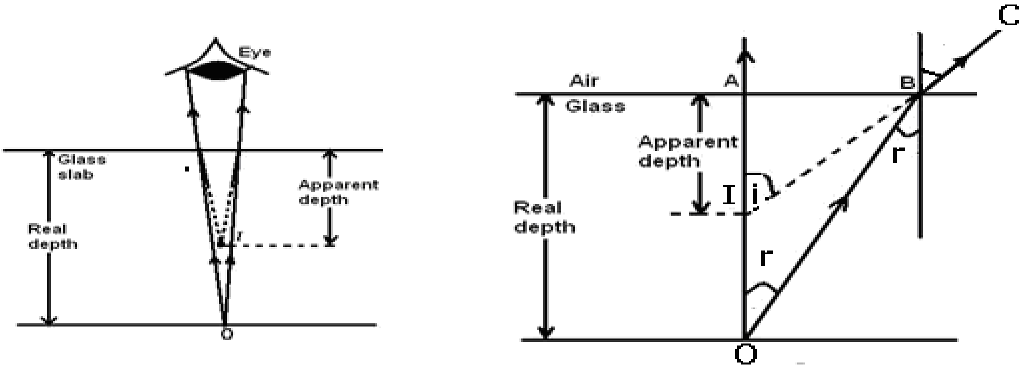
Figure 14.49

Explanation

Light rays from the object undergo refraction at the medium-air boundary. As the rays from optically more dense medium pass in to a less dense medium they are refracted away from the normal. On entering the eye, the eye automatically assumes the rectilinear propagation of light and then locates the position of the image above the actual point.

(b) Refractive Index related to Real and Apparent depth

Consider the diagram in figure 14.50 (b) on page 242 in which OBC is a ray very close to the normal which enters the eye from a point O at the bottom of the slab. The emergent ray BC appears to be coming from a virtual image I, so that AI is the apparent depth of the slab.



(a) *Figure 14.50* (b)

By using the principle of reversibility of light the refractive index of glass is given by

$$n = \frac{Sin\, i}{Sin\, r}$$

But $\angle AIB$ = i (corresponding angles) and $\angle AOB$ = r (alternate angles)
Substituting for i and r in the above formula, we have:

$$n = \frac{Sin\angle AIB}{Sin\angle AOB} = \frac{AB}{BI} \div \frac{AB}{BO} = \frac{BO}{BI}$$ Since B is very close to A,

$$n = \frac{AO}{AI}$$

$$\therefore \boldsymbol{n} = \frac{Real\ depth}{Apparent\ depth}$$

(c) Some effects of Refraction

(i) *Ponds, pools and liquids* in containers appear shallower than their actual depths.

(ii) *A stick or straight object appears bent* when partly immersed in a liquid as shown in figure 14.51 below.

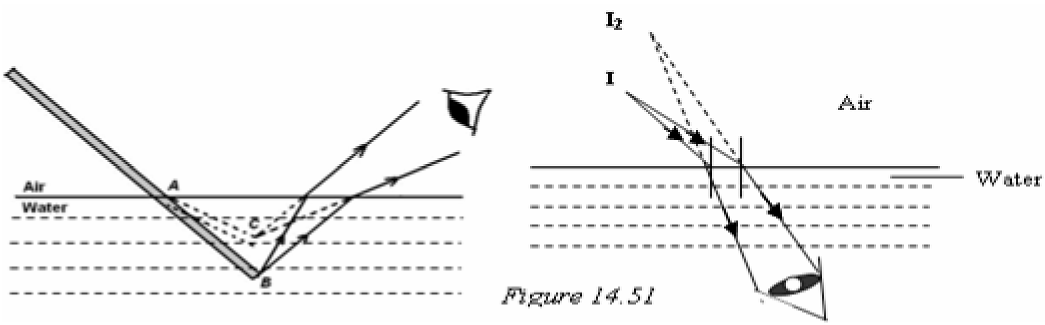


Figure 14.51

Note:

(i) *If an object is viewed from optically denser medium e.g water, the effect described above is reversed, thus the object appears farer than its actual distance.*

(ii) *The sun and stars seem to be higher* than they real are. This is because light from them is refracted when it leaves the vacuum of the space and enters the atmosphere.

(iii) *The sun and moon appear bigger* than their normal sizes when rising or setting.

(iv) *Radio waves:* Under total internal reflected in the ionosphere region and are reflected back to the earth surface.

(iv) Mirages

A mirage is a realistic image of an object that is either totally imaginary or that appears to be in a location other than the true one.

Mirages are formed in hot or cold places due to total internal reflection.
Light rays from an object are successively refracted at the boundaries of air layers of different densities until a point is reached when the ray is totally internally reflected. Since the eye interprets that light travels in a straight line, then the image of an object appears upside down. If the rays are from the space above, it appears as a pool of water.

Diagrams showing formation of mirages

Mirage in hot region

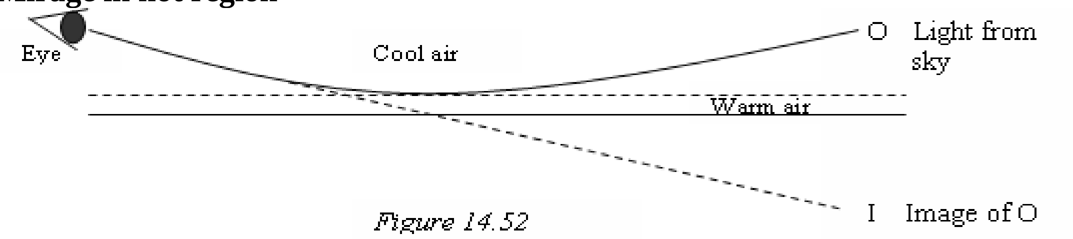


Figure 14.52

Mirage in cold region

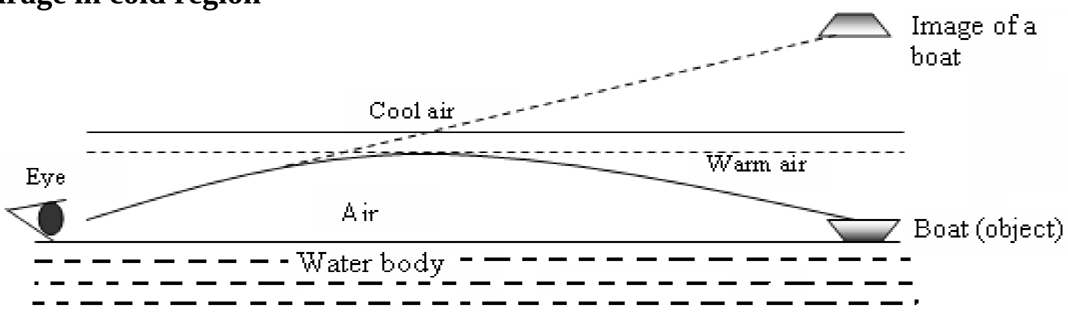
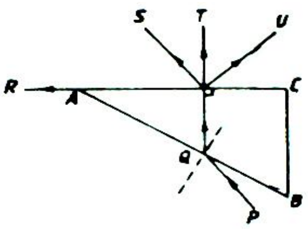


Figure 14.53

In the case of a mirage at sea or ocean during the day, the denser layers of air are next to the cool surface of the water, and the reflection takes place from the upper atmosphere. Thus the object appears distorted, elongated, inverted, and suspended in the air, producing a so-called *looming effect*.

Self-Check 14.4

1. A stick with one end immersed in water appears bent on the water surface because
   A. diffraction B. reflection C. interference D. refraction
2. A ray of light PQ is incident on the face AB of a glass prism ABC as shown in the diagram in the figure below.



Fiaure 14.54

Which one of the rays R, S, T, U indicates the correct direction of the emergent ray?
   A. R
   B. S
   C. T
   D. U
3. Which one of the following is a wrong formula in the study of optics?
   A. $m = \frac{u}{v}$ B. $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ C. $\textit{Sine}\ c = \frac{1}{n}$ D. $m = \frac{v}{u}$
4. A ray of light is incident on a glass block at an angle of incidence of 45^0 and the angle of refraction 21^0. Find the critical angle of the glass
   A. 42.0^0 B. 40.0^0 C. 33.8^0 D. 35.0^0

5. Total internal reflection occurs when

(i) the incident angle is equal to the critical angle

(ii) light travels from a dense to a less dense medium

(iii) light travels from a less dense to a dense medium

(iv) the incident angle in a dense medium is greater than the critical angle

A. (i), (ii) and (iii) only B. (ii) only

C. (iv) only D. (ii) and (iv) only

SECTION B

6. (a) State the laws of refraction.

(b) (i) What is meant by refractive index?

(ii) Describe an experiment you would use to measure the refractive index of glass using a glass block.

(c) (i) Light traveling in water is incident at a water-air surface at 30^0. What is the angle of refraction if the refractive index from air to water is 1.33? (3marks

(ii) The refractive index of a glass block is 1.62. Calculate its critical angle.

7. (a) Study the diagrams in figure 14.55 (a) and (b) below.

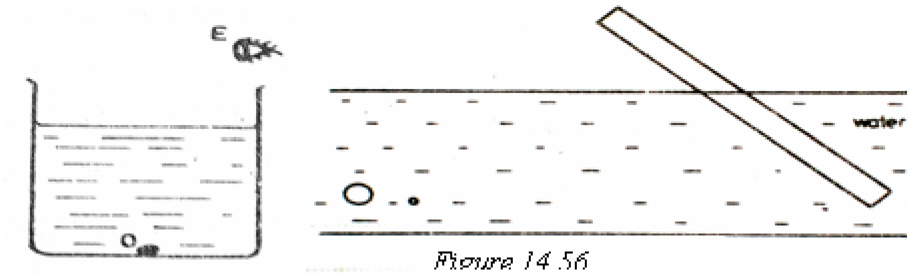


Figure 14.56

Copy and complete by drawing rays to show how the object appears to an observer at E and how the stick will appear to observer at O.

8. The diagram above shows rays of light in a semi-circular glass block.

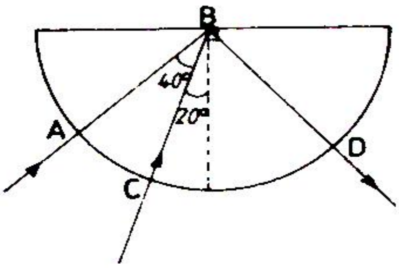


Figure 14.57

(a) Explain why ray AB:

(i) is not refracted on entering the block at A.

(ii) takes path BD on reaching B.

(b) Ray CB is refracted at B. Calculate the angle of refraction. (Refractive index of glass =1.5)

9. (a) What is meant by the following terms?

(i) Critical angle (ii) Total internal reflection

(b) (i) State two conditions for total internal reflection.

(ii) One application of total internal reflection.

(c) Draw a diagram to show how a fish in water attains a wide field of view.

10. A ray of light is incident in air on the surface of a glass block. If the angle of incidence is 30° calculate:

(a) the angle of refraction.

(b) the amount by which the ray is deviated on entering the glass. (*n* of glass = 1.5).

LENSES AND OPTICAL INSTRUMENTS

LEARNING OBJECTIVES

By the end of this topic you should be able to:

1. State (a) (i) The types of lenses. (ii) The terms used in lenses.
(b) (i) The principal rays used for drawing ray diagrams.
(ii) Draw ray diagrams using the principal rays.
(c) State the properties of images formed in lenses.
2. Solve problems using the lens formula.
3. Define the power of lens and state its unit.
4. Solve problems using the formula of power of lens.
5. (a) State the application of lenses.
(b) Draw and describe the action of:
(i) The camera. (ii) The projector.
6. Compare the camera and the human eye.

14.51 Lenses

A lens in optical systems refers to a glass or any transparent substance so shaped that it refracts light from any object and forms a real or virtual image of the object.

(a) Types of lenses

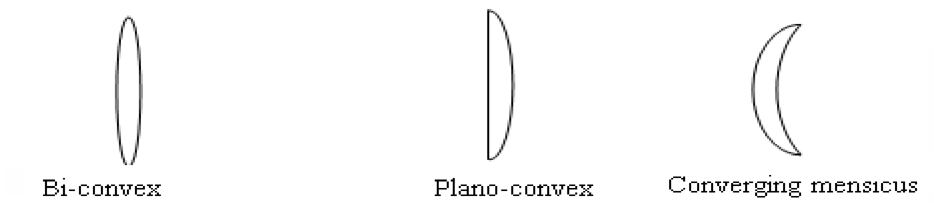
Lenses are made with a great variety of shapes and of different kinds. They all belong to two main groups namely: (i) Converging (Convex) lens
(ii) Diverging (Concave) lens

(i) Converging (Convex) lenses

A converging (convex) lens curves outward. It has a thick center and thinner edges.

The different types of converging lenses are bi-convex, plano-convex and converging meniscus.

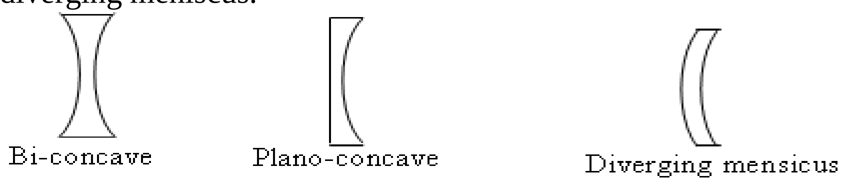
The diagrams showing the different types/shapes of converging lens.



(ii) Diverging lenses

Diverging, or concave, lens is curved inward, with a thin center and thicker edges.

The different types of diverginn lenses are bi-concave, plano-concave and diverging meniscus.



NB: *A meniscus lens has both a concave and convex surface and so, to avoid confusion, we shall refer to lenses by their converging and diverging property rather than their shape.*

(b) The principal focus and focal length of a converging lens

A parallel beam of light, parallel and close to the principal axis of a converging lens is converged or brought to focus at the principal focus or focal point. See figure 14.51 below.

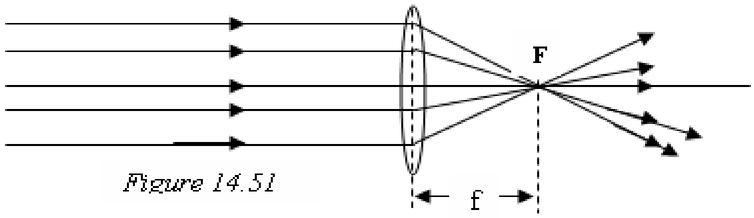


Figure 14.51

F = Principal focus (real) f = focal length (real)

(c) The principal focus F, and focal length, *f*, for a diverging lens

A parallel beam of light, parallel and close to the principal axis of a diverging lens is diverged and appear to be coming from the principal focus or focal point as shown in figure 14.52 below.

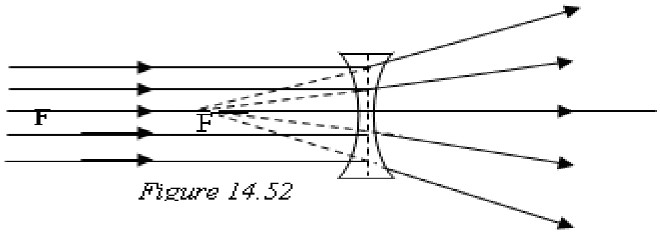


Figure 14.52

F = Principal focus (virtual) f = focal length (virtual)

Lens Definitions

(i) The *pole, P,* is the centre of the surface of the lens.

(ii) The *principle axis* of a lens is the imaginary line joining the centers of curvature of its surfaces.

(iii) The *optical centre*, C, is the point on the principal axis midway between the lens surfaces.

(iv) The *principal focus, or focal point, F* of a converging lens is the point on the principal axis to which rays parallel to the axis converge after refraction. The principal focus of a converging lens is real.

(v) The *principal focus, or focal point, F* of a diverging lens is the point on the principal axis from which all rays incident parallel appear to diverge after refraction. The principal focus of diverging lens is virtual.

(vi) The *focal length, f,* of a lens is the distance between the principal focus and the optical center of the lens.

(vii) The *focal plane* is the plane through the principal focus F perpendicular to the principal axis.

(viii) The *aperture* is the width of the lens from one edge to the other.

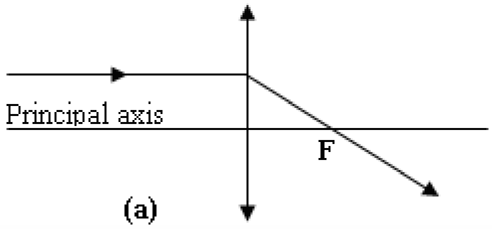
Notes:
- *A lens is considered to have two principal foci, depending upon whether light is incident from one direction or the opposite direction.*
- *For thin lenses, the principal foci are equidistant from the optical centre.*
- *All work subsequent connected with lenses in this book will assume that thin lenses are used.*

14.53 Ray diagrams

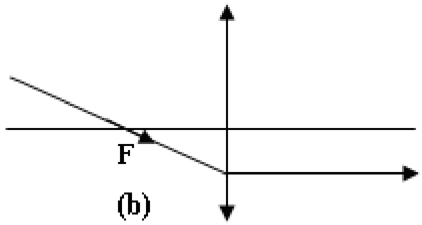
There are three special or particular rays which can be drawn accurately in lens ray diagrams. We choose the most convenient rays called *the principal rays* to find the position of the image formed by a lens.

(a) The principal rays for Converging lens

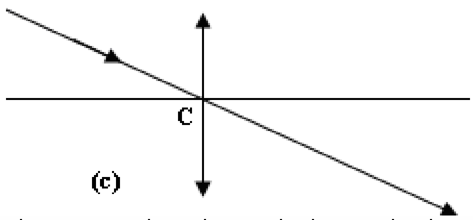
The figures 14. (a) – ((c) show the three principal rays for converging lens.



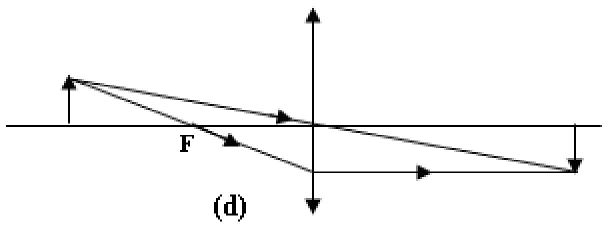
1. A ray parallel to the principal axis, is refracted (by the lens) to pass through the principal focus F.



2. A ray through the principal focus, F ir refracted parallel to the principal axis (i.e. ray 1. reversed)



3. A ray passing through the optical centre is undeviated or is not refracted.



All the principal rays combined

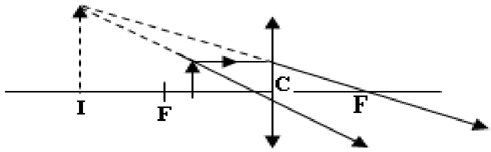
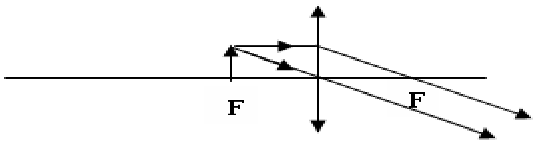
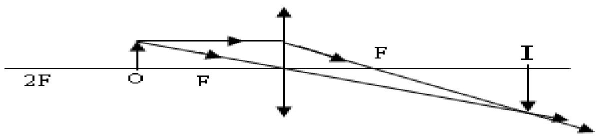
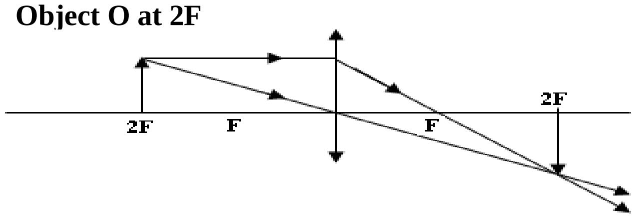
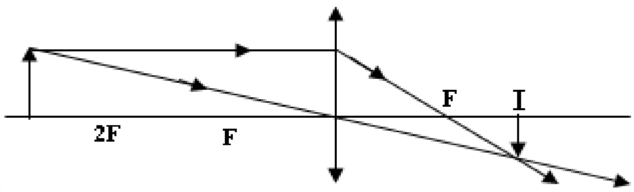
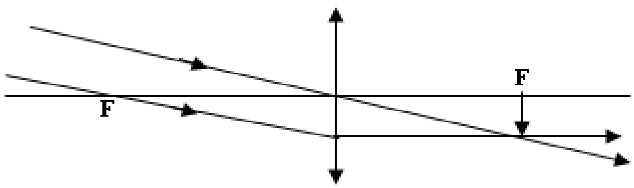
(b) Ray diagrams for Converging lens

In drawing the ray diagrams we choose any two of the principal ray to locate the position of the image formed by a lens. The distance of the object from the lens is critical in deciding the nature, size and position of the image and for this reason it is vital to mark clearly the position of the principal focus of a lens.

Remember, when drawing ray diagrams:

- *All real rays of light are drawn as solid lines with arrows indicating their direction.*
- *All virtual rays and images are drawn as broken lines.*
- *We draw rays from the tip of an object at a point off theprincipal axis so that we can find the size or magnification of the image and which way up it is.*
- *Images on the same side of a lens as the object are always virtual and erect.*
- *Images on the opposite side of a lens to the object are always real and inverted.*

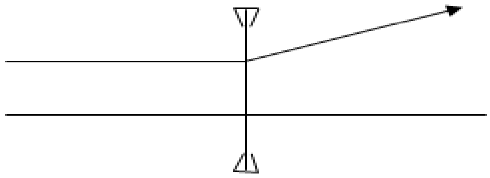
Images formed by Converging lenses

Ray diagram	Nature of image I
(a) Object O between F and C 	(i) Virtual (ii) Erect (iii) Magnified (iv) On same side of lens as O and further away
This arrangement is applied in: (i) Magnifying glasses (ii) Microscope eye piece. (iii) Spectacles correction for long sightedness.	
(b) Object O at F 	At infinity
Produces parallel beam of light as in a spot light with lamp at O.	
(c) Object O between F and 2F 	(i) Real (ii) Inverted (iii) Magnified (iv) On opposite side of lens to O, beyond 2F.
Applied in: (i) Projector (ii) Microscope objective lens	
(d) Object O at 2F 	(i) Real (ii) Inverted (iii) Same size as O (iv) On opposite side of lens, between F and 2F
(e) Object O beyond 2F 	(i) Real (ii) Inverted (iii) Diminished (iv) On opposite side of the lens to O, beyond 2F.
Applied in: (i) Camera (ii) The eye	
(f) Object O at infinity 	(i) Real (ii) Inverted (iii) Diminished On opposite side of lens at F.
Applied as objective lens in a telescope	

(c) The principal rays for Diverging lens

There are two principal rays of a diverging lens; they are:

1. A ray parallel to the principal axis, is refracted (by the lens) to appear to come from principal focus F.



2. A ray passing through the optical centre is undeviated or is not refracted.

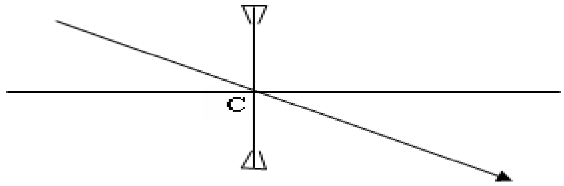
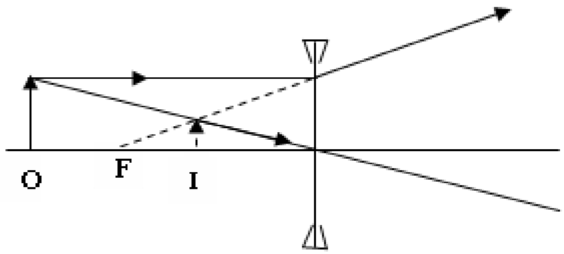


Image formed by Diverging lens

The ray diagram for a diverging lens is the same for all object positions.

Ray diagram	Nature of image I
	*Image I is:* *(i) Virtual* *(ii) Erect* *(iii) Diminished* *(iv) Between F and C* *(v) On same side of lens as O, but nearer*
Applied in: (i) eye piece in some instruments (ii) in spectacles to correct short-sightedness	

Experiment 14.51 To determine the Approximate focal length of a converging lens

– Focusing a distant object on to a screen.
– Move the lens to and fro until a sharp image is formed on a screen.
– The distance from the lens to the screen gives focal length, f.

Experiment 14.52 To determine the focal length of a converging lens using an illuminated object and a plane mirror

Apparatus

A ray box with a cross wire on its side, Source of light, a plane mirror, lens holder and a converging lens.

Procedure

- The apparatus are arranged as shown in figure 14.54 below.

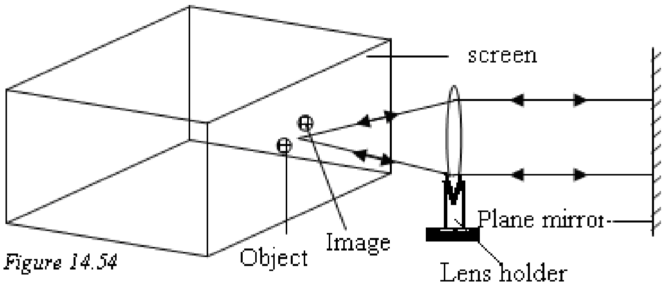


Figure 14.54

- Adjust the position of the lens until a sharp image of the cross wire is formed on the screen.

NB: *At this position of the sharp image, the rays from the lens emerge as parallel rays and strike the mirror at right angle and they are reflected back along their original path.*
The distance between the screen and the lens give the focal length of the converging lens.

(d) Scale drawing of ray diagrams

The ray diagrams can be drawn to scale using squared paper. This provides an accurate method of finding the position, size and nature of the image formed by a lens and may be used to answer lens questions. The following points should be noted:

(i) The lens is represented by a line at right angles to its principal axis.

(ii) The scale chosen for the distances from the lens does not necessarily need to be the same as for the object and image heights, if so, both scales should be stated on the diagram.

(iii) Choose a scale which is simple and convenient to use and that makes the diagram fill most of the sheet of squared paper.

Worked Example
Scale Diagram

1. An object 6 cm high is placed 20 cm from a converging lens of focal length 8 cm. Find by scale drawing the position, size and nature of the image.

Solution Object height = 6 cm, u = 20 cm, f = 8 cm.
Scale: 1 unit : 4 cm

Steps followed:

Now using the scale: Object height = $\frac{6}{4}$ = 1.5 cm, u = $\frac{20}{4}$ = 5 cm, f = $\frac{8}{4}$ = 2 cm

These values of object height, object distance, and the focal length are measured and drawn on a squared paper.
Measure the image height and the image distances using a ruler.
Multiply the distances by the scale to get the actual image height and the image distance

14.54 The Lens Formula

The lens formula is given by the relation: $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

Or When rearranged, we have $f = \frac{uv}{u+v}$

(a) The sign convention used in lens calculations

When solving problems on lenses (or curved mirrors), it is necessary to adopt a particular system of giving positive and negative values to distances.

The *real-is-positive* sign convention is used in this book:

(i) The distances from the lens to real objects, images and principal focus are positive values (i.e. *u*, *v* and *f*) are positive for real objects and images.

(ii) The distances from the lens to virtual objects, images and principal focus are negative values. (i.e. *u*, *v* and *f* are negative for virtual images).

NB: *All distances are measured from the optical centre of the lens.*

(b) The Magnification formula

The comparison of image size with object size is called *magnification*. The linear magnification, *m*, is defined by the formula:

Magnification, $m = \frac{Image\ \ Height}{Object\ \ height}$ Or Magnification, $m = \frac{v}{u}$

Note: *When using this formula for linear magnification, any negative signs for u and v should be ignored.*

Worked Examples

1. An object is placed 20 cm from a converging lens of focal length 15 cm. Find the nature, position and magnification of the image formed.

Solution: u = +20 cm, f = +15 cm, v = ?

(i) $\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow \frac{1}{20} + \frac{1}{v} = \frac{1}{15} \Rightarrow \frac{1}{v} = \frac{1}{15} - \frac{1}{20} = \frac{4-3}{60} = \frac{1}{60}$

$\therefore$ **v = 60 cm**

(ii) $m = \frac{v}{u} = \frac{60}{20} = \mathbf{3}$

Since v = positive, then a real and magnified image of magnification of 3 is formed 60 cm from the lens on side opposite to the object.

2. A four times magnified virtual image is formed of an object placed 12 cm from a converging lens. Calculate the position of the image and the focal length of the lens.

Solution m = 4, v = ? Substituting in $m = \frac{v}{u}$

v = mu = 4 x 12 = **48 cm**

u = 12 cm, v = - 48 cm (virtual image), f = ?

Substituting in $\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow \frac{1}{12} + \frac{1}{-48} = \frac{1}{f}$

$\therefore \frac{1}{f} = \frac{1}{12} - \frac{1}{48} = \frac{-1+4}{48} = \frac{3}{48}$ $\therefore$ **f = 16 cm**

(c) The power of a lens

The power, F, of a lens is defined as *the reciprocal of the focal length expressed in metres*.

The unit of power of a lens is Dioptres (D).

A diopter (D) is the power of a lens of focal length one metre.

The power of a lens is calculated from the formula:

$$Power = \frac{1}{focal\ \ length\ in\ metres}$$

Or $\mathbf{F} = \frac{1}{f}$

NB: The power of converging lens is positive while for a diverging lens is negative.

Worked Examples

A single lens

1. A converging lens has a focal lens 15 cm. Calculate the power of the lens.

Solution: f = 15 cm = $\frac{15}{100}$ = 0.15 m, F = ? Substituting in F $= \frac{1}{f} = \frac{1}{0.15}$ = 6.67D

2. Find the power of (a) a converging lens of focal length 20 cm.
(b) a diverging lens of focal length 10 cm.

Solution: (a) f = 20 cm = $\frac{20}{100}$ = 0.2 m, F = ? Substituting in F = $\frac{1}{f} = \frac{1}{0.2}$ = 5 D

(b) f = -10 cm = $\frac{-10}{100}$ = -0.1 m, F = ? Substituting in F = $\frac{1}{f} = \frac{1}{-0.1}$ = -10

D

Combination of two lenses

When two lenses are placed in contact, the power of the combination is obtained by adding or subtracting the powers of the individual lens.

Examples

1. Two converging lenses of focal length 10 cm and 20 cm are placed in contact. Find the power of the combination.

Solution: *Power of the first lens* f = 10 cm = $\frac{10}{100}$ = 0.1 m, F = ?

$$F1 = \frac{1}{f} = \frac{1}{0.1} = 10 \text{ D}$$

Power of the second lens f = 20 cm = $\frac{20}{100}$ = 0.2 m, F = ?

$$F2 = \frac{1}{f} = \frac{1}{0.2} = 5 \text{ D}$$

Power of the combination F = F1 + F2 = 10 + 5 = **15 D**

2. A converging lens of focal length 10 cm is placed in contact with a diverging lens of focal length 25 cm. Find the power of the combination.

Solution: *Power of the first lens* f = +10 cm = $\frac{10}{100}$ = 0.1 m, F =?

$$F1 = \frac{1}{f} = \frac{1}{0.1} = 10 \text{ D}$$

Power of the second lens f = -25 cm = $\frac{-25}{100}$ = -0.25 m (diverging lens), F = ?

$$F2 = \frac{1}{f} = \frac{1}{-0.25} = -4 \text{ D}$$

Power of the combination F = F1 + F2 = 10 + -4 = **6 D**

14.55 Applications of lenses

Lenses are of great use in every day life. They are used in the following:

- *the camera*, *projector*, *eye glasses*, *microscope*, *telescopes* and
- other optical instruments.

(a) The lens camera

Structure

The camera consists of light proof box with a converging lens in front and a light sensitive film at the back. The inner surface is painted black in order to prevent the reflection of stray rays of light. The construction of the camera is shown in the figure 14.55 below.

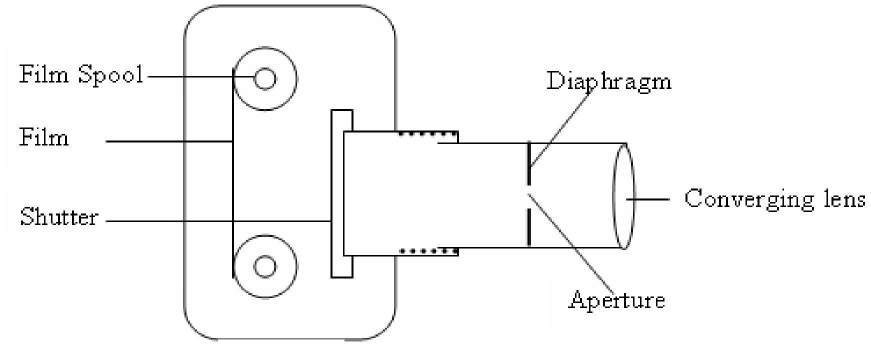


Figure 14.55

The parts of the camera and their functions

(i) *The converging lens* - focuses the image of the object on to the film.

(ii) *Diaphragm* - the diaphragm regulates the size of the aperture and thus controls the amount of light energy admitted through the lens.
It operates in conjunction with the shutter.

(iii) *Aperture* - the opening of a lens through which light enters.

(iv) *Shutter* - A spring loaded blind which covers the film. It admits or shuts off light from the film. The shutter has a variable speed. It also controls the amount of light entering the camera by length of time for which it is opened.

(v) *The film* - part of the camera on which images of objects are focused.

Note: *A camera is usually fitted with provision for adjusting the distance between the film and the converging lens.*

14.56 The Human Eye

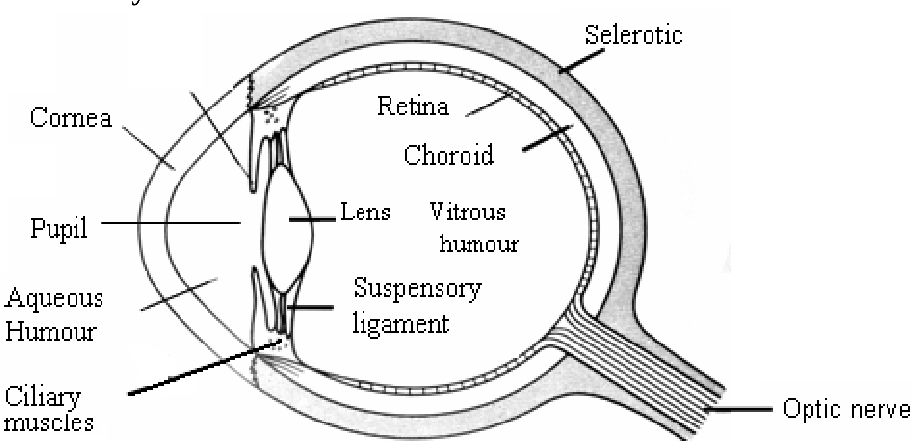


Figure 14.56

The human, eye, figure 14.56, in many respects is similar to the camera. The image is formed by the eye lens on a sensitive retina at the back of the eye and transmitted to the brain by means of *optic nerves.*

The amount of light entering the eye is controlled by the *iris*, the coloured portion of the eye which regulates the size of the *pupil* or circular opening in its centre.

In order for the eye to form an image on the retina, the ciliary muscles alter the focal length of the lens by changing its shape. This process by which the eye lens changes its size so that the image is formed on the retina is called accommodation. This process makes the eye to see both near and far objects.

The eye ball has a tough white wall called *sclerotic* and the transparent portion which is in front of the cornea. The cornea serves as a protective covering and also partly focuses light entering the eye. The space between the cornea and the lens contain a transparent liquid called *aqueous humour* and the rest of the eye ball is filled with a jelly like substance known as the *vitreous humour*. The eye lens is suspended by a *suspensory ligament.*

(a) Near point and far point

The closest point at which the eye can focus (comfortably) is called its *near point*; the most distant point is called the *far point*. For a normal eye these are 25 cm and infinity, respectively.

(b) The Eye Defects and Corrections

The eye defects are caused as a result of the eye loosing the ability of its power of accommodation hence resulting to the two defects, namely:

- Short Sighted (Myopia) and
- Long sight (Hypermetropia).

(i) Short Sighted (Myopia)

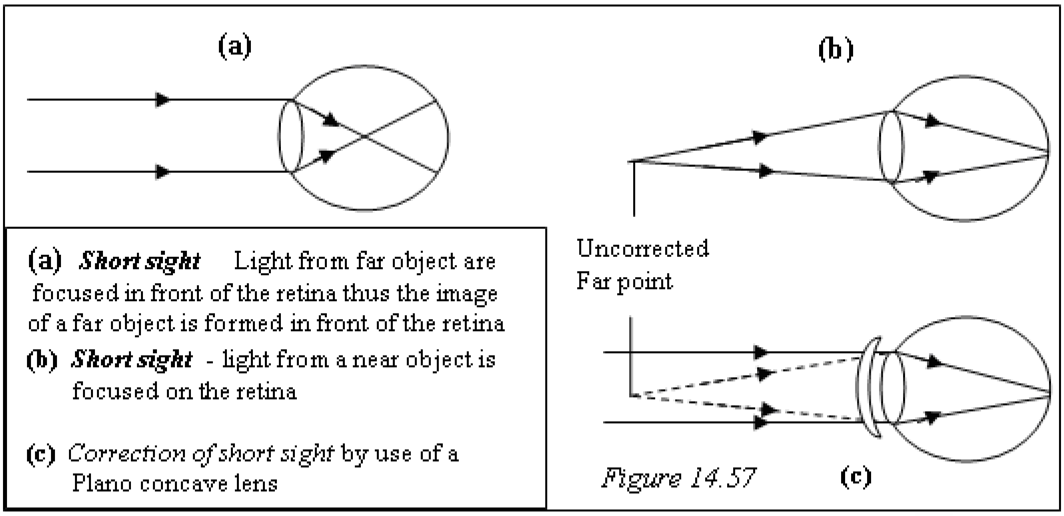
A short sighted person can see near objects clearly but not distant objects. For this person the far point is closer than infinity.

Light from a distant object is brought to focus in front of the retina.

Correction of Short sightedness

Short sightedness is corrected by using a suitable diverging lens. The diverging lens diverges the light rays such that they appear to come from the (uncorrected) far point of the eye.

Diagrams in figure 14.57 show Short sight and its correction



(a) ***Short sight*** Light from far object are focused in front of the retina thus the image of a far object is formed in front of the retina

(b) ***Short sight*** - light from a near object is focused on the retina

(c) *Correction of short sight* by use of a Plano concave lens

Figure 14.57

(ii) Long sight (Hypermetropia)

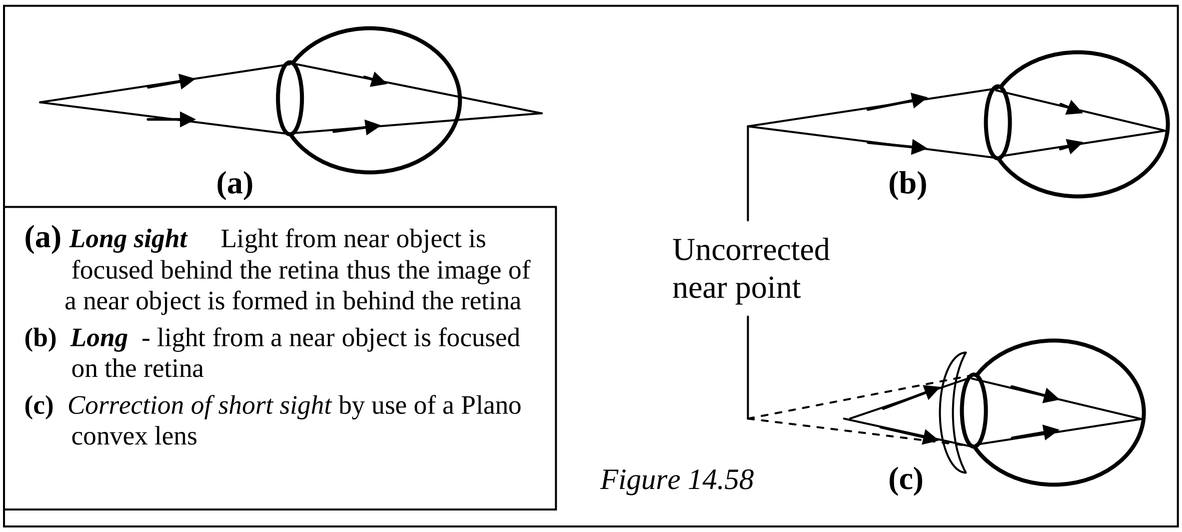
A long sighted person can see distant objects clearly but not those close-by. For this person his or her near point is greater than 25 cm from the eye.

Light from near by object focus behind the retina.

Correction of Long sightedness

Long sightedness is corrected by using a suitable converging lens. The lens must produce a virtual image at the (uncorrected) near point of the eye an object which is 25 cm.

Diagrams in figure 14.58 show long sight and its correction



(a) ***Long sight*** Light from near object is focused behind the retina thus the image of a near object is formed in behind the retina

(b) ***Long*** - light from a near object is focused on the retina

(c) *Correction of short sight* by use of a Plano convex lens

Figure 14.58

Comparison of the camera, the pin-hole and the human eye

	Eye	**Pin-hole Camera**	**Lens Camera**
Type of lens	Converging	None	Converging
Method of focusing	Change of lens Shape: thicker for near objects and thinner for far objects.	All distances focused if pin-hole is small.	Lens moves away from the film for near objects and near to the film for far objects.
Light control.	a) Iris. b) Sensitive of retina.	a) Hole size b) Exposure time c) Sensitivity of film.	a) Diaphragm. b) Exposure time or shutter speed. c) Sensitivity of the film.

14.57 The Projector or Enlarger

A projector is an instrument used to project image of a transparent slide on to a screen. It forms a magnified, real and inverted image of a film or slide. It is put in the projector upside, the image appears the right way up. Figure 14.59 show the diagram of the projector.

Structure

It consists of:
- Source of light
- Condenser
- Film or Slide
- Objective Lens

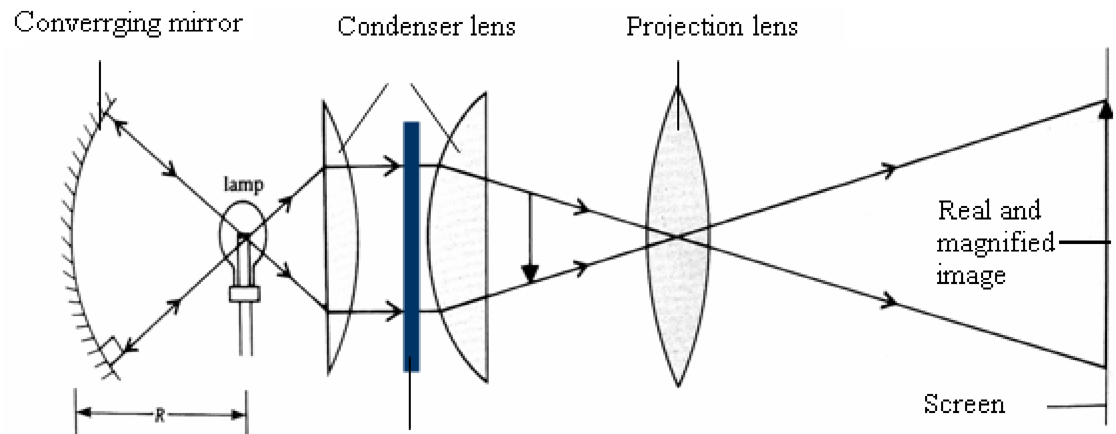


Figure 14.59

Parts of the projector and their functions

(i) Source of light

An electric bulb or electric arc forms a powerful beam of light. A concave mirror behind it reflects back any light rays that travel away from the film.

(ii) Condenser

One or two large convex lenses converge the light rays from the lamp or arc and concentrate them on the film.

(iii) Film or Slide

This is put into the projector both upside down and left to right. The image formed is then the right way up and not laterally inverted (i.e. turned about sideways).

(iv) Objective lens

The image is focused by a converging lens in a moveable tube. The object distance is slightly greater than the focal length of the lens, which therefore forms a real, inverted, magnified image. Moving the lens backwards or forward focuses the image. If it is moved closer to the film, the image is larger and is focused further away.

Self-Check 14. 5

1. A pin is placed in front of convex lens at a distance less than the focal length of the lens. What type of image is formed?

A. Real, inverted diminished. B. Virtual, erect, magnified
C. Real, erect, diminished. D. Virtual, inverted, magnified

2. An object 2 cm tall is placed 5 cm in front of a convex lens. A real image is produced 20 cm from the lens. Calculate the magnification of the lens.

A. 4 B. 2 C. 0.5 D. 0.25

3. An object is placed between a converging lens and its principal focus. The image formed is

A. real, magnified, upright B. real, magnified, inverted
C. virtual, diminished, upright D. virtual, magnified, upright

4. An image 5 cm high is formed by a converging lens. If the magnification is 0.4, find the height of the object

A. 2.0cm B. 4.6cm C. 5.4cm D. 12.5cm

5. An object is placed at a distance of 20 m from a convex lens of focal length 15cm. the type of image formed is

A. inverted and magnified B. inverted and diminished
C. upright and diminished D. upright and magnified

SECTION B

6. (a) State the types of the two common lenses.

(b) Define the following terms as used in the study of lenses.

(i) the principal focus and.

(ii) focal length of a converging lens.

(c) With the help of a ray diagram show how a converging lens can be used as a magnifying glass.

7. (a) Define:(i) real image. (ii) virtual image.

(b) State any two properties of an image of a real object formed in a diverging lens.

(c) An object 8 cm high is placed perpendicularly on the principal axis 12 cm away from the diverging lens. With the aid of a ray diagram, find the focal length of the lens, if the height of the image formed is 2 cm.

8. (a) The focal length of a converging 10.0 cm. What is its power?

(b) State two applications of converging lenses

(c) An object of height 4 cm is placed perpendicularly on the principal axis at a distance of 45 cm from a converging lens of focal length of 15 cm. By graphical construction, determine

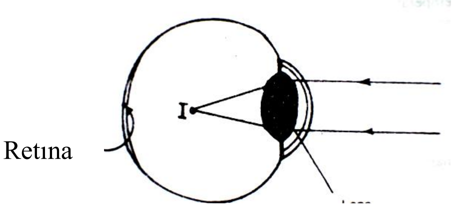
(i) the position of the image. (ii) the magnification.

9. (a) Define focal length of a converging lens.

(b) With the aid of a labeled diagram, describe a simple experiment to determine the focal length of a converging lens.

(d) Give one use of converging lenses

10. (a) The diagram in figure 14.60 figure a shows one of the eye defects in human.



Fiaure 14.60 *Lens*

(a) (i) State the type of eye defect shown in the diagram.

(ii) Explain with a help of diagram how the defect can be corrected.

(b) What is meant by accommodation?

(b) With a help of ray diagram, describe how a projector is used to project a magnified image on to a giant screen.

(c) With the aid of a labeled diagram, describe how a lens camera works.

REFRACTION OF LIGHT IN PRISMS

LEARNING OBJECTIVES

By the end of this topic you should be able to:

1. (a) Define deviation and minimum deviation.

(b) State the conditions for minimum deviation.

2. Solve problems involving refraction in prisms.

14.61 Prism

Prism, in geometry, is three-dimensional solid with two parallel triangular faces. See figure 14.61 below.

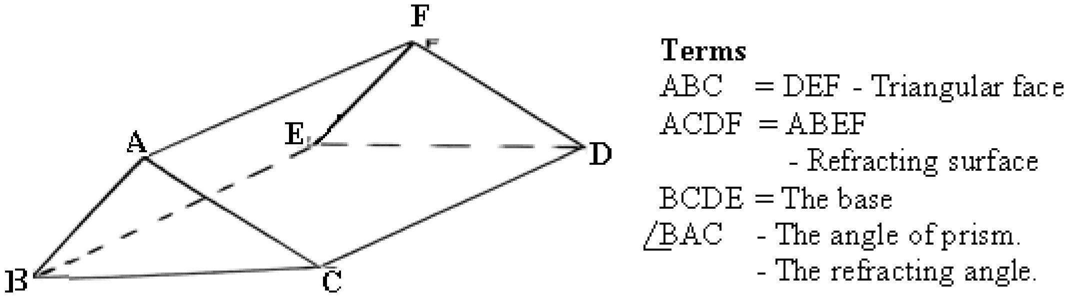


Figure 14.61

Note: For clarity, the triangular face is always considered.

14.62 Deviation by Prism

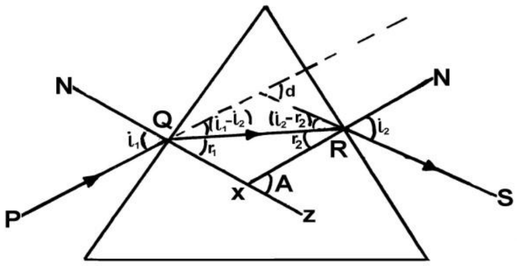


Figure 14.62

When light is passed through a prism, it is refracted at the two refracting surfaces as shown in figure 14.62 above.

The ray PQ is refracted at Q, it then passes along the path QR. It is then refracted again at R, and emerges by the path RS. The direction of the ray has been altered from the initial direction PQ to RS. By producing both lines so that they meet, the angle formed is called *angle of deviation, d*.

(a) Formula of Deviation, d

The deviation, *d*, is given by the formula: $\mathbf{d = (i_1 - r_1) + (i_2 - r_2)}$

Or $\mathbf{d = (i_1 + i_2) - (r_1 + r_2)}$

Derivation of the formula of deviation, d

From the diagram. One exterior angle = The sum of two interior angles

$$\mathbf{d = (i_1 - r_1) + (i_2 - r_2)} \quad \ldots\ldots\ldots\ldots \quad \mathbf{1}$$

From equation 1 $\quad d = (i_1 - r_1) + (i_2 - r_2)$

$= i_1 - r_1 + i_2 - r_2$

$= i_1 + i_2 - r_1 - r_2$

$$\mathbf{d = (i_1 + i_2) - (r_1 + r_2)} \quad \ldots\ldots\ldots\ldots \quad 2$$

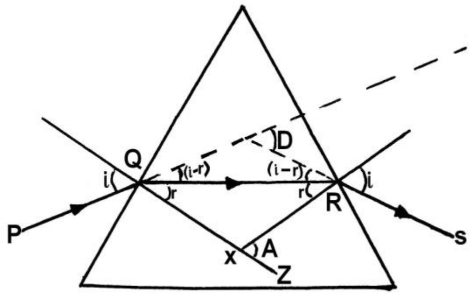
Also from the diagram,

$\angle RXZ = \angle XQR + \angle QRX$ (Sum of two interior angles = one exterior angle)

$$\mathbf{A = r_1 + r_2}$$

(b) Minimum Deviation, D

Consider the diagram in figure 14.63 below.



Fgure 14.63

Minimum deviation, D, is the deviation that occurs in a prism where the angle of deviation is at its minimum value. It occurs when the conditions of minimum deviation are fulfilled.

(c) Conditions for minimum deviation to occur

1. When the refracted ray in the prism is parallel to the base.
2. When the light ray passes symmetrically through the prism.
3. The angle of incident at the first refracting surface is equal to the emergent angle.

(d) The derivation of the formula of Minimum Deviation D

From the diagram, D $= (i_1 - r_1) + (i_2 - r_2)$ But $i_1 = i_2$ and $r_1 = r_2$

$= i - r + i - r$

$= i + i - r - r$

$= 2i - 2r$

$\therefore$ **D** $\mathbf{= 2(i - r)}$

Also from the diagram,

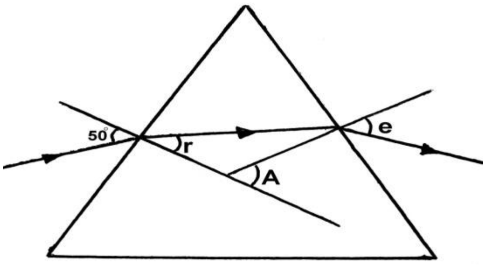
$\angle RXZ = \angle XQR + \angle QRX$ (Sum of two interior angles = one exterior angle)

A $= r + r$

A $\mathbf{= 2r}$

Worked Examples

1. (a) The diagram in the figure 14.65 shows a ray of yellow light incident at an angle of 50^0 on one side of an equilateral glass prism of refractive index 1.52.



(i) Calculate the angles marked r and e.

(ii) Sate and explain what could be observed if the ray above were of white light.

(b) Explain with the aid of a diagram why the writing on a piece of paper placed under a glass block appears raised when observed from above.

Figure 14.65

(c) State (i) the conditions necessary for total internal reflection to occur

(ii) One application of total internal reflection

Solution $n_g = 1.52$, $n_a = 1$

(i) Applying $n\text{Sin}i$ = Constant at the *first* refracting face, we have

$$n_a\text{Sin } i = n_g\text{Sin } r$$

$$1 \text{ x Sin}50° = 1.52 \text{ x Sin } r$$

$$\text{Sin } r = \frac{1 \text{ x Sin}50°}{1.52} = \frac{1 \text{ x } 0.766}{1.52} = 0.5040$$

$$r = \text{Sin}^{-1}0.5040$$

$$\therefore \mathbf{r = 30.26°}$$

(ii) Applying $n\text{Sin}i$ = Constant at the *second* refracting face, we have

$$n_a\text{Sin } i = n_g\text{Sin } r$$

$$1 \text{ x Sin } e = 1.52 \text{ x Sin } 29.74°$$

$$\text{Sin } e = \frac{1.52 \text{ x Sin } 29.74°}{1} = \frac{1.52 \text{ x } 0.4961}{1} = 0.7540$$

$$e = \text{Sin }^{-1}0.7540$$

$$\therefore \mathbf{e = 48.94°}$$

2. The figure 14.66 below shows a ray of light incident at an angle of 46.46° on one side of an equilateral triangular prism immersed in a liquid of refractive index n_l. Given that the refractive index of the glass is 1.5, and the angle of refraction on the first face is 40°, calculate;(i) the value of n_l,
   (ii) the value of i_2 and
   (iii) the angle of deviation.

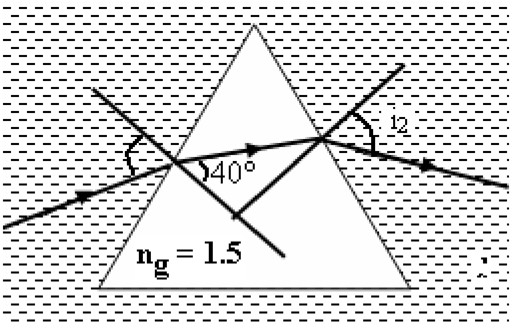
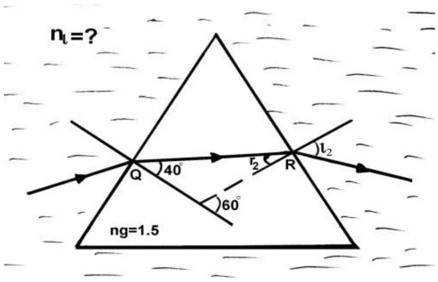


Figure 14.66

Solution: **(i)** Applying n Sin i = Constant at B we have



$$n_l \text{ Sin } 46.46° = n_g \text{ Sin } 40°$$

$$n_1 = \frac{n_g Sin 40°}{Sin 46.46°} = \frac{1.5 \ x \ 0.6428}{0.7249} = \frac{0.9642}{0.7249} = 1.33$$

(ii) Applying $r_1 + r_2 = A$

$$40° + r_2 = 60°$$

$$r_2 = 60° - 40$$

$$\mathbf{r_2 = 20°}$$

Applying n Sin i = Constant at C we have

$$n_g\text{Sin}r_2 = n_l \text{ Sin } i_2$$

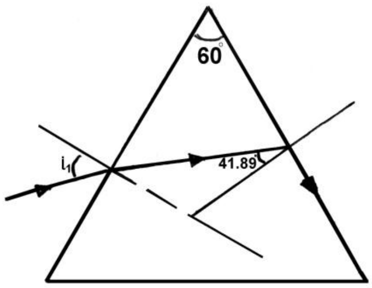
$$\text{Sin } i_2 = \frac{n_g Sin r_2}{n_l} = \frac{1.5 \ x \ Sin 20°}{1.33} = \frac{1.5 \ x \ 0.3420}{1.33} = 0.3857$$

$$i_2 = \text{Sin}^{-1}0.3857 = 22.69°$$

(iii) Applying $d = (i_1 + i_2) - (r_1 + r_2)$

$$= (46.46° + 22.69°) - (40° + 20°) = 69.15° - 60° = \mathbf{9.15°}$$

3. Study the diagram below and answer the questions that follow.

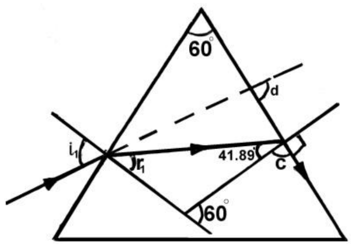


Calculate; (i) the refractive index of the glass.

(i) The value of i_1

(ii) The angle of deviation.

Solution:



(i) Applying $n = \dfrac{1}{SinC}$

$$= \frac{1}{Sin41.8°} = \frac{1}{0.6665}$$

$\therefore\ n = 1.5$

(ii) Applying $r_1 + r_2 = A$

$r_1 + 41.8° = 60°$

$r_1 = 60° - 41.8°$

$\therefore\ \mathbf{r_1 = 18.2°}$

Applying n Sin i = Constant at B we have

$n_a \text{ Sin } i_1 = n_g \text{ Sin } r_1$

$$\text{Sin } i_1 = \frac{n_g Sin i_1}{n_a} = \frac{1.5\ x\ Sin18.2°}{1} = 1.5 \text{ x } 0.3123 = 0.4685$$

$i_1 = Sin^{-1}0.4685$ **= 27.94°**

(iii) Applying d $= (i_1 + i_2) - (r_1 + r_2)$

$= (27.94° + 90°) - (18.2° + 41.8°)$

$= 117.94° - 60°$

$\therefore$ **d = 57.94°**

Self-Check 14.6

1. (a) Explain what is meant by minimum deviation in a glass prism.
   (b) State the conditions for minimum deviation occur.
2. The figure shows a ray of light incident at an angle of *i* on one side of an equilateral triangular prism of refractive index 1.5. The angle of emergency on the second face is 45°
   Calculate: (i) the value of i. (ii) the angle of deviation..
3. (a) Complete the diagram in figure 14.67 to show the path of the light ray PQ through the prism.

   (b) (i) State the conditions for total internal reflection to occur.
   (ii) State one application of total internal reflection.
   (iii) Calculate the critical angle for an air-glass interface if refractive index of glass is 1.5.

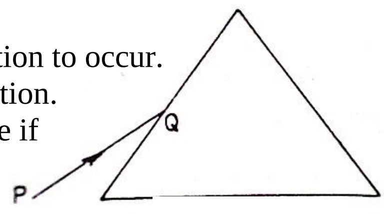


Fig. 14.67

DISPERSION OF WHITE LIGHT AND COLOURS

14.71 LEARNING OBJECTIVES

By the end of this topic you should be able to:

1. (a) Define dispersion of white light by a prism.
(b) Describe an experiment to produce pure spectrum.
2. State the primary and secondary colours.
3. Name the secondary colours formed when primary colours are mixed by addition.
4. (a) Define and state complementary colours.
(b) State the colour of objects in different colours of light.
5. State (i) The effect of filters on colours of light.
(ii) The appearance of colours in electric lamps.

14.72 Dispersion of White light

Dispersion is the separation of white light into its constituent colours. When a narrow beam of white light is passed through a glass prism, it is refracted and spreads into a band of colours of the rainbow as shown in figure 14.71 below. The band of the colours so formed is called *spectrum*.

The diagram showing the formation of a spectrum

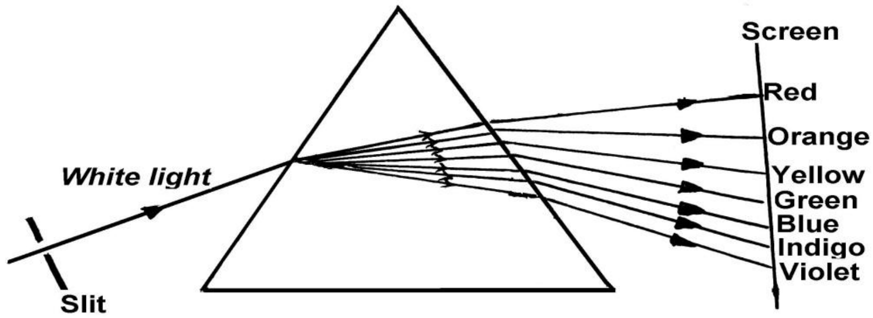


Figure 14.71

Notes: The order of the colours can easily be recalled by the statement: ***Richard Of York Gain Battle In Vein***.

Conclusion

From the above result, we can conclude that:

(i) White light is a mixture of seven colours.
(ii) The prism has different refractive indices for the different colours.
(iii) The prism separates the colours by deviating red least and the violet light most, and other colours are deviated to varying intermediate extends.

(a) Recombining the Spectrum

The spectrum obtained from a white light can be recombined. This may be done by using the following methods.

Method 1. Using two prisms

The first prism deviates the white light into spectrum. The second prism placed in inverted position reverses the deviation and dispersion of the first prism so that the colours of the spectrum recombine and the emergent ray obtained is white light as shown in figure 14.72 on page 263.

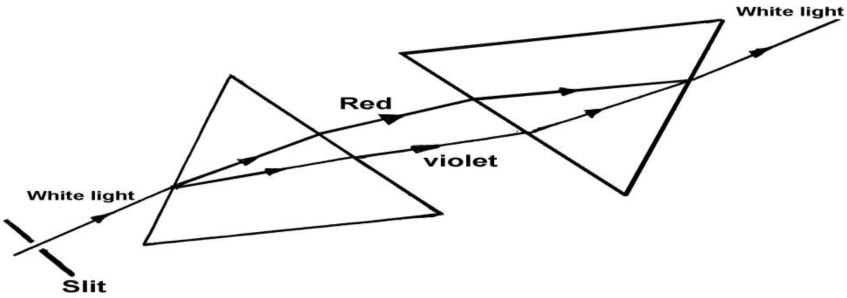


Figure 14.72

Method 2 Using Newton's disc

Newton's disc is a card coloured with all the colours of the spectrum in equal areas. If the card is spun round rapidly the colours of the spectrum are added together and appear as nearly white. The off-white colour is due to the imperfect reflection of colours from the disc.

(b) The Colours of the Spectrum of White Light

The colours of the spectrum of white light are those seen in a rainbow. They are usually named in order from top to bottom as: *Red*, *Orange*, *Yellow*, *Green*, *Blue*, *Indigo* and *Violet*. The colours overlap. That is there are no clear boundaries (no gaps or breaks) between the colours. See figure 14.73 below.

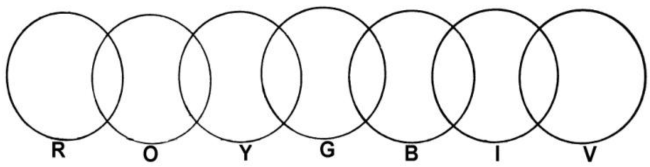


Figure 14.73

Because of the overlaping of the colours, we describe the spectrum of white light as continuous. Which means that there is a complete range of colours from the red end to the violet end of the spectrum. This makes the spectrum impure. However, a pure spectrum can be produced in the laboratory.

(c) Pure Spectrum

A pure spectrum is one in which the colours in it do not overlap. That is to say one colour only forms each part of the image on a screen.

To produce a pure spectrum the following conditions should be fulfilled:

(i) the source of white light must be restricted by a very narrow slit,

(ii) The prism is placed such that there is minimum deviation thus making the white light arriving at the glass prism a parallel beam, and

(iii) The parallel beams of different colours of light emerging from the prism should be focused by a lens onto a screen so that they do not overlap.

Experiment 14.71 To produce a pure spectrum

Apparatus: A source of white light, a slit, two converging lenses, a glass prism and a screen.

Procedure

Place the source of light at the focal length of the first lens, L1.
Arrange the rest of the apparatus as shown in the figure 14.74 on page 265.

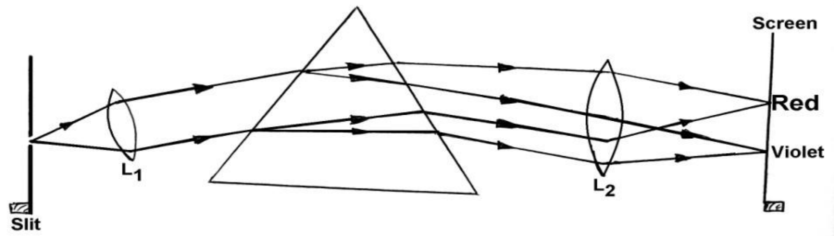


Figure 14.74

This produces parallel emergent beam of white light which is incident on the first refracting surface of the glass prism. The second converging lens, L_2, reverses the effect of the first lens so that the parallel beam of each separate colour is focused at different points on the screen thus forming a pure spectrum.

14.73 Sums with Colours

The human eye has three types of light sensitive cells or colour detectors called cones. Each type of cone detects part of the spectrum of white light.

For example:

The red sensitive cones detect:	Red, Orange and Yellow
The green sensitive cones detect:	Yellow, Green and Blue
The blue sensitive cones detect:	Blue, indigo and Violet.

The simplified plan is shown in figure 14.75 below.

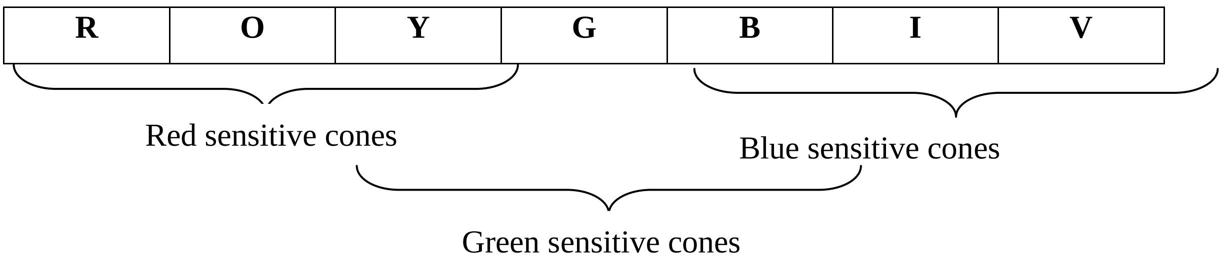


Figure 14.75

As seen from above, there is overlapping of the zones of the spectrum detected by the three types of cones. If the eye receives red light, only the red sensitive cones will be stimulated, but yellow light will stimulate both red and green sensitive cones.

Notes: *Although the eye has only three different colour detectors, it can distinguish all the colours of the spectrum from the variety of combinations of responses produced by overlapping of the zones.*

(a) Primary and Secondary Colours

Colours of light are grouped in two main categories. These are:

(i) Primary Colours and

(ii) Secondary Colours

(i) Primary Colurs

Primary clours are colours which cannot be made by adding (or mixing) any other colours of light together. These are: Red, Green and Blue.

Note: *With the eye having three types of cones, or colour detectors, it is not surprising to find that the three primary colours can be used to stimulate the other colours in the spectrum.*

(ii) Secondary Colours

Secondary colours are colours of light that are made or obtained by adding two primary colours together.

These are: Yellow (Y), Magenta (M) and Cyan (C), (some times called *peacock blue*).

(b) Addition of Colours of light

Addition of colours of light on white screen can be done by using two or thre projectors or light boxes. The colour filters of the colours to be added are placed in each of the window of the projector or light box and are then directed onto a white screen.

(i) Using three projectors or light boxes

An effective way of showing all the possible combinations of primary colours, RGB, on a screen is shown in the figure 14.76 below.

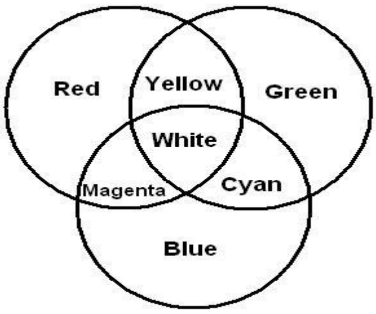


Figure 14.76

Notes:

- ❖ *The circles represent the circular images of the projector windows with the three primary colours of light on the screen.*
- ❖ *All the three colours of light intersect to make white light.*
- ❖ *Two overlapping primary colours make a secondary colour, CMY. This can also be obtained by using two projectors or light boxes.*

(ii) Using two projectors or light boxes

This can be summarized s shown in the table 14.71 below.

Primary Colours of light added	**Secondary Colour made**
Red + Green	Yellow
Red + Blue	Magenta
Green + Blue	Cyan

Table 14.71

Note:

- ❖ *Light of the secondary colour yellow (made of red and green) is indistinguishable to the eye from the true yellow in the spectrum of white light. This is because the red and green sensitive cones in the eye are equally stimulated by a mixture of red and green light or pure yellow light.*
- ❖ *True yellow light is a single colour and can not be separated by a glass prism.*

(c) Complementary Colours

Complementary colours of light are a pair of a primary colour and a secondary colour which when mixed together make white light. Since there are three primary colours of light, then there are three pairs of complementary colours. These are given in the table 14.72 below.

Colours of light mixed			Colour produced
Primary Colour		Secondary Colour	
G + R + B		-	White
R	+	C	White
G	+	M	White
B	+	Y	White

Table 14.72

(d) Mixing colours by subtraction

According to the results of mixing colours by addition, blue and yellow are complementary colours, and when mixed form white. But a student in art lesson learns that blue paint and yellow paint, when mixed together, form green. This would not be possible if the paints were pure blue and pure yellow. The success of this method of making green paint depends on the fact that the pigments in common use are impure colours.

Yellow paint is a compound yellow so that, when illuminated by white light, it reflects red, yellow and green light and absorbs the blue. Similarly, blue paint is not a pure colour: it reflects blue and green and absorbs red and yellow.

When the two paints are mixed, then between then they absorb red, yellow and blue. The only colour they both reflect is green. Consequently, the mixture looks green. This process is called colour mixing by subtraction to distinguish it from the effect of mixing coloured lights by reflection from a white surface which is called *colour mixing by addition*.

14.74 The colour of an object in light

When light is incident on a surface of any object or medium, three things happen to it in varying proportions.

(i) The light may all be transmitted or reflected by the body and the body appears white,

(ii) Only some colours of the white light may be reflected while others are absorbed and the object appears coloured,

(iii) The light may all be absorbed and the body appears black and the energy absorbed is converted into internal energy, the body becomes slightly warm hence, a slight rise in temperature.

The apparent colour of an object is the sum of the colours it is reflecting. Thus:

(a) A white object appears white in white light because it reflects all the colours of the spectrum which add up to give it a white appearance.

(b) A black body appears black in any colour of light because it absorbs all the colours of the spectrum and reflects none.

(c) A white object appears green in green light because it can only reflect the green light that shines on it.

The table 14.73 below summarizes the appearance of the colours of objects.

Colour of object	Colour of object incident on the object	What happens	Appearance of object
White	White	Reflects all colours	White
Black	Any colour of light	Absorbs all colours and reflects none	Black
White	Green	Reflects green light	Green

Table 14.73

Colour of objects can appear to change colour in other colours of light as seen in the following examples.

- ❖ A red object appears red in white or red light because it can reflect red but:
- ❖ A red object illuminated with green light will appear black because it absorbs the green light and reflects none.
- ❖ A yellow object appears yellow in white or yellow light; it reflects yellow light and usually also two primary colours, red and green, which together appear yellow (R + G = Y).
- ❖ If however, a yellow object is seen in any colour of light which contains some red, green or yellow light, it can reflect those colours and may appear to change its own colour, for example:
- ❖ A yellow (R + G) object seen in cyan (G + B) light will appear green because it absorbs the blue light and reflects green.

(a) Colours of light and filters

Filters can be used to produce a particular colour of the spectrum.
When light is passed through filters of various colours; the following happen to the light.

- Very little reflection occurs,
- Some small amounts of colours are transmitted and
- Some colours are absorbed.

For example a red filter absorbs all the other colours of white light. But red light passes through it. The green filter transmits only green light and a blue filter transmits blue.
However, yellow filter has a different characteristic. In that it can transmit red and green lights as well as yellow light.

(b) Various Colours from an electric lamp

The source of light of various colours can be obtained from an ordinary electric lamp placed behind gelatine sheets of different colours. By this means the appearance of coloured objects in different colours of light can be examined.
Red objects look red in red light, while blue and green look black because they absorb the red light. Similarly a red object appears black in green light. However, a yellow object appears black only in blue or violet light. It appears yellow only in yellow light or white light. In red light it apperas red and in green light green.

14.75 Infrared and Ultra-violet light

The spectrum from the sun has both visible and invisible spectrum.

Ultra-violet	V I B G Y O R	Infrared

Visible spectrum

Both ultra-violet and infrared are detected by a thermopile connected to a galvanometer which shows a deflection. Ultra-violet ray is found at the extreme end of the violet light colour while infrared is found beyond the red light colour. The ultra-violet can also be detected by a photographic paper which darkens when the spectrum falls on it.

Self-Check 14.7

1. A piece of white cloth viewed through a blue glass appears blue because
   A. Blue light is absorbed by the glass
   B. the glass adds blue light to the light coming from the cloth
   C. the glass transmits only blue light and absorbs all other colours
   D. the colour of the glass is reflected onto the cloth.

2. A student is holding a white paper with green printing on it. If she enters a room with red lights, she will see
   A. black printing on a red paper
   B. blue printing on a red paper
   C. yellow printing on a red paper
   D. a red printing on a white paper.

3. Which one of the following is correct?
   (i) green light shone on the green surface is all absorbed
   (ii) green light added equally to red light appears yellow
   (iii) green light passes through a red filter
   A. (ii) only B. (i) and (ii) only C. (ii) and (iii) only D. (i), (ii) and (iii)

4. White light is separated into its component colours due to
   A. absorption B. dispersion C. reflection D. Transmission

5. Which of the following are secondary colours only?
   A. Red, green and yellow.
   B. Blue, yellow and magenta.
   C. Yellow, cyan and magenta.
   D. Red, green and blue.

6. Which of the following statements are true?
   (i) surfaces which reflect all colours of light appear white
   (ii) red surfaces absorb all colours and reflect only red light
   (iii) black surfaces appear black because they reflect all colours
   A. (i) only B. (i) and (ii)only C. (i) and (iii) only D. (ii) and (iii) only

7. The secondary colours of light are cyan, magenta and yellow. Which of the following sets of addition of colours of light will produce white light

(i)	cyan	+	blue and mangeta	+	red
(ii)	cyan	+	red and mangeta	+	green
(iii)	yellow	+	red and mangeta	+	blue
(iv)	cyan	+	green and yellow	+	blue

   A. (i) only B. (ii) only C. (iii) only D. (iv) only

8. Which of the following statements are true?
   (i) A magenta filter absorbs green light and transmits red and blue lights
   (ii) A magenta filter absorbs blue light and transmits red and green lights
   (iii) A cyan filter absorbs red light and transmits blue and green lights
   (iv) A cyan filter absorbs blue light and transmits red and green lights

   A. (i) only B. (ii) only C. (ii) and (iv) only D. (i) and (iii) only

9. Dispersion of light is
   A. the rectilinear propagation of light.
   B. the spreading of light around an obstacle.
   C. the splitting of white light into its constituent colours.
   D. the changing of direction by light when it moves from one media to another.

10. When a yellow dress with blue dots is placed in a room lit with pure red light, the dress appears
    A. red with black dots B. yellow with blue dots
    C. green with red dots D. black with yellow dots.

SECTION B

11. (a) Explain the phenomenon of dispersion applied to white light.
    (b) Draw a ray diagram to show the dispersion of white light by a glass prism
    (c) With the aid of a diagram, explain briefly how a pure spectrum may be produced.
    (d) Explain briefly what happens when white light falls on a green body.

12. (a) Distinguish between secondary and primary colours. Give one example of each.
    (b) Name the colour that would be obtained when the following coloured lights are mixed.
        (i) Green and red.
        (ii) cyan and red.
    (c) Explain why an object illuminated by white light appears.
        (i) coloured
        (ii) black
    (d) State why most hind car registration number plates are printed black on a yellow background.

CHAPTER FIFTEEN
THERMAL PROPERTIES OF MATTER

LEARNING OBJECTIVES

By the end of this chapter, you should be able to:

1. (a) Define: Expansion and Contraction of a material.
(b) Describe: The ball & ring/The bar & the gauge experiments to demonstrate expansion and contraction in solids.
(c) State (i) The applications of expansion in:
(ii) The problems caused by expansion and their solutions.
2. (a) Define:- Linear expansivity
(b) Solve problems involving linear expansivity.
3. (a) Describe: Experiments to show expansion in Liquids and Gases.
(b) Explain the anomalous behaviour of water and give its importance to aquatic animals.
4. (a) State: (i) Any two thermometric liquid.
(ii) The properties of thermometric liquid.
(iii) Advantages of mercury over alcohol as used a thermometer.
(b) Define: The lower and upper fixed points.
(c) Solve problems involving
(i) Conversion of temperature from one scale to another.
(ii) Calculation of temperature for unmarked thermometer.

15.1 Expansion and Contraction

Expansion is the increase in size or an object when it gets hotter. While contraction is the decrease in size of an object when it becomes colder.

(a) Expansion of solids

All solids expand when heat is applied to them. However, they do so in varying amounts.

Some solids expand very little while others expand greatly.

Expansion in solids can be investigated by using the following experiments.

(i) The ball and ring.
(ii) The bar and gauge

Experiments to investigate the effect of heat on solids

Experiment 15.1 Ball and ring experiment

Apparatus/Requirements A ball and ring, source of heat

Procedure Part I

Pass the ball through the ring when cool as shown in figure 15.1 below.

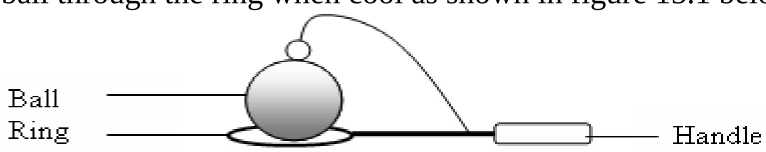


Figure 15.1

Observation: The ball passes through the ring easily.

Part II

- Heat the ball on a Bunsen burner for some time.
- Try to pass it through the hole again.

Observation: The ball does not pass through.

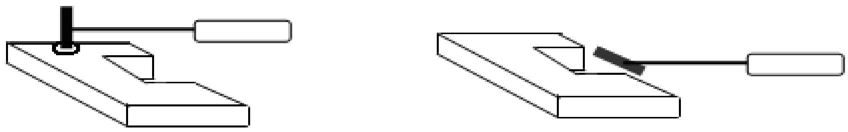
Explanation: When the ball is heated, it expands i.e. the size becomes bigger. As such, it can not pass through the ring.

Expedient 15.2 The bar and gauge experiment

Apparatus/Requirements A bar and gauge, source of heat

Procedure: Part I

- Pass one end of the bar through the hole on the gauge.
- Remove the bar and fix it in between the ends of the gauge as shown in figures 15.2 (a) and (b).



(a) *Figure 15.2* (b)

Observation: The head of the bar passes through the hole and also fits in the gauge easily.

Part II

- Heat the bar on a Bunsen burner flame for some time.
- Try to pass it through the hole and also try to fix it in the gauge.

0bservation:

The head of the bar does not pass through the hole and does not fit in the gauge.

Allow the bar and the ball to cool and repeat the above procedures of passing the ball through the ring and fixing the bar in the hole and the gauge respectively.

Results: When the ball and the bar cool:

(i) The ball passes easily.

(ii) The bar fits in the gauge easily.

Explanation

When the bar is heated, it expands both side ways and length ways (i.e. the size becomes bigger and longer). As such, it can not pass through the hole and fit in the gauge.

Note: *The gauge shows that the bar has increased in length, which is called linear expansion. The ring shows that the diameter of the ball has increased in all directions. The expansion in area of a solid is known as superficial expansion and the expansion in volume is called cubical expansion.*

15.11 Applications of expansion in solids

(a) Bimetallic strip A bimetallic strip is a strip made of two different metals welded or riveted together. When cold the double strip is straight as shown in the figure 15.3 below.

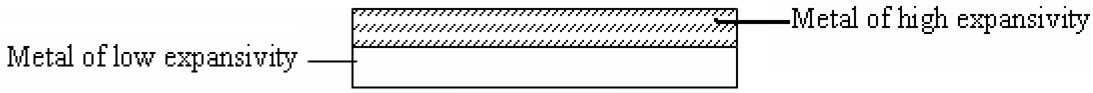


Figure 15.3

(b) Effect of heat on bimetallic strip

When a bimetallic strip is heated, the metals expand with the metal of higher expansivity expanding more than the one of low expansivity. The strip bends to form a curve with the metal of high expnasivity on the inside as shown in figure 15.4 below.

Figure 15.4

Examples of a bimetallic strip are;

Brass and iron - Brass having higher expansivity than iron.

Aluminium and iron - Aluminium having higher than iron.

Question: *State which metal would be on the out side and inside if the two bimetalic strips in the example above are heated*.

(c) Uses of expansion in bimetallic strip

Bimetallic strip are used as electrical switches, in thermostats and many other mechanical switching devices.

Thermostat

A Thermostat is a device that automatically regulates the temperature of a system by maintaining it constant or varying it over a specific range.

A bimetal thermostat uses a special strip of metal to open and close a circuit as temperature fluctuates. Two metals with different expansion rates are bonded to make the strip. The thermostat is arranged so that when the metals are hot, the strip bends upward (toward the metal with the lower expansion rate) and disconnects the circuit. In this particular case, the thermostat will activate a heater when the circuit is closed and electricity is flowing.

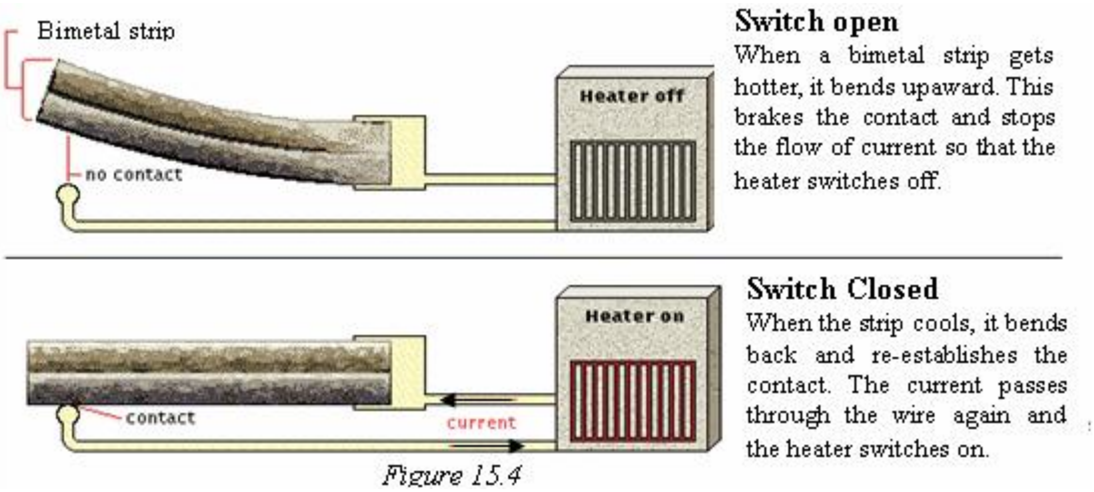


Figure 15.4

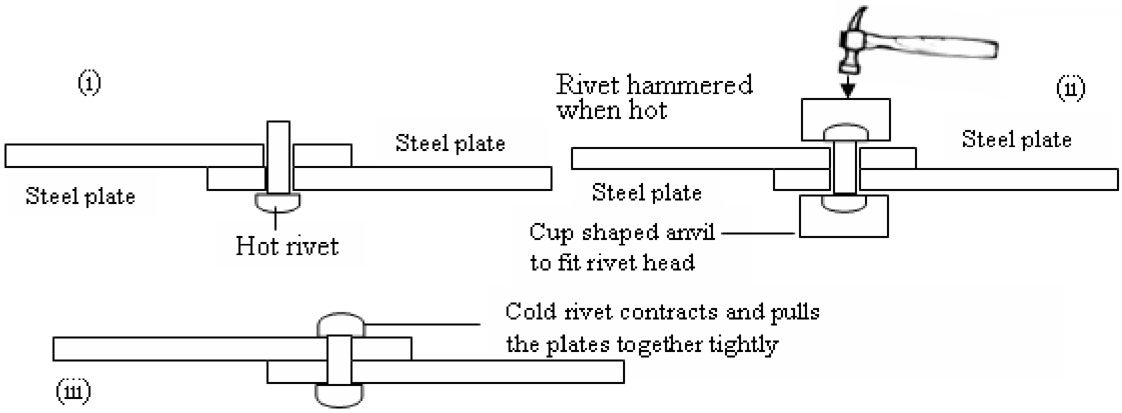
Note: *Increasingly, the use of bimetallic strip for this purpose is being replaced by electronic circuits with no moving parts.*

(a) Tight fitting of

(i) Riveting

Expansion and contraction is used in riveting to get a tight joint of two or more metal plates.

- A hot rivet (expanded) is pushed through a hole in the two plates or rods
- to be joined.
- The end of the hot rivet is hammered to form another.



Results: As the rivet cools it contracts and pulls the two plates more tightly.

(ii) Steel tyres onto cart and train wheels

The steel trye is designed to just fit when it is red hot. As it cools down, it tightens and grips on the wheel.

(iii) Wheel and axles

The same method is also used in fitting wheels on to axles.

(b) Steel bridges

Girder bridges made of steel. During cold weather the bridge contracts and becomes shorter. And during hot weather, it expands and becomes longer. In order to allow for expansion and contraction, one end of the bridge is fixed and the other end is placed on rollers as shown in figure 15.6 below. This enhances the *to* and *fro* movement during expansion and contraction.

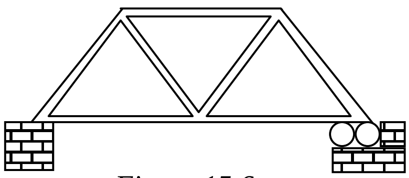


Figure 15.6

(c) Railway lines

Railway tracks have been bent and seriously damaged due to expansion during very hot days where the gap allowed for expansion was too small. Now to allow room for expansion and contraction, fairly large gaps are left between the sections of rails. And the sections are held together by fish plates fixed by bolts in oval shaped holes.

(d) Pipelines

Pipelines carry steam, liquids and gases from one point to another. The pipes contract when cold and expand when hot. To avoid breakage of the pipes due to the force of expansion and contraction, the pipes are fixed fitted with loops or expansion joints. The joints and loops allow the pipes to expand and contract when steam passes through and when it cools down.

(e) Electricity and telephone lines

During cold weather especially at night hours, telephone wires contract and when it is hot during the days, they expand. During cold weather (from evening up to early morning hours) electricity/telephone wires contract. The wires become shorter and taut. And during the hot afternoon hours, the wires become longer and slack.

To avoid the wires from cutting, they are fixed loosely to allow contraction and expansion.

15.2 Linear expansivity

The change in the length of a substance during expansion is called *linear expansion*. And
the measure of the tendency of a particular material to expand is called its *expansivity*. The lengthways expansivity of a material is called its *linear expansivity* and is given by the symbol α (alpha).

Formula of linear expansivity

Definition:

Linear expansivity is defined as: *The increase in length of a unit length of a material for a degree rise in temperature*.

I.e. Linear Expansivity $= \dfrac{Linear\ Expansion}{Original\ length \text{ x } temperature\ rise}$

S.I Unit of Linear expansivity

The S.I unit of linear expansivity is a derived unit.

$$= \frac{m}{m\ x\ °C}$$

$$= °C^{-1} \text{ or } K^{-1}$$

By re arranging the formula for linear expansion we obtain a formula which can be used to calculate expansion of things like bridges and railway lines.

Linear expansion = Linear expansivity* x *Original length* x *temperature rise

Factors which determine linear expansivity

The linear expansion or change in length Δl of a material depends on three things:

(a) The length of the material, *l*.
(b) The change (rise) in temperature
(c) The linear expansivity of the material

Worked example

1. Calculate the linear expansion of aluminium cable 50 m between two electric poles when the temperature rises by 40°C. The linear expansivity of aluminium is $2.6 \text{ x } 10^{-5}$/°C

Solution Δl = ? α = $2.6 \text{ x } 10^{-5}$/°C, l = 50m

$$\Delta l = \alpha\, l \Delta\theta$$
$$= 2.6 \text{ x } 10^{-5} \text{ x } 50 \text{ m x } 40\ °C$$
$$= 5.2 \text{ x } 10^{-2} \text{ m}$$

Answer: The aluminium will increase in length by $5.2 \text{ x } 10^{-2}$ m or 0.052 m or 5.2 cm.

15.3 Expansion in liquids

All liquids like solids expand with varying amounts. Some expand more than others.

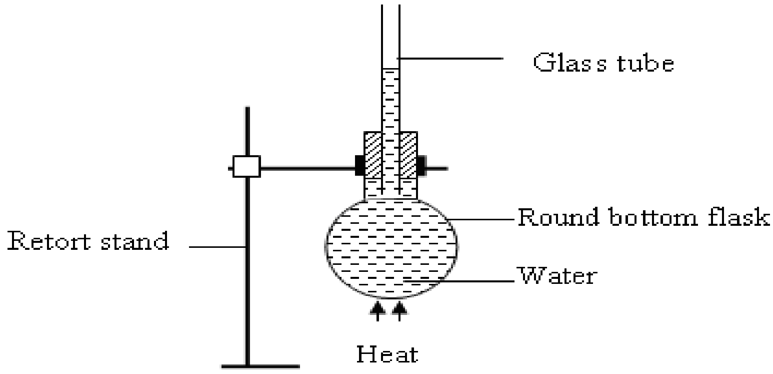
Experiment 15.3 To demonstrate expansion of water

Apparatus: A round bottom flask, capillary tube, source of heat, a cork and a liquid.

Procedure

- Fill a round bottom flask up to the brim with liquid (e.g. water).
- Insert a capillary tube through a cork and cork the flask.
- Mark the level of liquid in the glass tube.

- Set up the apparatus as shown in figure 15.7 below.



Figur 15.7

- Heat the flask as you observe the level of the water in the tube.

Observation: First the level of the water falls and then starts to rise again.

Explanation

(i) The fall of water level

The initial fall of water level is due to the expansion of the glass flask, which gets heated first and expands. The expansion of the flask results to increased volume of the flask. So water moves in to fill the extra volume.

(ii) The rise of water level

Finally, the heat reaches the water and starts to expand, thus rising up the glass tube.

Note that: Water expands faster than glass.

15.31 Comparison of expansion of different liquids.

Expansion in different liquids can be compared by filling the liquids in different flasks of
the same size and type.

Experiment 15.4 To compare the expansion of different liquids e.g. water, alcohol, methylated spirit,ether, benzene

Apparatus: Four identical flasks, trough, water, stirrer, source of heat

Procedure

- Fill the flasks with the liquids to the same height/level and then place them in water in a trough as shown in figure 15.8 on page 276.

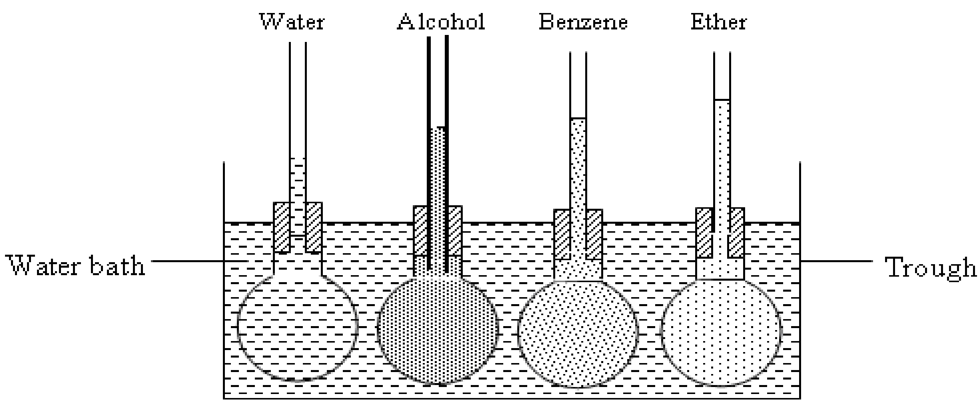


Figure 15.8

- Heat the water bath while stirring.
- Observe the levels of the liquids in the flasks

Observation: The levels of the liquids in the tubes first fall and then rise by different amounts as the heating continues.

Result: The result shows that ether expands most followed by benzene while water being the least.

Conclusion: The above observation shows that some liquids expand more than others for a given rise in temperature.

15.32 The expansion of water

Most liquids contract steadily as they cool, and contract further on reaching their freezing point. Water contracts as it cools down from 100°C to 4°C. However, between 4°C and 0°C, water behaves unusually in that it expands as it gets cooler with its minimum volume at 4°C. This behaviour of water is described as anomalous (irregular). When water freezes its volume increases by about 8%, which is a much larger increase in volume than occurs between 4°C and 0°C.

The change in volume/density of water with temperature as shown in the figures 15.9 (a) and (b) on page 277.

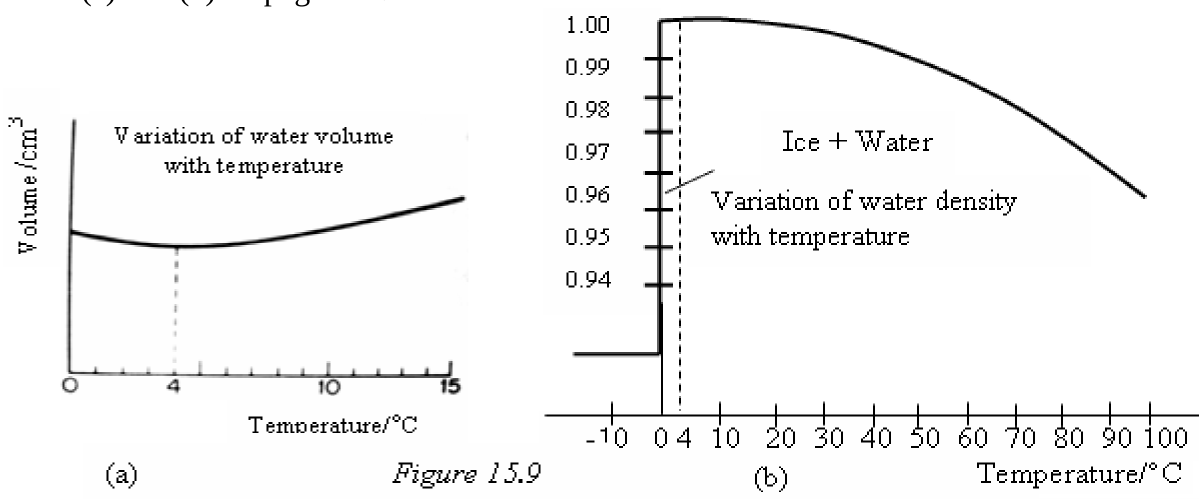


Figure 15.9

The importance of the anomalous expansion of water

When a pond is freezing over, density of water at 4°C remains at the bottom of the pond.
The less dense (but lower temperature) water, between 3°C and 0°C, floats in layers above it. The water on the surface is frozen, but floats because it is less dense than the water below it. The different density layers stop convection currents spreading the heat. Ice is a bad conductor of heat so that the layer of ice on the top of a pool acts like an insulating blanket and slows further loss of heat from the water below.

Aquatic animals and plants make use of this phenomenon, by living in the liquid layer when the water freezes over in the winter.

15.4 Expansion in gases

Experiment 15.5 To show expansion in gases

Apparatus: A round bottom flask fitted with glass tube, water, trough and source of heat from Bunsen burner.

Procedure

- Fill a trough with clean water.

- Pass a glass tube through a cork and invert the cork into a round bottom flask.
- Dip the tube in the water in the trough.

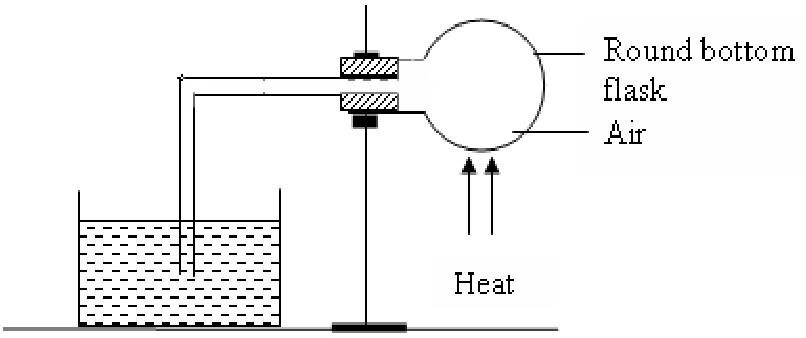


Figure 15.10

- Heat the flask by directing a Bunsen burner flame to it for a short time.
- Observe the water level in the tube.
- Allow the flask to cool while the tube is still in the water. Observe what happens.

Observation: When the flask is hot, the level of the water in the glass tube falls and air bubbles are released.
On cooling, the water level rises up the glass tube.

Explanation
- When the flask is heated the air inside expands. This forces some air out of the flask. Thus bubbles of air are seen as the air escapes.
- As the air in the flask cools, it contracts creating more space and reducing the pressure, then the atmospheric pressure acting on the surface of the water pushes water into the glass tube.

15.5 HEAT AND TEMPERATURE

Heat and measurement of temperature

Heat - *is a form of energy which when absorbed by an object makes it hotter and when lost by an object leaves it colder.*

Temperature - *is the degree of hotness or coldness of a body*.

Words such as warm, hot, tepid, cool and cold tell u about the temperature of an object. The words are not very precise, so if we need to be more accurate about the temperature of an object, we use a thermometer graduated with some scales called temperature scales.

NB: *Temperature is the measure of how hot or cold a body is and should* ***not*** *be confused with the amount of heat the body contains.*

(a) Thermometers

Definition: *A thermometer is an instrument used to measure temperature*.

A number of different types of thermometer are available. Each type of thermometer makes use of a particular thermometric property i.e. a property that changes with temperature. Examples of such properties are:

(i) Change in length of liquid column

(ii) Increase in electrical resistance

The most common type of thermometer is the liquid-in-glass thermometer. Thermometric liquids used in liquid-in-glass are:

- mercury and alcohol.

(b) Properties of thermometric liquids

A thermometric liquid should have the following properties:

(i) Should be opaque for easy reading.
(ii) Good conductor of heat.
(iii) High and uniform expansivity.
(iv) High boiling point.
(v) Low freezing point and
(vi) Should not wet glass.

Mercury and alcohol compared

Mercury	Alcohol
(i) Opaque and therefore can easily be read	- Not opaque but can be coloured.
(ii) Good conductor of heat therefore sensitive to small temperature changes.	- Poor conductor of heat therefore not so sensitive to small temperature changes
(iii) Has uniform expansivity	- Expansion not so regular
(iv) Has high boiling point (357 °C) and therefore can measure high temperatures	- Low boiling point (78 °C) and therefore not suitable for measuring high temperatures
(v) Not very low freezing point (-39 °C) therefore can not measure temperatures below its freezing point.	- Low freezing point (-115 °C)
(vi) Does not wet glass	- Wets glass

(c) Advantages of alcohol over mercury as thermometric liquid

1. It has higher expansivity (about six times) that of mercury.
2. It has lower freezing point than mercury. Alcohol freezes at -115 °C while mercury freezes at -39 °C. The high freezing point of mercury makes the measurement of temperatures lower than -39 °C to be impossible.

Apart from the disadvantage, mercury is preferred to alcohol for the following reasons.

(i) It is a better conductor of heat than alcohol and therefore responds more readily to changes in temperature.
(ii) It is opaque and makes reading easy.
(iii) It has a high boiling point, 357 °C and where as alcohol has low boiling point, 78 °C, can easily vaporize to fill the upper part with vapour.

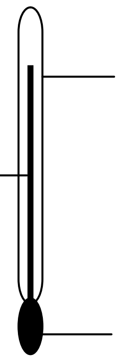
(d) Reasons why water is not used as a thermometric liquid

Water is unsuitable for use in thermometers because of the following reasons.

(i) It freezes at 0 °C. (ii) It has irregular expansion.

15.41 Structure of a liquid in glass thermometer

A *liquid-in-glass* thermometer consists of the following features.



Stem made of uniform diameter glass capillary with a fine bore, which increases the sensitivity of the thermometer. The stem carries temperature scale from which temperature readings are taken.

Bulb The bulb contains the bulk of the thermometric liquid.

(f) How to use a thermometer

The bulb is inserted or placed in contact with the object /substance whose temperature is to be measured.
The liquid in the bulb will either:

(i) Acquire heat energy from the substance and expand making its level rise or

(ii) Loses heat energy to the substance making its level to fall.
The temperature value is then read at the liquid level which directly corresponds on the temperature scale.

15.42 Graduation or Calibration of a thermometer

In order to establish a temperature scale, we choose two *fixed points*. A fixed point is a definite temperature at which a change of state occurs.
The two fixed points chosen are called *lower fixed point* and *upper fixed point*.

(a) Lower fixed point - *is the temperature of pure melting ice at standard atmospheric pressure (i.e. at* pressure 760mmHg).

(b) Upper fixed point -*is the temperature of steam from pure water boiling under standard atmospheric pressure.*

NB: *The ice and the water must be pure because the presence of impurities lowers melting point of ice and elevate the boiling point of water.*

Finding the fixed points of a thermometer

(a) Lower fixed point

- Freeze some pure (distilled) water.
- Crush the ice into small pieces and fill a filter funnel with the pieces and wait for the ice to begin melting.
- When the ice begins to melt (i.e. at 0 °C) insert the bulb of the thermometer so that it is covered with ice, figure 15.11 (a). The melting ice cools the mercury to 0 °C.
- When the mercury stops shrinking (i.e. when the level becomes constant), mark the stem of the thermometer at the mercury level.
- This point is the lower fixed point, or the ice point on Celsius scale.

(b) Upper fixed point

- Remove the thermometer and insert it through a two holed cork.
- Fill a flask with pure water and cork it with the cork carrying the thermometer and a delivery tube such that the bulb is just above the water surface, figure 15.11 (b).
- Heat the water in the flask to a boiling point.

- When the mercury stops expanding (i.e. when the level becomes constant) mark its level on the thermometer stem.

  This point is the upper fixed point, or steam point (100°C) on the Celsius. Then divide the difference between the two points into 100 equal points. Mark the points as a scale along the stem either in Celsius scale or Kelvin or both.

Finding the fixed points of a thermometer

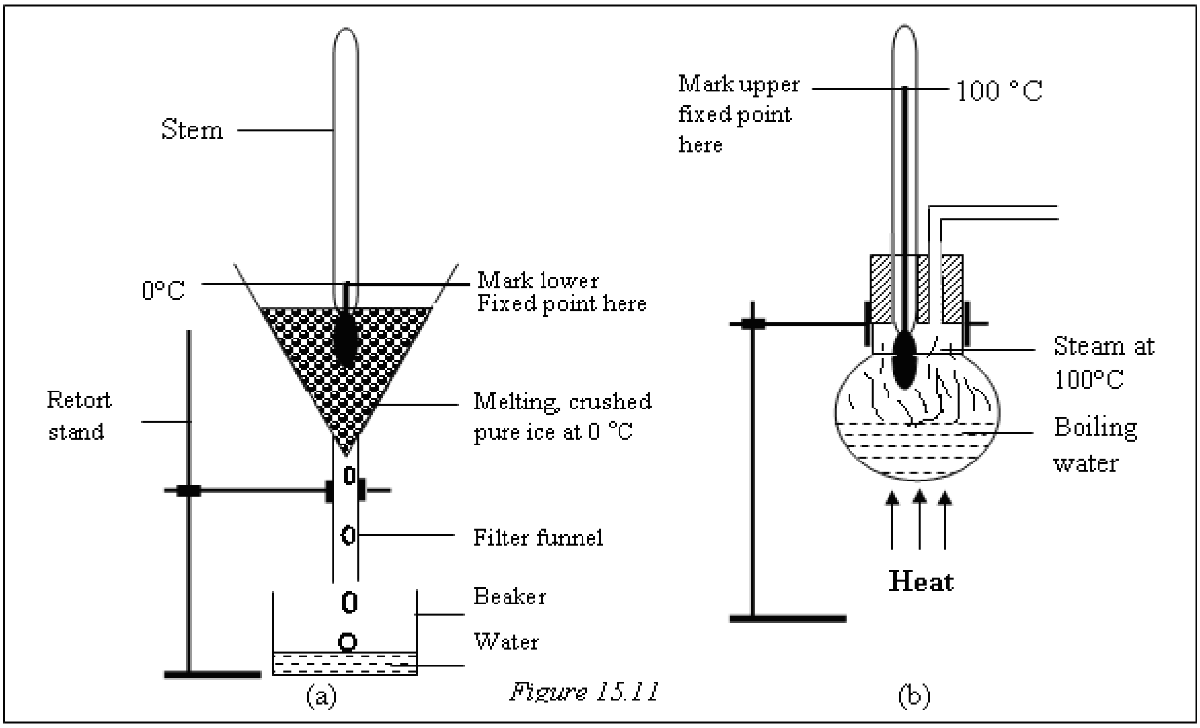


Figure 15.11

15.5 Temperature Scales

There are two common scales of temperature, namely:

(i) Celsious Scale and
(ii) Kelvin Scale

(a) Celsius scale (Centigrade)

The centigrade or Celsius scale assigns a value of 0 °C to the freezing point and 100 to the boiling point of pure water. It is defined by dividing the fundamental difference (the difference between the upper and lower fixed points) into 100 °C equal degrees. The temperatures on this scale are called "Degree Celsius".

(b) Kelvin scale (Thermodynamic Scale)

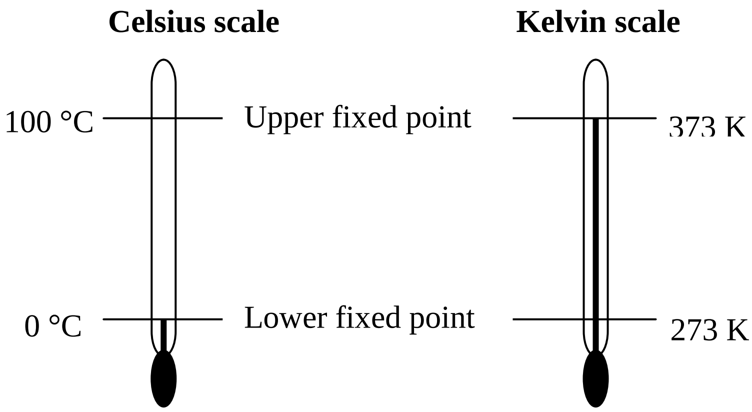
In the Kelvin or absolute scale, the lower fixed point is 273.15 K and the upper fixed point is 373.15 K. The *Kelvin* (K) is the S.I unit of temperature.

Note:
(*i*) *The dgree intervals are identical to those measured on the Celsius scale.*
(ii) Absolute zero is approximately -273.15 °C or zero (0 K) degree on the Kelvin scale.
(iii) Absolute zero is the lowest possible temperature. It is characterized by complete absence of heat energy.
(iv) Temperatures on Kelvin do ***not*** *have degree symbol (°).*
(v) One Kelvin is the same as one Celsius. I.e. 1 K = 1 °C

15.51 (a) Relationship between Celsius scale and Kelvin scale

The two temperature scales are related to each other as shown below.

Fixed point	**Celsius scale**	**Kelvin scale**
Lower fixed point	0	273
Upper fixed point	100	373



(b) Conversion of temperature from one scale to another

The temperature value, θ, on Celsius scale is related to the temperature value, T on Kelvin scale by the formula:

$$\boldsymbol{\theta = T - 273}$$

Or

$$\boldsymbol{T = \theta + 273}$$

Worked Examples

1. Convert the following temperature readings to Celsius scale.
   (a) 1010 K(b) 233 K (c) 373 K

2. Convert the following temperature readings to Kelvin scale.
   (a) 240 °C (b) 30 °C (c) 120 °C

Solution

(a) θ = ?, T = 1010 K,

$\theta = T - 273$
$= 1010 - 273$
$\therefore \theta = 737$ °C

(b) θ = ?, T = 233 K,

$\theta = T - 273$
$= 233 - 273$
$\therefore \theta = -40$ °C

(c) θ = ?, T = 273 K K,

$\theta = T - 273$
$= 373 - 273$
$\therefore \theta = 100$ °C

2. (a) θ = 240 °C, T = ?

$T = \theta + 273$
$= 240 + 273$
$\therefore T = 513$ K

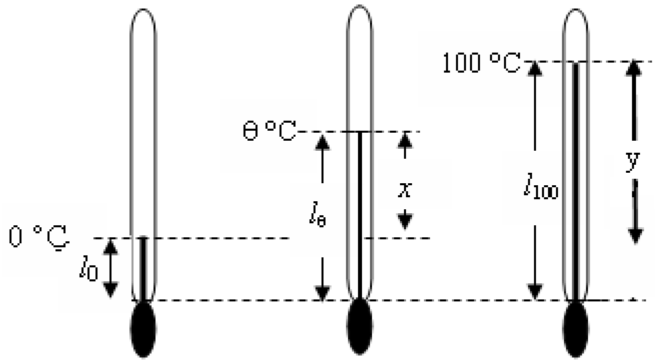
(b) θ = 30 °C, T = ?

$T = \theta + 273$
$= 30 + 273$
$\therefore T = 303$ K

(c) θ = 120 °C, T = ?

$T = \theta + 273$
$= 120 + 273$
$\therefore T = 393$ K

(c) Calculating the temperature values when the lengths of the thermometric liquid for the lower and upper fixed points are given



From the diagram: l_0 = the length of mercury column at 0 °C.

l_{100} = the length of mercury column at 100 °C.

l_θ = the length of mercury column at unknown temperature, θ °C.

$x = (l_\theta - l_0)$

$y = (l_{100} - l_0)$

$$\text{Temperature, } \theta, \text{ in °C} = \frac{\textit{Mercury length above the lower fixed point}}{\textit{Fundamental difference}} \times 100$$

$$= \frac{l_\theta - l_0}{l_{100} - l_0} \times 100$$

$$\therefore \theta = \frac{x}{y} \times 100$$

Worked Examples

1. A mercury thermometer is calibrated by immersing it in melting pure ice and then in boiling pure water. If the mercury columns are 6 cm and 16 cm respectively, find the temperature when the mercury column is 8 cm long.

Solution l_0 = 6 cm, l_{100} = 16 cm, l_θ = 8 cm, θ = ?

$$\theta = \frac{l_\theta - l_0}{l_{100} - l_0} \times 100 = \frac{8-6}{16-6} \times 100 = \frac{2}{10} \times 100 = \mathbf{20\ °C}$$

2. The length of mercury column of a thermometer at ice point and steam point are 2.0 cm and 22.0 cm respectively. The reading of the thermometer when the mercury column is 9.0 cm long is

A. 45.0 °C B. 40.9 °C C. 35.0 °C D. 31.8 °C

Solution l_0 = 2.0 cm, l_{100} = 22.0 cm, l_θ = 9.0 cm, θ = ?

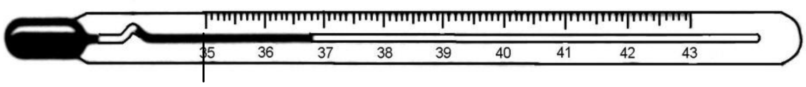
$$\theta = \frac{l_\theta - l_0}{l_{100} - l_0} \times 100 = \frac{9.0-2.0}{22.0-2.0} \times 100 = \frac{7.0}{20.0} \times 100 = \mathbf{35\ °C}$$

Therefore the answer is **C.**

Other types of thermometers

Other types of thermometers include:

(i) Clinical thermometer - Used by doctors in hospitals and clinics.



Constriction - Causes break to the mercury thread and stops the thread above it from moving back in to the bulb.
- It enables the doctors to take the reading at their own time.

NB: *Before use on another patient the thermometer is shaken to let the mercury thread to move in to the bulb.*

(ii) Thermocouple
(iii) Resistance thermometer
(iv) Thermister
(v) The constant-volume gas thermometer
(vi) The maximum six fixed thermometer

Self-Check 15.0

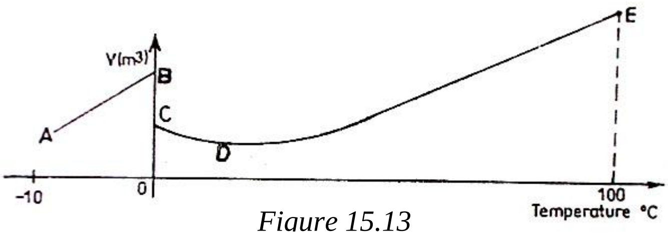
1. The distance between the fixed points on mercury in glass thermometer is 25cm. What is the temperature in degrees Celsius if the mercury thread is 8cm long?

A. $\frac{100 \times 25}{8}$ B. $\frac{100 \times 8}{25}$ C. $\frac{25 \times 8}{100}$ D. $\frac{100 \times 25}{10}$

2. Which one of the following fluids is the best conductor of heat?

A. Air B. alcohol C. water D. mercury

3. The graph in the figure shows water being heated from -10^0c to 100^0c.



Fiaure 15.13

At what point does the substance have maximum density?

A. E
B. C
C. D
D. B

4. A bimetallic strip operates on the principle that metals

A. are heat controllers. B. are good heat conductors.
C. have different rates of expansion. D. have the same rates of expansion.

5. In order to make a mercury thermometer more sensitive, the

A. degree markings must be further a part.
B. diameter of capillary tube must be reduced.
C. volume of the mercury bulb must be reduced.
D. capillary tube must be open to air.

6. A tight bottle top becomes easier to unscrew when hot water flows over it because the

A. cap expands more than the glass
B. glass in the neck of the bottle contracts
C. hot water acts as oil between the glass and the bottle
D. increased pressure of the air in the bottle causes the cap to expand

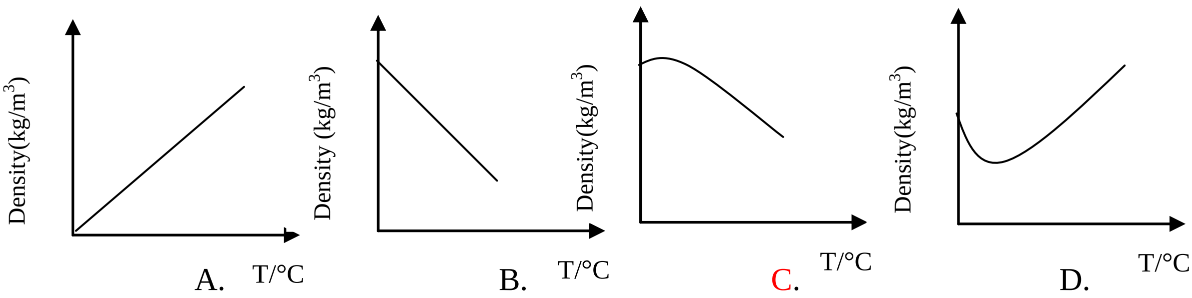
7. Which of the following changes occur when a metal block is heated?

	Volume	Mass	Density
A	increases	remains the same	decreases
B	increases	increases	increases
C	remains the same	remains the same	decreases
D	increases	remains the same	increases

8. The distance between the lower and the upper fixed points on the Celsius scale in unmarked mercury-in- glass thermometer is 25cm. If the mercury level is 5cm below the upper fixed point, then the temperature is
A. 5^0C B. 20^0C C. 80^0C D. 95^0C

9. The unusual expansion of water when it is cooled between 4^0C and 0^0C is due to
A. water molecules coming closer together to form a compact structure
B. formation of a new arrangement of molecules which requires a large volume
C. the increased repulsive forces between the water molecules
D. differences in the sizes of water and ice molecules

10. Which one of the following graphs shows how the density of water varies with temperature between 0^0C and 100^0C?



SECTION B

1. (a) Define the following terms.
   (i) Expansion
   (ii) Contraction of a material.
   (b) Describe an experiment to demonstrate expansion and contraction in solids.
   (c) State any one application of expansion in solids.

2. (a) Define linear expansivity of a material.
   (b) Calculate the linear expansion of concrete bridge of span 100m when the temperature rises by 20°C. The linear expansivity of concrete is 1.2×10^{-5}/°C

3. (a) Describe an experiments to show expansion in Liquids.
   (b) Explain the anomalous behaviour of water and give its importance to aquatic animals.

4. (a) State: (i) Any two thermometric liquids you know.
   (ii) The properties of a thermometric liquid.
   (iii) Advantages of mercury over alcohol as used a thermometer.

   (d) Define the following terms.
   (i) Lower fixed point.
   (ii) Upper fixed points.
   (c) Describe how the fixed points of a thermometer are determined in the laboratory.

5. The interval between the ice and steam points on a thermometer is 192 mm. Find the temperature when the length of the mercury thread is 67.2 mm from the ice point.

6. The distance between the lower and upper fixed points on the Celsius scale in unmarked mercury-in-glass thermometer is 25 cm. If the mercury level is 5 cm below the upper fixed, calculate the temperature value.

7. Convert the following temperature readings to Celsius scale.
   (a) 750 K (b) 400 K (c) 973 K

8. Convert the following temperature readings to Kelvin scale.
   (a) 340 ℃ (b) 130 ℃ (c) 20 ℃

9. (a) Name any two physical properties, which change with temperature.
   (b) Explain why gaps are left between rails in a railway line.
   (c) Why do gases expand much more than solids for the same temperature change?
   (d) Name one application of a bimetallic strip.
   (e) Mention any three reasons for not using water as a thermometric liquid.

10. Figure 15.13 shows strips of copper and iron bonded together.

Figure 15.13

   (a) Redraw the diagram to show what happens when the strip is heated.
   (b) Why does the change you have shown in (a) take place?

ANSWERS TO SELF-CHECK QUESTIONS

CHAPTER	SELF-CHECK NUMBER	NEMERICAL ANSWERS
2	**2.0**	**1. (b)** 24.8 mm or 2.48 cm or 0.0248 m **(c) (i)** 4.27 cm or 0.0427 m **(ii)** 10.63 cm or 0.1063 m
		2. (b) (i) 6.68 mm **(ii)** 7.47 mm **(iii)** 9.74 mm
	2.1	**1. (a) (i)** 10 km **(ii)** 2 000 cm **(b) (i)** 25 000 g **(ii)** 2 kg **(c) (i)** 43 200 s **(ii)** 900 s **(d) (i)** 0.02 m^3 **(ii)** 50 000 000 cm^3 Or 5×10^7 cm^3
	2.2	**1. (a)** 3 **(b)** 1 **(c)** 4 **(d)** 4 **(e)** 1 **2.** 34.1 mm **3.** (a) 15 m^2 **(b)** 80 m^3
	2.3	**(a) (i)** 2.22×10^{-4} **(ii)** 2.5×10^{-3} **(b) (i)** 5.62×10^{3} **(ii)** 7.5×10^{4} **(c) (i)** 1.2×10^{11} **(ii)** 2×10^{1} **(d) (i)** 5×10^{0} **(ii)** 1.1×10^{3} **(e) (i)** 2.5×10^{4}m **(ii)** 2.5 kg **(iii)** $11 \times 10^{-2}(10^{-2})$
	2.4	**1**. A. **2**. B. **3**. D. **4**. C. **5**. D. **6**. B. **7**. B. **8**. D. **9**. D. **10**. B. **11**. B. **12**. B. **13**. D. **14**. A. **15**. B.
3	**3.0**	**1**. D. **2**. D. **3**. C. **4**. B. **5**. C. **6**. C. **7**. D. **8**. B. **9**. A. **10**. B. **11**. A. **12**. A. **13**. B. **14**. B. **15**. B. **16**. D. **17**. B. **18**. D. **19**. A. **20**. C.

4	**4.0**	**1.** A. **2.** D. **3.** C. **4.** D. **5.** C. **6.** A. **7.** C. **8.** A. **9.** A. **10.** B. **11.** B. **12.** C. **13**. C. **14.** C. **15.** B. **16**. A. **17**. D. **18.** C. **19.** C. **20.** A.
	4.1	**1**. **D**. **2.** **(b)** 1.2×10^{-6} cm **3.** **(i)** 0.00002 cm^3 or 2×10^{-5} cm^3 **(ii)** 1.77×10^{-7}cm **4**. 1.2×10^{-5} mm or 1.2×10^{-6} cm or 1.2×10^{-8} m **5**. 3.18×10^{-6} mm or 3.18×10^{-7} cm or 3.18×10^{-9} m
5	**5.0**	**1**. C. **2**. A. **3**. B. **4**. B. **5**. C. **6**. A. **7**. C. **8**. C. **9**. D. **10**. D.
	5.1	**1.** C. **2.** D. **3.** B. **4.** C. **5**. C. **6.** B. **7.** D. **8.** A. **9.** B. **10.** C. **11.** C. **12.** C. **13.** B. **14.** D. **15.** D. **16.** A. **17.** A. **18.** D. **19.** A. **20.** D. **21.** B. **22.** C. **23.** A. **24**. **A**. **25**. A.
		SECTION B **26.** (b) (i) 120 N (ii) 20 N **28**. **(b)** **(i)** 50 N **(ii)** 5 ms^{-2} **(c)** **(i)** 86.8 N **(ii)** 100N **(d)** **(i)** 4 N **(ii)** 8 ms^{-2} **29.** **(a)** 5 N **(b)** 1 ms^{-2}
6	**6.0**	**1.** D. **2.** C. **3.** A. **4.** B. **5.** C. **6.** B. **7.** C. **8.** C. **9.** C. **10**. B.
		SECTIONS B **11.** **(b)** **(i)** 60 N **(ii)** 80 N **12.** **(c)** **(i)** 17.54 N **(ii)** 0.46 N **14.** **(c)** **(i)** 2.5 N **(ii)** 10 N

7	**7.0**	**1.** B. **2.** D. **3.** D. **4.** D. **5.** B. **6.** B. **7.** D. **8.** B. **9.** D. **10.** C. **11.** C. **12.** B. **13.** C. **14.** **C**. **15.** D. **16.** C. **17.** D. **18.** B. **19.** B. **20.** D. **21.** **A**. **22.** C. **23.** D. **24.** D. **25.** C.
		SECTION B **26. (c) (i)** 17 500 J **(ii)** 3 500 W **27. (c)** 60 W **28**. (c) **(i)** 3 240 J **(ii)** 648 W
8	**8.0**	**1**. **A**. **2**. **A**. **3**. **C**. **4**. **A**. **5**. **B**. **6**. B. **7**. B. **8**. B. **9**. B. **10**. C. **11**. A. **12**. C. **13**. D. **14**. B. **15**. B.
		SECTION B **16. (c) (i)** 4 **(ii)** 75% **18. (b) (ii)** 3 **(iii)** 0.8 m **(iv)** 400 N **(v)** 80% **20. (a)** 4 **(b)** 75% **21. (i)** 2 **(ii)** 1 N **(iii)** 100% **22. (a) (i)** 4 **(ii)** 75% **(b) (i)** 0.25 **(ii)** 1 200% **23**. **(a)** 200 N **(b)** 3
9	**9.0**	**1**. B. **2**. C. **3**. B. **4**. C. **5**. C.
	9.1	**1**. A. **2**. C. **3**. D. **4**. B. **5**. A. **6**. B. **7**. B. **8**. B. **9**. D. **10**. B. **11.** D. 12. A. **13.** B. **14.** B. **15.** C. **16.** B. **17.** A. **18.** A. **19.** C. **20.** A. **21.** D. **22.** C. **23.** D. **24.** B. **25.** B.
		SECTION B **26. (b) (i)** 4 : 5 **(ii)** 15 cm **27. (a) (iv)** 30 800 N **28. (b) (ii)** 1 250 N m^{-2} **29. (b) (ii)** 800 m

10	**10.0**	**1.** B. **2.** D. **3.** A. **4.** A. **5.** A. **6.** D. **7.** A. **8.** D. **9.** B. **10.** D.
		SECTION B **11. (c)** 0.8 **12. (c)** 500 kg m^{-3} **13.** 21.52 kg
11	**11.0**	**1.** **(a)** 40 m/s **(b)** 80 m **2.** **(a)** 10 s **(b)** 220 m **3.** **(a)** 2 ms^{-2} **(b)** 25 m **4.** **(a)** 20 m/s **(b)** 30 m/s **(c)** 50 m/s **(d)** 80 m/s
	11.1	**1. (a)** 40 m/s **(b)** 80 m **2. (a)** 10s **(b)** 80 m **3. (a)** 2 m/s^2 **(b)** 25 m **4. (a)** 20 m/s **(b)** 30 m/s **(c)** 50 m/s **(d)** 80 m/s
	11.2	1. C 2. C 3. B 4. B 5. B 6. C 7. D 8. D 9. D 10. B 11.C 12. D 13. A 14. A 15. D 16. C 17. C 18. A 19. A 20. B **SECTION B** **21. (b)** 2.5 m/s^2 **22. (a)** 10 m/s **(b)** 25 m/s
	11.3	1. **C**. 2. D. 3. C. 4. D. 5. D. 6. A.
	11.4	1. (a) 100m/s (b) 500m 2. (a) 180m (b) 60m/s 3. (a) 80m (b) 4s 4. (a) 4500m (b) 60s 5. (a) 100m (above the ground) (b) 2 s (c) 6.47s
	11.5	1. A 2. B 3. (a) 500m (b) 500m (c) 1000m 4. (a) 2000m (b) 6000m (c) 200m/s 5. (i) 5m (ii) 20m (iii) 45m (iv) 125m

12	**12.0**	**1**. A. **2**. B. **3**. C. **4**. B. **5**. B. **6**. D. **7**. B. **8**. A. **9**. D. **10**. A.
		SECTION B **1. (e)** 70 000 N **2**. **(b)** **(i)** 420 ms^{-1} **(ii)** 1 760 J **3**. **(c)** **(i)** 12 ms^{-1} **(ii)** 33.33 N **4**. 1 m/s **5. (b)** 2 ms^{-1} **6**. 10m/s
13	**13.0**	**1** . D. **2**. B. **3**. C. **4**. D. **5**. A. **6** . C. **7**. D. **8**. D. **9**. D. **10**. B. **1 1**. D. **12**. D. **13**. C. **14**. B. **15**. A.
		SECTION B **3.** **(c)** 175 N
14	**14.1**	**1**. C. **2**. B. **4**. **(a) (iii)** 1.4
	14.2	**1**. C. **2.** B. **3**. C. **4**. C. **5**. D.
	14.3	**1**. A. **2**. A **3**. A. **5**. **(a)** $8\frac{4}{7}$ cm **(b)** $\frac{3}{7}$
	14.4	**1**. D. **2**. C. **3**. A. **4**. A. **5**. D. **SECTION B** **6. (c)** 38.12° **8. (b)** 30.87° **10. (a)** 19.47° **(b)** 10.53°
	14.5	**1.** B. **2**. A. **3**. D. **4.** D. **5**. A. **SECTION B** **8. (a)** 10 D
	14.6	**2. (i)** 52.37° **(ii)** 37.37° **3**. 41.8°
	14.7	**1**. C. **2**. A. **3**. A. **4**. B. **5**. C. **6**. B. **7**. B. **8**. D. **9**. C. **10**. A.
15	**15.0**	**1.** B. **2.** D. **3**. C. **4**. C. **5**. B. **6**. A. **7**. A **8.** B. **9**. B. **10**. C. **SECTION B** **2**. 2.4×10^{-2} m **5**. 35 °C **6**. 20 °C **7**. **(a)** 477 °C **(b)** 127 °C **(c)** 700 °C **8**. **(a)** 613 K **(b)** 403 K **(c)** 293 K